ASSIGNMENT #3: due TODAY

if you need extension, contact me

ASSIGNMENT #4: due FRIDAY 3/18

PROBABILITY II

Chapters 1+2 of Practical Statistics for Astronomers

Bayesian Inferences with Probability

GOAL: estimating the parameters of assumed probability distributions, i.e., we are assuming a model for our data and wish to find out how this model is characterized. In other words, we are *data modeling*.

We have a probability distribution (the likelihood) $f(data|\bar{\alpha})$ and we wish to know the parameter vector α . In the Bayesian route, we need to compute the posterior distribution of α

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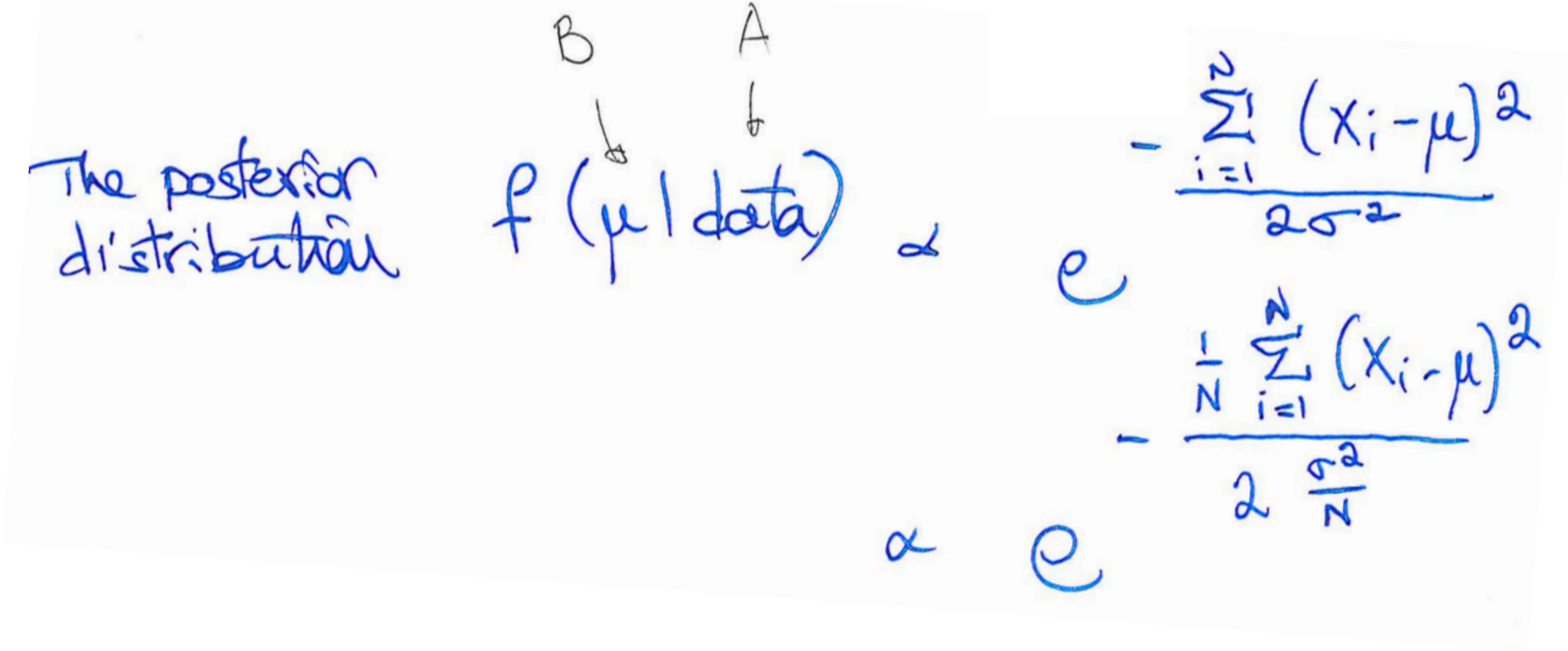

