

ASTRONOMY 51/151 - SPRING 2022

Exercise Sheet 3

DUE by Friday, March 11, 2022

300 points

A. Bayes' Theorem 1

There are N red balls and M white balls in an urn; we know the total $N + M = 10$. We draw $T = 5$ times (putting the balls back after drawing them) and get $R = 3$ red balls. Calculate the posterior probability of how many red balls there are in the urn, and plot it as a function of the true number of red balls. *Hint: see example on page 27 of the book "Practical Statistics for Astronomers", ed. 2.* Repeat for $T = 50$ and $R = 30$.

B. Bayes' Theorem 2

Before 1987, four naked-eye supernovae had been recorded in 10 centuries. Using the Bayes' theorem, calculate the posterior probability of the supernova rate per century ρ . Assume that our prior on ρ is uniform between 0 and 1 supernovae per century. Plot the derived posterior probability distribution for ρ as a function of the number of supernovae per century. Calculate the peak and mean of the posterior probability *Hint: see example on page 29 of the book "Practical Statistics for Astronomers", ed. 2.* Repeat adopting a Haldane prior instead of a uniform prior.

C. Gaussian distribution 1

Given a standard Gaussian distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$, calculate the probability that a measurement will deviate from the mean by less than 1.5σ . *Hint: You need to numerically calculate an integral.*

D. Gaussian distribution 2

Given a Gaussian distribution $prob(x)$ with mean $\mu = 2.5$ and standard deviation $\sigma = 0.4$, calculate the value x_{15} such that 15% of measurements are at $x < x_{15}$, and the value x_{85} such that 15% of measurements are at $x > x_{85}$. Express these values both in absolute units, and in units of standard deviations.