name

Do the Pre-Lab BEFORE the Lab

Pre-Lab: Astrometry & Kinematics

Velocities and Distances of Comets and Meteors

In this lab you'll be learning some basic techniques of how to measure angles, positions, distances and velocities. In the Astronomy Program here at CSI, one of the major research areas is on "Asteroid Hunting," led by Prof. Irving Robbins. He spends endless nights in the observatory, even in January and February when is it absolutely freezing, and takes picture after picture of designated regions of the sky. He then compares these pictures (by "blinking" them on the computer) to find the asteroids. He also calculates their orbital elements (positions, distance and velocities) in the same way as in part II of this lab. His results get published in a professional journal along with other data obtained from national observatories. If you enjoy this lab, check out our web site or stop by the observatory.

Comets and Meteors – explain the difference between the two

Now go and grab them…

1) Read and understand the Appendix to this Lab.
   Read Page 57, page 169 section 9-1; understand Figure 9-4 on page 170.

2) What is an angular size?

   ____

   ____

3) How does the angular size differ from a linear size?

   ____

   ____

4) If you put an object at twice its **distance**, how does the **angular** size change? ____________
   If you put an object at four times its **distance**, how does the **angular** size change? __________
   If you put an object at five times its **distance**, how does the **LINEAR** size change? __________

5) What is the relationship between angular size, linear size, and distance? Explain **IN WORDS**.

   ____

   ____

6) Circle the correct statement, based on what you just said

   a) angular size is proportional to linear size
   b) linear size is proportional to distance
   c) angular size is proportional to distance
   d) angular size is inversely proportional to linear size
   e) linear size is inversely proportional to distance
   f) angular size is inversely proportional to distance

7) Abbreviate what you have just said and turn it into a formula. Use the term "proportional," or "inversely proportional;" use the symbol "$\propto$" for the word "proportional."

   ____

8) Write down the Small Angle formula.

   ____

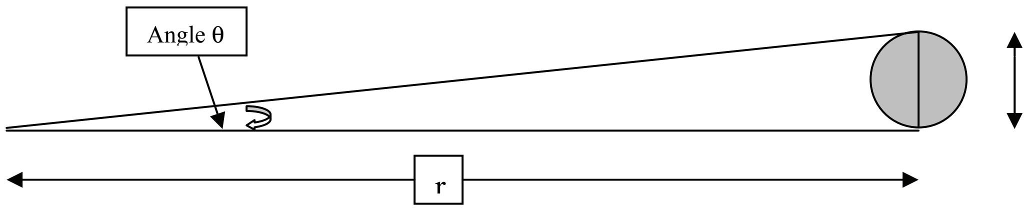
9) Solve the Small Angle formula for the distance (re-arrange so that distance=…)

   ____

10) Is this formula consistent with the formula of question 8? ______________________
    Is this formula consistent with the statement of question 9? ______________________

OPTIONAL Exercise for Extra Credit – **NO** calculation needed, only analyze the Table

The Small Angle Formula is only applicable to small angles. Why? It is an approximation. Check the drawing in the Appendix. The *linear* size is actually not linear, it is an arc of a circle. Therefore, the mathematically more correct method would be to deal with real triangles like we did in Lab #1.



Using trigonometry we have: $\tan\theta = \frac{s}{r}$ → $s = r \cdot \tan\theta$. The small angle formula is: $s = \frac{r \cdot \theta}{57.3^o}$

So how accurate is the small angle formula? The easiest way to go about that is to try some numbers. To make it easy, let's take a distance, r, of 1 pc, and then calculate the linear size, s, for several angles. This is done for you. The last column shows the percentage discrepancy between the values of s.