The posterior distribution $f(\overset{B}{\mu} \mid \overset{A}{data}) \propto e^{-\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{2\sigma^2}}$

$$\propto e^{-\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}{2\frac{\sigma^2}{N}}}$$

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This method is related to the classical technique of MAXIMUM LIKELIHOOD. If the prior is diffuse, then the posterior probability is proportional to the likelihood term $f(data|\bar{\alpha})$. Maximum likelihood picks out the mode (i.e., the peak) of the posterior, i.e., the value of α which maximizes the likelihood. We will learn more on this later…

EXAMPLE 2: Suppose we make an observation at a randomly selected position in the sky. Our model of the data, an event D consisting of a single measured flux density f, is that it is distributed in a Gaussian way about the true flux density S with variance σ^2 .

The extensive body of source counts tells us the a-priori distribution of S, prob(S)=KS$^{-5/2}$ (this is the prior) describing our prior state of knowledge. K normalizes the counts to 1, i.e., there is presumed to be one source in the beam at some flux-density level.

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Suppose that the source counts extend from 1 to 100 units, the noise level was $\sigma = 1$, and the data were 2, 1.3, 3, 1.5, 2, 1.8, then determine the posterior probability of the flux for the first 2, 4, and 6 measurements.
NOTE: The increase in data gradually overwhelms the prior but the prior affects the conclusions markedly when there are a few measurements.

