

Reading assignment TUESDAY 10/20: Chapter 10.1 / 10.2 / 16.3

Homework Assignment #3 due by: TUESDAY 10/27 before beginning of class

The Pressure Integral:

The microscopic source of pressure in a perfect gas is particle bombardment, resulting in transfer of momentum, hence a force (F=dp/dt). The average force per unit of area is the PRESSURE.

In thermal equilibrium in the stellar interior, the angular distribution of particle momenta is isotropic, i.e., particles are moving with equal probability in all directions.

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NOTE: the relation between p and v_p depends upon relativistic considerations, whereas n(p) depends on the type of particles and quantum statistics.

$$F_{L} = \frac{2\pi c^{2} h}{\lambda^{5}} \frac{1}{e^{hc/\lambda KT}} \xrightarrow{F_{V}} F_{V} = \frac{2\pi h p^{3}}{c^{2}} \frac{1}{\frac{h^{3}/\kappa T}{c^{2}}}$$
[erg s⁻¹ cm⁻² A⁻¹]
$$\int e^{hc/\lambda KT} \frac{1}{p = c/\lambda} = \frac{c}{\lambda^{2}} d\lambda$$

$$F_{L} = \frac{2\pi c^{2} h}{\sqrt{5}} \frac{1}{e^{hc/\sqrt{KT}}} \xrightarrow{4} F_{y} = \frac{2\pi h p^{3}}{c^{2}} \frac{1}{\frac{h^{3}/KT}{c^{2}}}$$
[erg s⁻¹ cm⁻² A⁻¹]
$$p = c/\lambda$$

$$dp = \int_{1}^{\infty} d\lambda$$

$$u(x) dy = \frac{4}{c} F_{x} \int energy density of photons of frequency
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ENERGY
$$M^{\circ} = \int_{0}^{\infty} \mu(x) dx = \frac{8\pi^{5} k^{4}}{15 c^{3} h^{3}} T^{4} = a T^{4}$$

DENSITY $M^{\circ} = \int_{0}^{\infty} \mu(x) dx = \frac{8\pi^{5} k^{4}}{15 c^{3} h^{3}} T^{4} = a T^{4}$
 $w/a = 7.565 \times 10^{-15} \frac{0.00}{cm^{3}} \frac{0.00}{k^{4}}$

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$$N_{\lambda} d\lambda = \frac{M(\lambda) d\lambda}{E_{\lambda}} = \frac{M(\lambda) d\lambda}{hc/\lambda}$$

$$\rightarrow N := \int_{0}^{\infty} N_{\lambda} d\lambda = 8\pi \left(\frac{KT}{hc}\right)^{3} \int_{0}^{\infty} \frac{x^{2} dx}{e^{2} - 1} = 19.232912\pi \left(\frac{KT}{hc}\right)^{3}$$

$$Number density = 2.404114$$

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PHOTON HAS ENERGY E = hP & MOMENTUM P= E = hP -> PHOTONS CON EXERT A RADIATION PRESSURE

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$$Number = \frac{19.232912\pi}{hc} \left(\frac{KT}{hc}\right)^{3}$$

If in thermodynamic equilibrium, then the radiation flux is isotropic PHOTON HAS ENERGY E = hP & MOMENTUM P= E = hP -> PHOTONS CON EXERT A RADIATION PREJSURE

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$$Number = \frac{100}{2.404114}$$

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PHOTON HAS ENERGY
$$E = hP q$$
 MOMENTUM $P = \frac{E}{C} = \frac{hP}{C}$
 \rightarrow PHOTONS CON EXERT A PADIATION PREJUDE
 $P = \frac{1}{3} \left(\int_{0}^{\infty} p N_{p} n(p) dp = \frac{1}{3} \int_{0}^{\infty} \frac{hP}{C} c n(y) dP = \frac{1}{3} \int_{0}^{\infty} hP n(y) dp$
 $e norry doughty doughty of Photons $M = 2T^{4}$
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 $\Rightarrow P_{R} = \frac{1}{3} M = \frac{1}{3} OT^{4}$
 $\Im = 7.565 \times 10^{-15} \frac{erg}{m^{3} \cdot K^{4}}$$

