

Naked-Eye Astronomy Lab

Distance and Size of Moon

Needed Math: a little geometry and a little algebra.

The diameter and distance of the Moon can be inferred from carefully observing a lunar eclipse. Basically, the idea behind this lab is that since a lunar eclipse occurs because the Moon enters into the Earth's shadow, one can directly see and hence compare the size of the Moon with that of the Earth's shadow. While the Moon is partway into the shadow, you will take a picture, using an ordinary camera and film. The picture that you get will look something like Figure 1, below (which I have

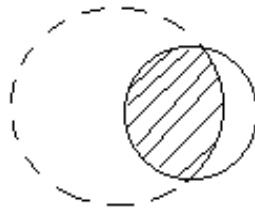


Figure 1

intentionally not drawn to scale--so don't use this figure for your analysis). Using the curvature of the Earth's shadow you can extrapolate to make a complete circle, like the dashed line in Figure 1. Then, you can measure the size of the Moon and the size of the Earth's shadow on the scale of the picture; the ratio that you measure on your picture is the same ratio as in reality. For discussion sake, let's assign this ratio the variable X , that is $X = D_{sh} / D_M$, where D_{sh} is the diameter of the Earth's shadow on the Moon and D_M is the diameter of the Moon.

Now, the trick is figuring out the relation between the size of the Earth's shadow and the size of the Earth--this is the hardest part of this lab. Let's start by considering the geometry of a lunar eclipse. The situation, as seen from the North Pole, is depicted in Figure 2 below. Note that the Earth's shadow is really a cone in space. It comes to a point because the Sun is so much bigger than the Earth. The Moon orbits the Earth at a distance of d_M .

Now consider Figure 3 which is similar to Figure 2 but with some distances and lengths labelled with variables. The only variables you need to follow are θ , D_s , D_e , d_s , and L_{sh} . θ is the angle that the Sun's rays meet at the end of the shadow, D_s is the diameter of the Sun, D_e is the diameter of the Earth, d_s is the distance of the Sun, and L_{sh} is the length of the Earth's shadow. Note that θ is also the angular size of the Earth as

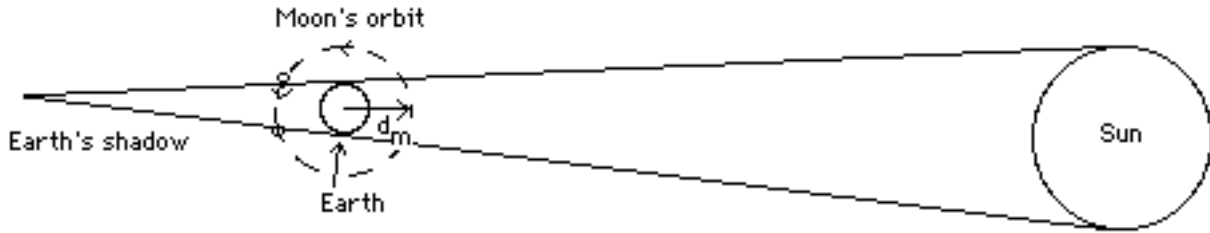


Figure 2

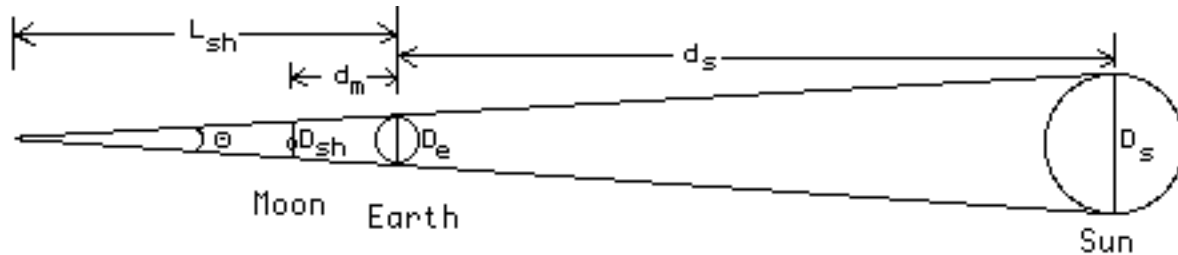


Figure 3

seen from the end of the Earth's shadow. Consider the relative sizes of the Sun and the Earth. Since the Sun's diameter is so much much larger, the distance from the Earth to the end of the shadow is a tiny fraction of the distance from the Sun to the end of the shadow. Therefore, the distance from the Sun to the end of the shadow is essentially the same as the distance of the Sun from the Earth (these distances differ by only 1%). Think about what this means--that the angle θ is essentially the same as the angular size of the Sun as seen from the Earth. And, since we get such beautiful solar eclipses we also know that the angular size of the Sun must equal the angular size of the Moon. Therefore, the angle θ also equals the angular size of the Moon. This is extremely convenient, for then we can draw Figure 4, below.