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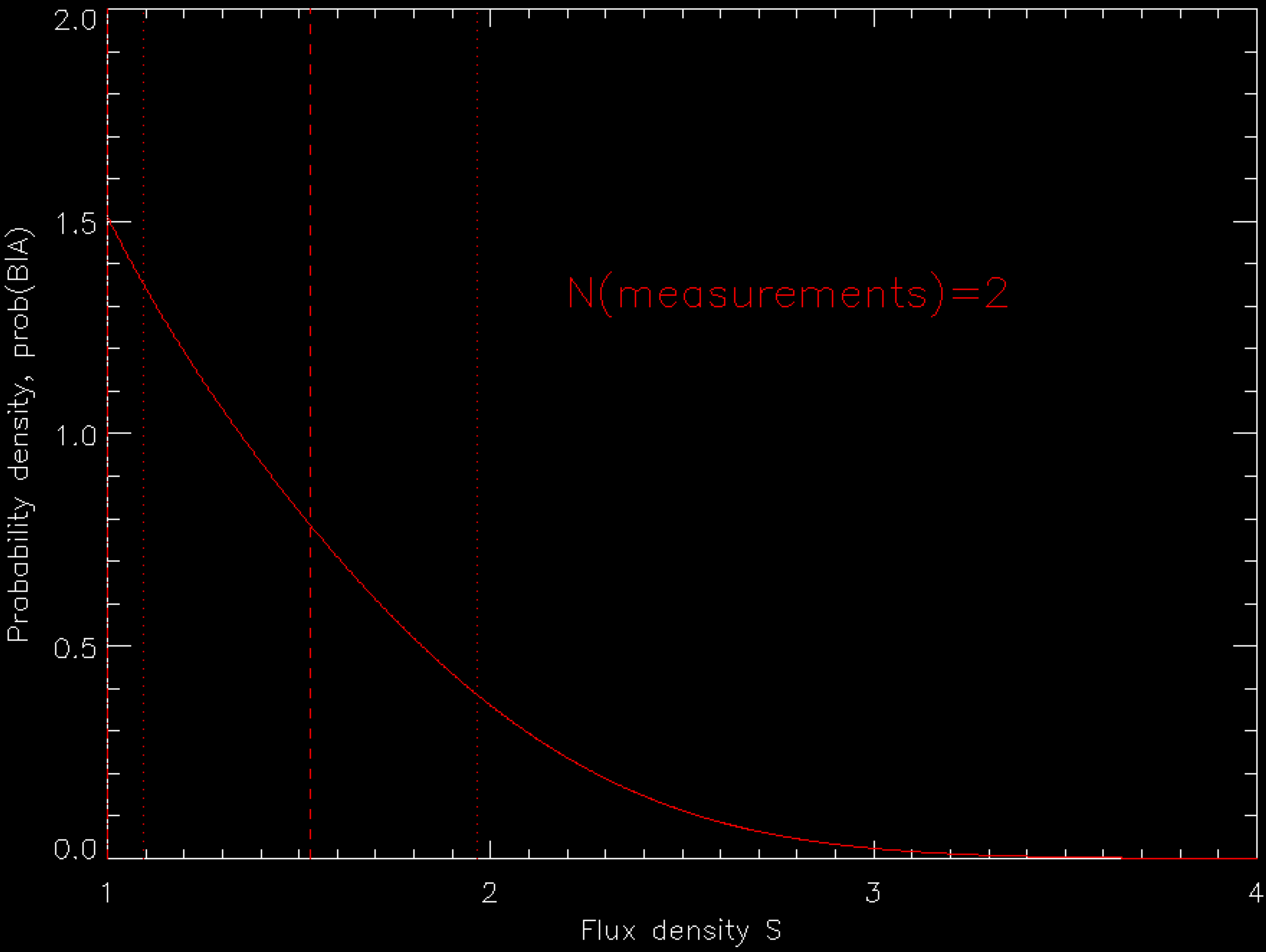
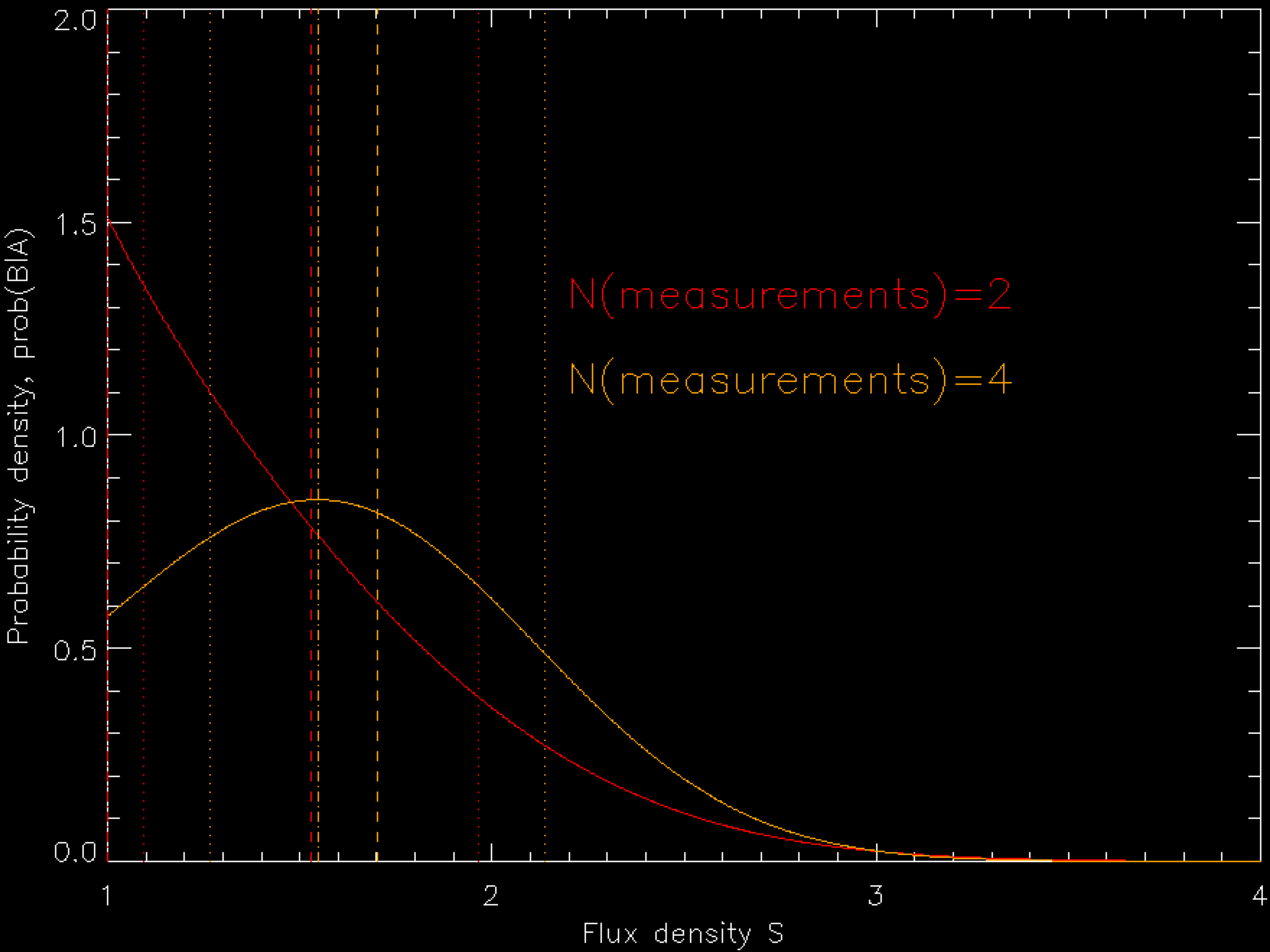
NOTE: If I knew nothing about the prior, the mean and sigma of the measurements [2, 1.3, 3, 1.5, 2, 1.8] are: $\mu = 1.93 \quad \sigma = 0.59$. From the posterior probability f(x):