angle	trigonometry $s_{trig} = r \cdot \tan\theta$	small angle formula $s_{saf} = \frac{r \cdot \theta}{57.3^o}$	percentage error $\frac{s_{trig} - s_{saf}}{s_{trig}}$	angle ± accuracy
60 deg	1.7320508075689	1.0471975511966	39.5400211921927	60 ± 23.724 deg
50 deg	1.1917535925942	0.8726646259972	26.7747434184320	50 ± 13.3874 deg
40 deg	0.8390996311773	0.6981317007977	16.7999037470397	40 ± 6.71996 deg
30 deg	0.5773502691896	0.5235987755983	9.3100317882891	30 ± 2.79301 deg
20 deg	0.3639702342662	0.3490658503989	4.0949458126391	20 ± 0.81899 deg
10 deg	0.1763269807085	0.1745329251994	1.0174594391758	10 ± 0.10175 deg
5 deg	0.0874886635259	0.0872664625997	0.2539768208274	5 ± 0.0127 deg
1 deg	0.0174550649282	0.0174532925199	0.0101541202028	1 ± 0.0001 deg
30 min	0.0087268677908	0.0087266462600	0.0025384913861	30 ± 0.00076 min
10 min	0.0029088902913	0.0029088820867	0.0002820533256	10 ± 2.8E-05 min
5 min	0.0014544420689	0.0014544410433	0.0000705133015	5 ± 3.5E-06 min
1 min	0.0002908882169	0.0002908882087	0.0000028205317	1 ± 2.8E-08 min
30 sec	0.0001454441054	0.0001454441043	0.0000007051329	30 ± 2.1E-07 sec
10 sec	0.0000484813681	0.0000484813681	0.0000000783481	10 ± 7.8E-09 sec
5 sec	0.0000242406841	0.0000242406841	0.0000000195870	5 ± 9.8E-10 sec
1 sec	0.0000048481368	0.0000048481368	0.0000000007835	1 ± 7.8E-12 sec

Comment on the accuracy of the small angle formula. Would you use the small angle formula for an angle of 10^o? If your answer is "no," explain under which conditions you would nevertheless use the small angle formula for an angle of 10^o? What about an angle of 1^o, 10' or 1'?

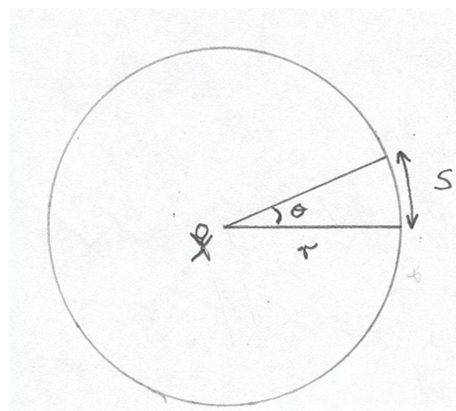
Appendix: The Small Angle Formula

You have to know how to use the small angle formula (boxed equations).
Understanding the derivation is optional.

Let's look at the stars. Since they all seem to be equally far away, it looks to us as if they are painted onto a circular dome like in the picture below. You look at two stars and see the angular distance, θ, between the stars. The distance between the stars, "s," depends on both the angle, θ, and the distance to the stars. The bigger the angle, the bigger the size "s." An angle of 90^o corresponds to one quarter of a circle, and an angle of 360^o corresponds to the circumference of the circle. The circumference of a circle is $C=2\pi r$, where r is the radius of the circle. This corresponds to an angle of 360^o. However, we are only interested in a fraction of the circle, the distance "s" that corresponds to the angle, θ. If you increase the angle θ, "s" increases in a similar fashion. This means that the ratio remains the same, or in terms of a formula it means that

$$\frac{s}{\theta} = \text{constant}$$

The largest angle is 360°, but the ratio is still the same, *i.e.*,



$$\frac{C}{360^o} = \text{constant} = \frac{s}{\theta}$$

We can measure the angle, θ; however, we have to calculate the linear distance "s" by solving the above equation. We get

$$s = \frac{C}{360^o} \cdot \theta$$

Since the circumference C is equal to $2\pi r$, this gives

$$s = \frac{2\pi r}{360^o} \cdot \theta$$

Dividing this by 2π gives

$$\boxed{s = \frac{r}{57.3^o} \cdot \theta}$$

Since the angles in astronomy are generally very small, we often prefer to quote the angles in arc seconds rather than degrees. Since there are 60 arc minutes in one degree, and again 60 arc seconds in one arc minute this gives

$$s = \frac{r \cdot \theta}{57.3 \cdot 60 \cdot 60''}$$

And the final relationship we will use in class then is

$$\boxed{s = \frac{r\theta}{206265''}}$$