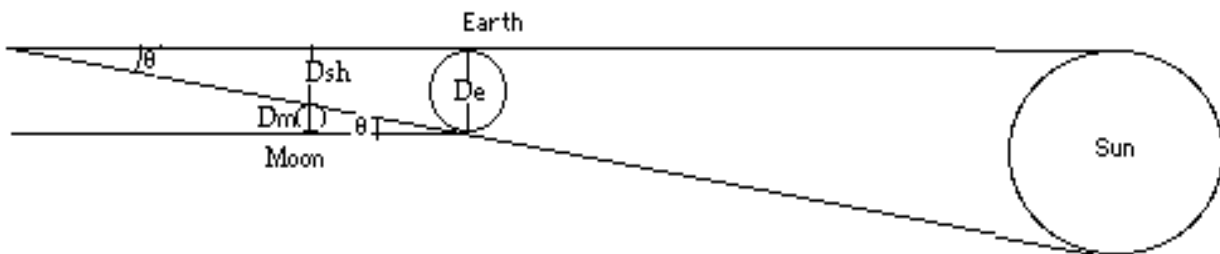


Figure 4

In Figure 4, the Moon is at the edge of the Earth's shadow, as if this were a snapshot of Figure 2 taken at the time the Moon has just left the Earth's shadow. Additionally, another line is added--the line on the bottom going from the edge of the Earth to the edge of the Moon (and beyond). Now note that we have two lines going from the lower edge of the Earth to either side of the Moon: one is the lower edge of the Earth's shadow and the other is the line that has been added. What we have, then, is a

representation of the angular size of the Moon as seen from the Earth (when the Moon is on the horizon). Now recall that the angular size of the moon equals the angular size of the Sun, which equals angle θ , the vertex angle of the Earth's shadow. The fact that these two angles are equal, by the geometrical law of "alternate interior angle", means that in Figure 4, the top and bottom lines extending from the edge of the Earth to the left must be parallel. Therefore, the perpendicular distance between these two lines at any two points will be equal. And now for the final point: this means that the distance between these two lines at the position of the Moon is equal to the distance between the two lines at the Earth. This means that the sum of D_{sh} , the diameter of the Earth's shadow, which is the distance from the top line to the diagonal line, plus D_M , the diameter of the Moon, which is the distance from the diagonal line to the bottom line, equals D_e , the diameter of the Earth, which is the total distance between the parallel lines at the position of the Earth. That is,

$$\mathbf{D_{sh}+D_M=D_e.} \quad (1)$$

In this lab, you will measure the relative size of the Moon's shadow and the Earth's shadow, i.e. you will measure the ratio D_{sh}/D_M . As before, let's say that this equals X . Then, you know that $D_{sh}/D_M=X$ and hence that $D_{sh}=X(D_M)$. Take this expression for D_{sh} and plug it into equation (1). You'll get then a relatively simple equation that involves only D_M and two knowns: X , which you'll measure, and D_E , which you have just measured in your first lab. Manipulate this equation to get an expression for D_M and solve and you'll have a measure of the diameter of the Moon.

Now let's discuss how to infer the distance, d_M , of the Moon. Well, the diameter of the Moon, D_M , and the distance of the Moon, d_M , are related to the angular size of the moon, θ_M , by

$$\theta_M(\text{radians})=D_M/d_M .$$

You need then only to measure the angular size of the Moon and then manipulate this equation so that it gives you the distance of the Moon, i.e.

$$\mathbf{d_M= D_M/q_M(\text{radians}).} \quad (2)$$

To measure the angular size of the Moon you make a "cross-staff." Imagine holding up a ruler to the sky at arm's length, which is about 70 cm, and measuring the apparent distance between two objects. Suppose you measured the apparent distance to be 2 cm. What you found is that the angular separation between the two objects is the same as that of the 2 cm markings on the ruler held at a distance of 70 cm. Since you know the actual size and distance of the latter, you know what that angle is. By the small angle approximation it is $\Theta(\text{radians}) \sim 2\text{cm}/70\text{cm}$. To make this measurement more reliable,

rather than measuring the distance from the ruler to your eye each time, thumbtack the ruler to the end of a stick whose length you know and then make the measurement by holding the other end of the stick against your cheek just below your eye.

(You can be clever about your choice of stick. Since you can cut a stick to any length, you can choose a length that would be most convenient. For example, choose a length where one centimeter on the ruler equals one degree of angle. That is $1\text{cm}/(\text{length of stick})=1$ degree. The small angle approximation must be in radians, so convert the 1 degree to radians, and solve for the length of the stick. What do you get? _____. Another way of viewing this is to imagine that the ruler at the end of the stick is part of a circle that is centered on your eye. If one centimeter on the ruler is to correspond to one degree, then this circle must have 360 cm around the circle, meaning that its circumference is 360 cm. Since the center of the circle is at your eye the length of the stick is equal to the radius of the circle. Therefore, the length of the stick that you want is equal to the radius of a circle of circumference of 360 cm. Using the fact that the circumference of a circle is $2\pi R$, you can solve for the R, to find the desired length of the stick. You should find the same answer that you got above.)

If you look hard enough, you can probably find a nicer picture of a lunar eclipse on the web.