$$peak = 1.69 \quad \mu = 1.93 \quad \sigma = 0.39$$

$$\mu = \int_{-\infty}^{+\infty} xf(x)dx \qquad \sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$$



N(measurements)=2
Probability density, prob(B|A)
Flux density S
0.0
0.5
1.0
1.5
2.0
1
2
3
4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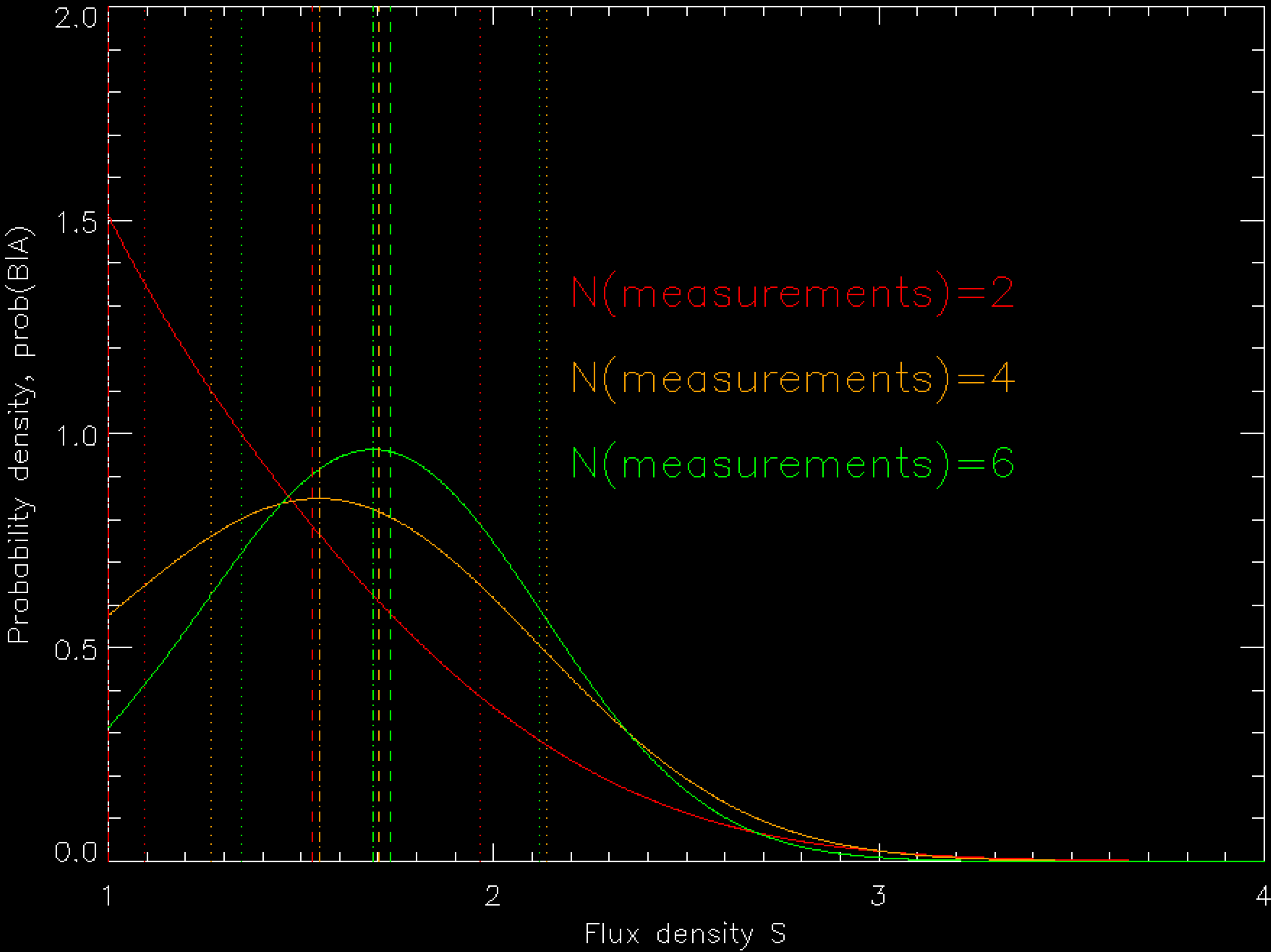
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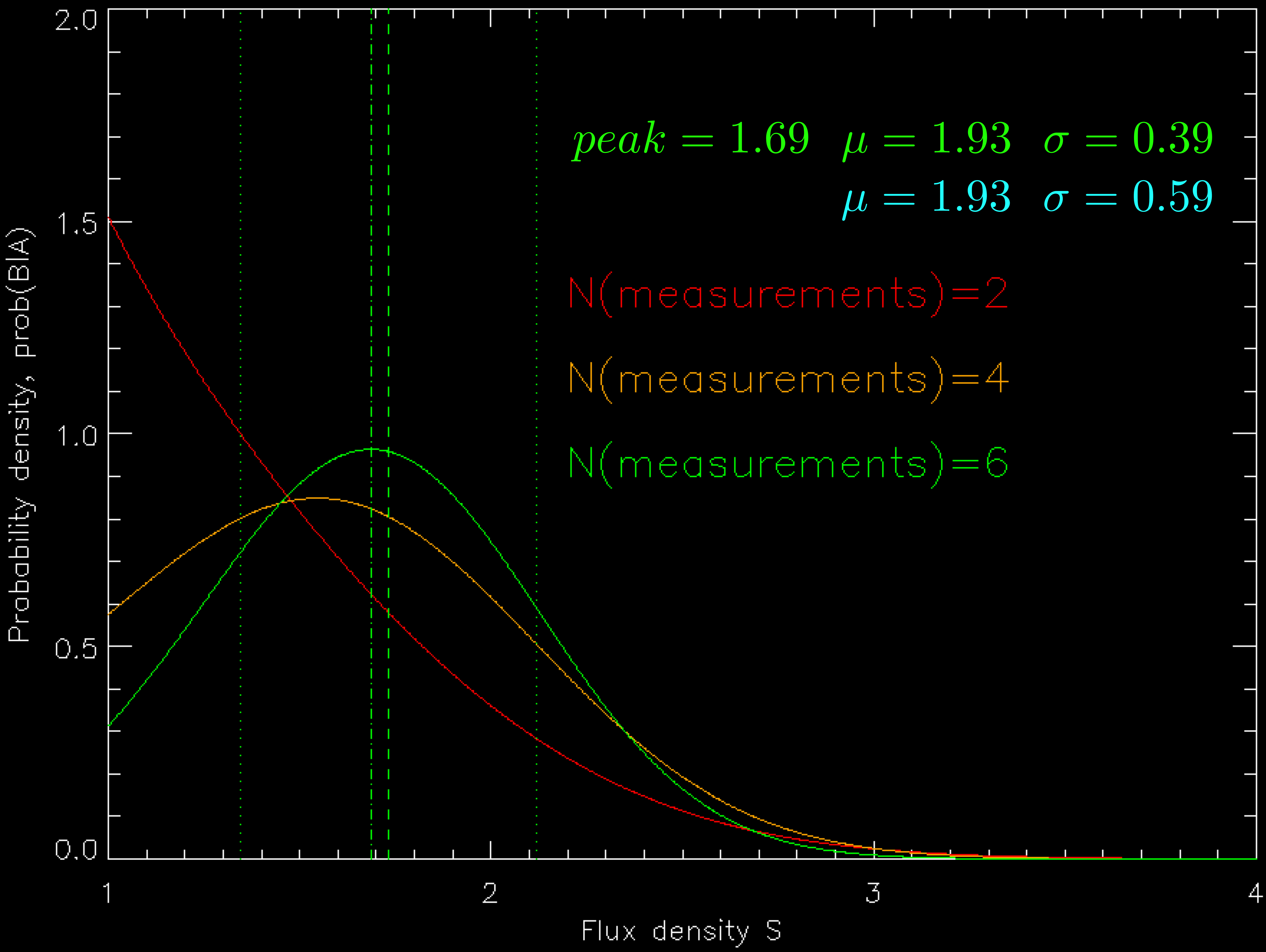
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There are a number of methods to generate random numbers. Key issuesL