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$$Number = \binom{N_{\lambda}}{density}$$

$$2.404114$$

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 $P = \frac{1}{3} \left(\int_{0}^{\infty} p \cdot n(p) dp = \frac{1}{3} \int_{0}^{\infty} \frac{hv}{C} c \cdot n(p) dP = \frac{1}{3} \int_{0}^{\infty} \frac{hv}{P} \cdot n(p) dv$
 $e \cdot norgy doughty$
 $= 2 T^{4}$
 $\Rightarrow P_{2} = \frac{1}{3} M = \frac{1}{3} O T^{4}$
 $2 = 7565 \times 10^{-5} \frac{2}{3} \frac{v}{V}$
Radiation pressure
for a blackbody
radiation

cm

Stellar Opacity



The Sun spectrum deviates substantially from the shape of the blackbody Planck function due to solar absorption lines removing light from the Sun's continuous spectrum (a.k.a., line blanketing)

Let's consider the photosphere of the Sun, and the temperature varies from 5580K to 5790K over a distance of 25 km.

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TEMPERATURE
$$H_T := \frac{T}{|dT/dr|} = \frac{5685 \text{ }^\circ\text{K}}{(5790-5580)/25} = 677 \text{ Km}$$

SCALE HEIGHT $\frac{dT}{dr} \simeq \frac{\Delta T}{\Delta r} = \frac{T_F - T_i}{f_F - r_i}$

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Characteristic distance over $\frac{dT}{dr} \simeq \frac{\Delta T}{dr} = \frac{T_{P}-T_{L}}{Qr-r_{L}}$

 $g = 2.1 \times 10^{-4} \frac{K_{9}}{m^{3}}$ Mass density of photosphere, primarily made of H in the ground state $n = \frac{g}{m_{H}} = 1.25 \times 10^{23} \text{ m}^{-3}$ Number density of atoms $g = 2.1 \times 10^{-4} \frac{k_0}{m^3}$ Mass density of photosphere, primarily made of H in the ground state $n = 3/m_H = 1.25 \times 10^{-3} \text{ m}^{-3}$ Number density of atoms Weak free path $l = \frac{1}{NC}$ w/C cross sector $S = Tr(2a_0)^6 = 3.52 \times 10^{-20} \text{ m}^2$ a_0 radius of Bohr's Rom $Q = 2.1 \times 10^{-4} \frac{K_0}{m^3}$ Mass density of photosphere, primarily made of H in the ground state $n = S/m_{H} = 1.25 \times 10^{23} \text{ m}^{-3}$ Number density of atoms

mean free path
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between collisions $l = nC$ $C = T(2a_0)^2 = 3.52 \times 10^{-20} \text{ m}^2$
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i.e., the atoms see a constant kinetic temperature between collisions, hence LTE is valid for H atoms in the photosphere Consider now a beam of parallel light rays traveling through a gas; absorption is the ensemble of processes of removing photons (including scattering).

For pure absorption, the intensity declines exponentially falling by a factor of e⁻¹ over a characteristic distance:

W/ K1 opscity (or absorption coefficient) [K1] = m²/kg cross section for absorbing photons of wavelength I per unit of mass of stellar material. = -

In the photosphere:

 $k_{500 \text{ nm}} = 0.03 \frac{\text{m}^2}{\text{kg}} \rightarrow l = 160 \text{ km}$ NOT TOO DIFFERENT FROM HT $\simeq 677 \text{ km}$ Consider now a beam of parallel light rays traveling through a gas; absorption is the ensemble of processes of removing photons (including scattering).

For pure absorption, the intensity declines exponentially falling by a factor of e⁻¹ over a characteristic distance:

W/ Ky opacity (or absorption coefficient) [Ky] = m²/kg cross section for absorbing photons of wavelength & per unit of mass of stellar material. l = $k_{500 \text{ nm}} = 0.03 \frac{\text{m}^2}{\text{kg}} \rightarrow l = 160 \text{ km}$ NOT TOO DIFFERENT FROM HT ~ 677 km In the photosphere:

i.e., the photospheric photons do not see a constant temperature, and so LTE is not strictly valid in the photosphere. LTE must be used with caution in stellar atmosphere.

Equivalent width (EW, or W) of spectral line



 N_a = number of absorbing atoms per unit of area (this is obtained from the Boltzmann and the Saha equations once the temperature and the density are known).

