

ASTRONOMY 51/151 - SPRING 2022

Exercise Sheet 4

DUE by Friday, March 18, 2022

300 points

A. Non-uniform Deviates 1

An astrophysicist has developed a code to evolve the trajectories of photons produced by a perfect black-body at a temperature $T=6\times 10^3$ K. As initial conditions, she will need to specify for each photon its energy and direction of propagation. Provide her with the initial distribution of 10,000 black-body photons with frequency ν between 0 and 5×10^4 Hz, to which she will then assign a random initial direction of propagation. Recall that the number of density of photons $N(\nu, T)$ at temperature T and with frequency between ν and $\nu + d\nu$ is:

$$N(\nu, T) = \frac{8\pi\nu^2}{c^3} \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]^{-1}, \quad (1)$$

and where you can set $h = c = k = 1$. Use the rejection method to generate the 10,000 photons with frequency ν between 0 and 5×10^4 Hz. Produce (i) a plot of Eq. 1 as a function of frequency ν between 0 and 5×10^4 Hz; on this plot, also over-plot the 10,000 non-rejected photons. Produce (ii) a plot of the photon distribution function, i.e., number of generated photons in bins of frequencies between 0 and 5×10^4 Hz (use 1000 Hz as the bin size).

B. Non-uniform Deviates 2

Using the rejection method, generate 10,000 random values of $-1 < x < 1$ and $-1 < y < 1$ such that $x^2 + y^2 < 1$, i.e., within the unit circle and centered in $x = y = 0$. Plot these points in the 2-D diagram y vs x .