1. How long is it before the pseudo-random cycle is repeated? Or how many random numbers do you need? —> need to understand the characteristics of the generator
2. Follow the prescribed implementation precisely
3. The routines generate pseudo-random numbers, i.e., run them again from the same starting point and you will get the same set of numbers.

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The simplest distribution function for random numbers is a **constant probability distribution, a.k.a., uniform deviate**. Uniform deviates are the building blocks of random number generation and Monte Carlo techniques.

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Example: usually, a uniform deviate generator produces N random numbers between 0 and 1 from a uniform distribution. If I want N random numbers between 0 and 10 from a uniform distribution, I multiply those generated from the previous example by 10. If I want N random numbers between 2 and 12 from a uniform distribution, I multiply those generated from the first example by 10 and then I add 2.

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Then $a(x) = \displaystyle\int^{x} f(x)dx$ ★

From whence the required transformation x=x(a), the inverse of ★ , i.e., solve ★ for x.

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$a \in (0,1)$ → x=x(a) other random distribution

random uniform

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In general,

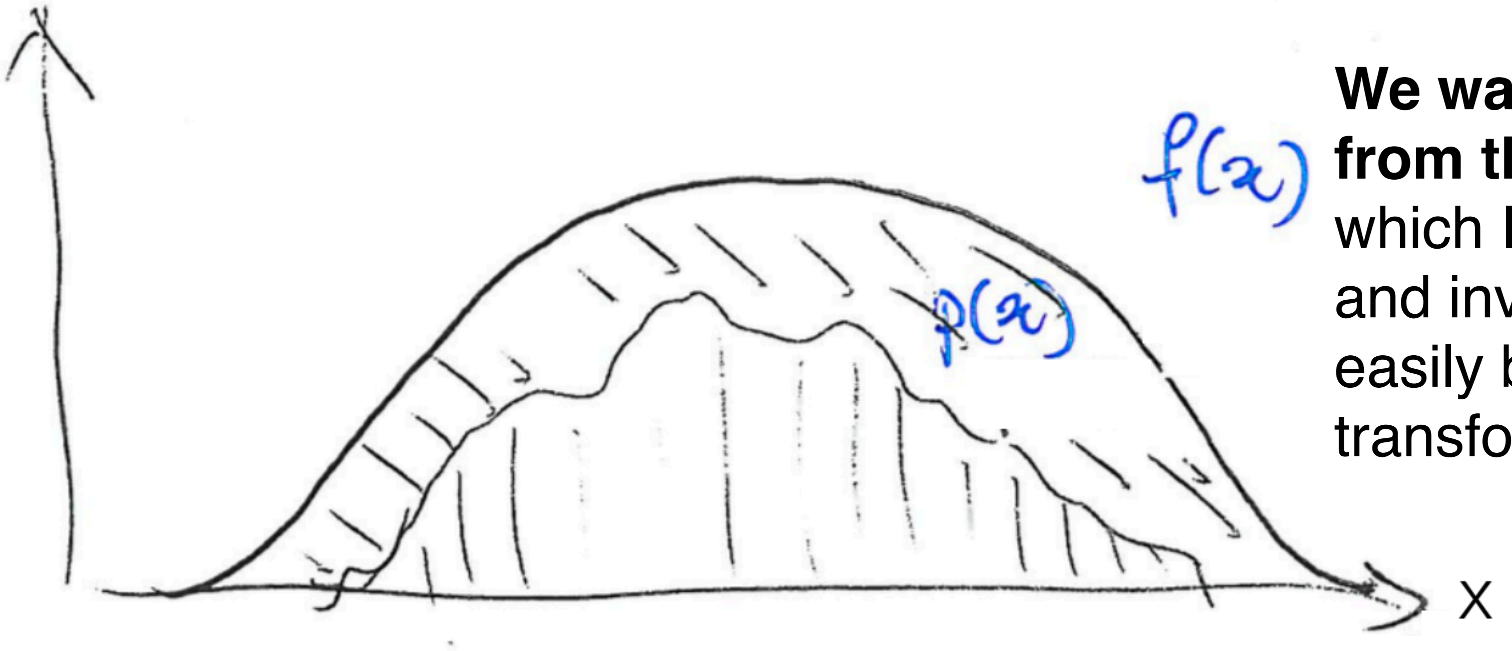
you want a set of random numbers drawn from a distribution f(x) ➡ $\left|\frac{da}{dx}\right| = f(x)$

Taking the integral, $a(x) = \int^{x} f(x)dx = F(x)$ primitive function

So that, x=x(a), x=F^{-1}(a) inverse function of the primitive (ex., F = e^{-} ; F^{-1}=-ln)

The transformation method has a limited validity: it is limited by the knowledge of $F^{-1}(a)$; this is known analytically for the exponential and a normal (Gaussian) deviates.

Q: What if $F^{-1}(a)$ cannot be calculated?
A: We use the rejection method (general, but not as efficient as the transformation method)



We want to produce random numbers drawn from the distribution p(x). f(x) is a Gaussian which I know to construct (it should be integrable and invertible, so that a random sample can easily be obtained from f(x) through the previous transformation method).

If I can construct a distribution function that follows f(x) and that incorporates p(x), then I can reject the excess and be left with the desired deviate.

Of course, there is an overhead = rejected points = $\int f(x)dx - \int p(x)dx$

The problem therefore is that of generating random numbers below f(x).