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 fN_a = effective number of atoms lying above each square meter of photosphere actively involved in producing a given spectral line

Growth curve method



This is used to determine the value of N_a , hence the abundances of elements in the stellar atmosphere.



For small abundances, when the absorption line is has not saturated, EW is proportional to N_a . In the central region is when the line saturates and the EW changes little because of only the wings becoming deeper and so little change in the EW. Increasing Na even further increases the importance of pressure broadening, and the wings are significantly affected, with the EW growing as $N_a^{1/2}$.



For a given element, the EW is measured from the spectrum (e.g., of the Sun). Using the appropriate curve of growth, I can obtain fN_a . Known fN_a and f, I can derive the total number of atoms of a specific element using the Boltzmann and Saha equations.

Repeating this for many elements, transitions, **I can derive the chemical composition of the photosphere of stars**.

TABLE 9.2 The Most Abundant Elements in the Solar Photosphere. The relative abundance of an element is given by $\log_{10}(N_{\rm el}/N_{\rm H}) + 12$. (Data from Grevesse and Sauval, *Space Science Reviews*, 85, 161, 1998.)

	Atomic	Log Relative Relative abundar	
Element	Number	Abundance	by number
Hydrogen	1	12.00	
Helium	2	10.93 ± 0.004	→ 8.5% N _H
Oxygen	8	8.83 ± 0.06	→ 0.068% N _H
Carbon	6	8.52 ± 0.06	→ 0.033% N _H
Neon	10	8.08 ± 0.06	→ 0.012% N _H
Nitrogen	7	7.92 ± 0.06	
Magnesium	12	7.58 ± 0.05	
Silicon	14	7.55 ± 0.05	
Iron	26	7.50 ± 0.05	
Sulfur	16	7.33 ± 0.11	
Aluminum	13	6.47 ± 0.07	
Argon	18	6.40 ± 0.06	
Calcium	20	6.36 ± 0.02	→ 0.00023% N _H
Sodium	11	6.33 ± 0.03	
Nickel	28	6.25 ± 0.04	

lsotope \$	A .	mass fraction in parts ¢ per million	number fraction in parts \$ per million
Hydrogen-1	1	705,700	909,964
Hydrogen-2	2	23	15
Helium-3	3	35	18
Helium-4	4	275,200	88,714
Carbon-12	12	3,032	326
Carbon-13	13	37	
Nitrogen-14	14	1,105	102
Oxygen-16	16	5,920	477
Neon-20	20	1,548	100
Neon-22	22	208	. 12
Sodium-23	23	33	
Magnesium-24	24	513	28
Magnesium-25	25	69	4
Magnesium-26	26	79	
Aluminum-27	27	. 58	· 5
Silicon-28	28	653	30
Silicon-29	29	34	2
Silicon-30	30	23	
Sulfur-32	32	396	16
Argon-36	36	77	3
Calcium-40	40	60	2
Iron-54	54	72	2
Iron-56	56	1,169	27
Iron-57	57	28	1
Nickel-58	58	49	1
Other element		3 870	140

H-I: 705700/1000000 = 0.7057 = 70.6%

He-4: 275200/1000000 = 0.2752 = 27.5%

Metals: 198100/1000000 = 0.0191 = 1.9%



A = mass number







NUCLEOSYNTHESIS:

- Nucleosynthesis occurs in the natural evolution of stars
- The initial H and He are fused into heavier nuclei, dispersed in the ISM in the terminal phases of stellar evolution
- During the first 3 minutes after the Big Bang, H (75% by mass), He (25%) and traces of D, ³He, Li, Be, B were produced (primordial nucleosynthesis)
- ¹²C, ¹⁶O, ²⁰Ne most abundant elements after H and He (produced by He fusion)
- ⁵⁶Fe: nuclear binding energy per nucleon has a maximum at ⁵⁶Fe, i.e., successive nuclear fusion reactions cease to liberate energy when all light nuclei have been fused into ⁵⁶Fe. Farther fusion requires energy, hence the termination of energygenerating stages of nuclear fusion.
- Very heavy elements can be formed efficiently by the capture of free neutrons liberated as a by product of reactions between light charged particles.



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