ASTRONOMY 51/151 - SPRING 2022
Exercise Sheet 2
DUE by Friday, February 25, 2022
300 points

A. Closed Quadratures

Consider the following integral:

$$I_1 = \int_0^{\pi/2} sin^{2m-1}\theta cos^{2n-1}\theta d\theta = \int_0^{\pi/2} f(\theta) d\theta, \quad (1)$$

where $m > 0$, $n > 0$. Although the code to calculate this integral must be written so that any value of m and n can be input, use $m = 2$ and $n = 4$ for outputting specific values.

(i) I_1 can be calculated analytically. Use that book of tabulated integrals you never use to derive the analytical solution of I_1. After writing the general solution for any choice of m and n, calculate the analytic solution of I_1 for $m = 2$ and $n = 4$ (*Hint: it should be 0.025*).

(ii) Plot the integrand $f(\theta)$ as a function of θ

(iii) Calculate I_1 numerically using the trapezoidal extended closed formula. (*Hint: to check that the algorithm works, test it with the integrant function $f(x) = x + 1$ over the interval [0,1]; calculate numerically the value of I_1 as a function of N points used to sample the interval [0,1], and plot the resulting values of I_1 as a function of N – you should find the same exact value (corresponding to the analytic solution) for any N used*).

(iv) Calculate I_1 numerically using Simpson's extended closed formula. (*Hint: to check that the algorithm works, test it with the integrant function $f(x) = x^3 + x^2 + x + 1$ over the interval [0,1]; calculate numerically the value of I_1 as a function of N points used to sample the interval [0,1], and plot the resulting values of I_1 as a function of N – you should find the same exact value (corresponding to the analytic solution) for any N used*)

(v) Compare the results obtained in *(iii)* and *(iv)*. Specifically, on the same figure, plot $\log_{10}(error)$, where $error = |I_{1,\text{exact}} - I_{1,\text{numeric}}|$, as a function of $\log_{10} N$, for both the trapezoidal and Simpson's extended closed formulas. Which is more precise?