STEP 1: choose a random number with uniform deviate $\bar{a} \in [0, A]$

STEP 2: calculate $\bar{x}$ $so\ that$ $\int_0^{\bar{x}} f(x)dx = \bar{a} = F(\bar{x})$

STEP 3: once $f(\bar{x})$ in known, I choose a random number $\bar{a}$ from a uniform deviate between 0 and $A = f(\bar{x})$, i.e., $\bar{a} \in [0, A = f(\bar{x})]$

STEP 4: if $\bar{a} \leq p(\bar{x})$ → KEEP

if $\bar{a} > p(\bar{x})$ → REJECT

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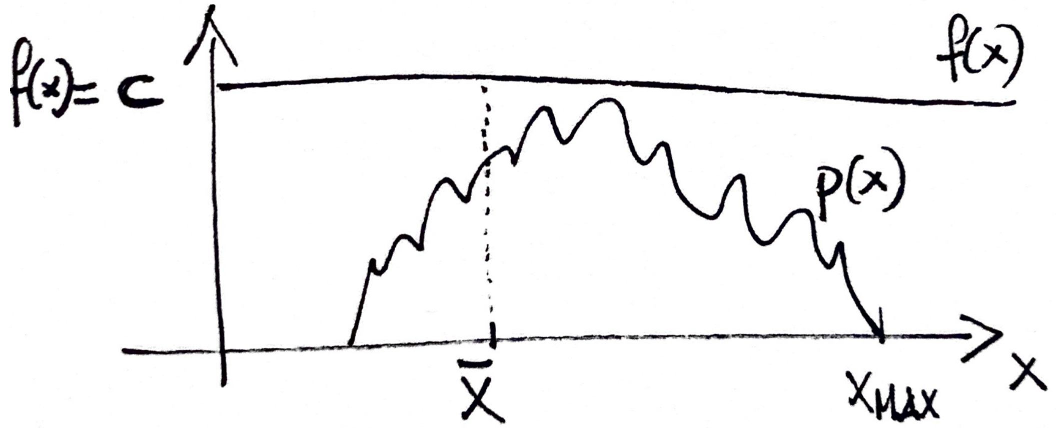
EXAMPLE:



STEP 1: I pick a number $\bar{x}$ from a uniform deviate between 0 and x_{max}

STEP 2: I pick a random number from a uniform deviate between 0 and C, which I call $\bar{a}$

STEP 3: If

$$\bar{a} \leq p(\bar{x}) \quad \rightarrow \quad accept$$
$$\bar{a} > p(\bar{x}) \quad \rightarrow \quad reject$$

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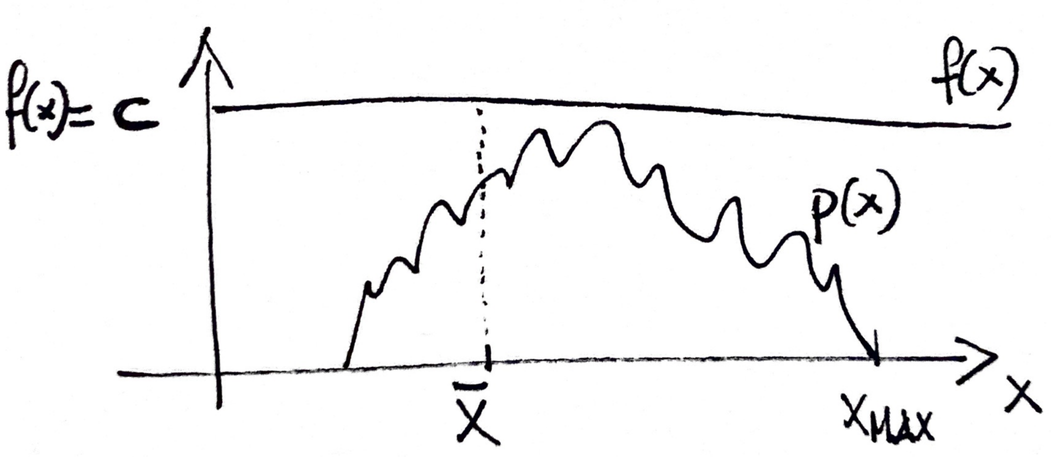

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$$\begin{aligned} \bar{a} \leq p(\bar{x}) &\rightarrow accept \\ \bar{a} > p(\bar{x}) &\rightarrow reject \end{aligned}$$

The REJECTION METHOD is easy to implement, but it can have large overheads, and the smarter f(x) is chosen, the less overheads it will have.