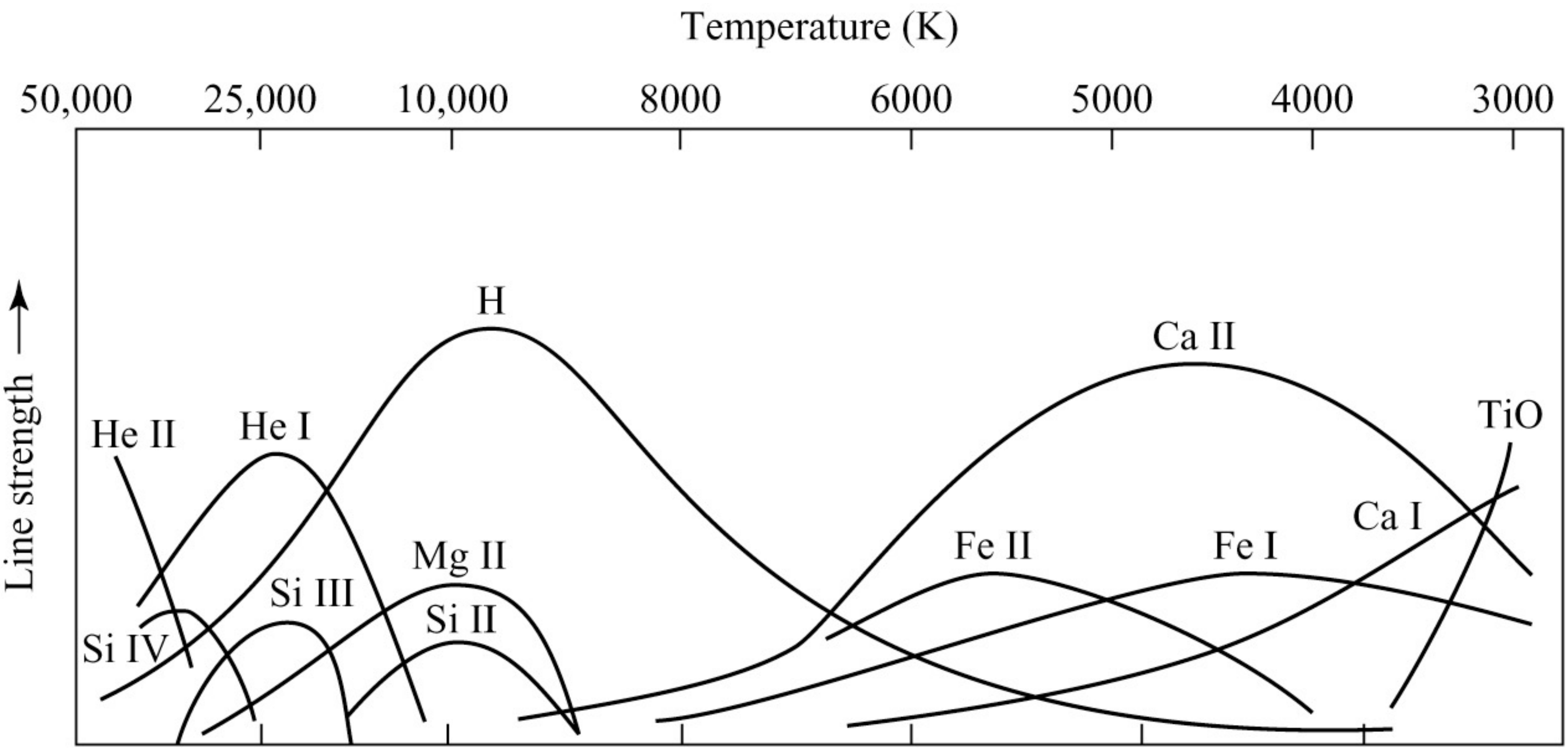
The Classification of Stellar Spectra

Reading assignment
TUESDAY 10/13: Chapter 9

Strength of various spectral lines with temperature



T_{ion}: the ionization temperature is obtained from the distribution of atoms in the ionized levels from the Saha's equation. By comparing the strengths of the absorption lines of two different states of ionization of the same atom, it is possible to determine the relative number densities of the two stages of ionization of the atom (e.g., H and K lines of CaII with the line of neutral Ca @ 422.6 nm).

Taking the log of the Saha's equation:

$$\log \frac{N_{i+1}}{N_i} = -\frac{5040}{T(K)} \chi_i(eV) + 2.5 \log T - 0.18 + \log \frac{Z_{i+1}}{Z_i} - \log P_e$$

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The one-parameter Harvard classification of spectral type is directly explained by the above, since the dominant dependence is with temperature. However, P_e is also to be considered to properly characterize stars with ionized atoms in their atmospheres. P_e is smaller in cooler stars than in hotter stars, but I can consider it a constant at first approximation for main-sequence stars on the H-R diagram.

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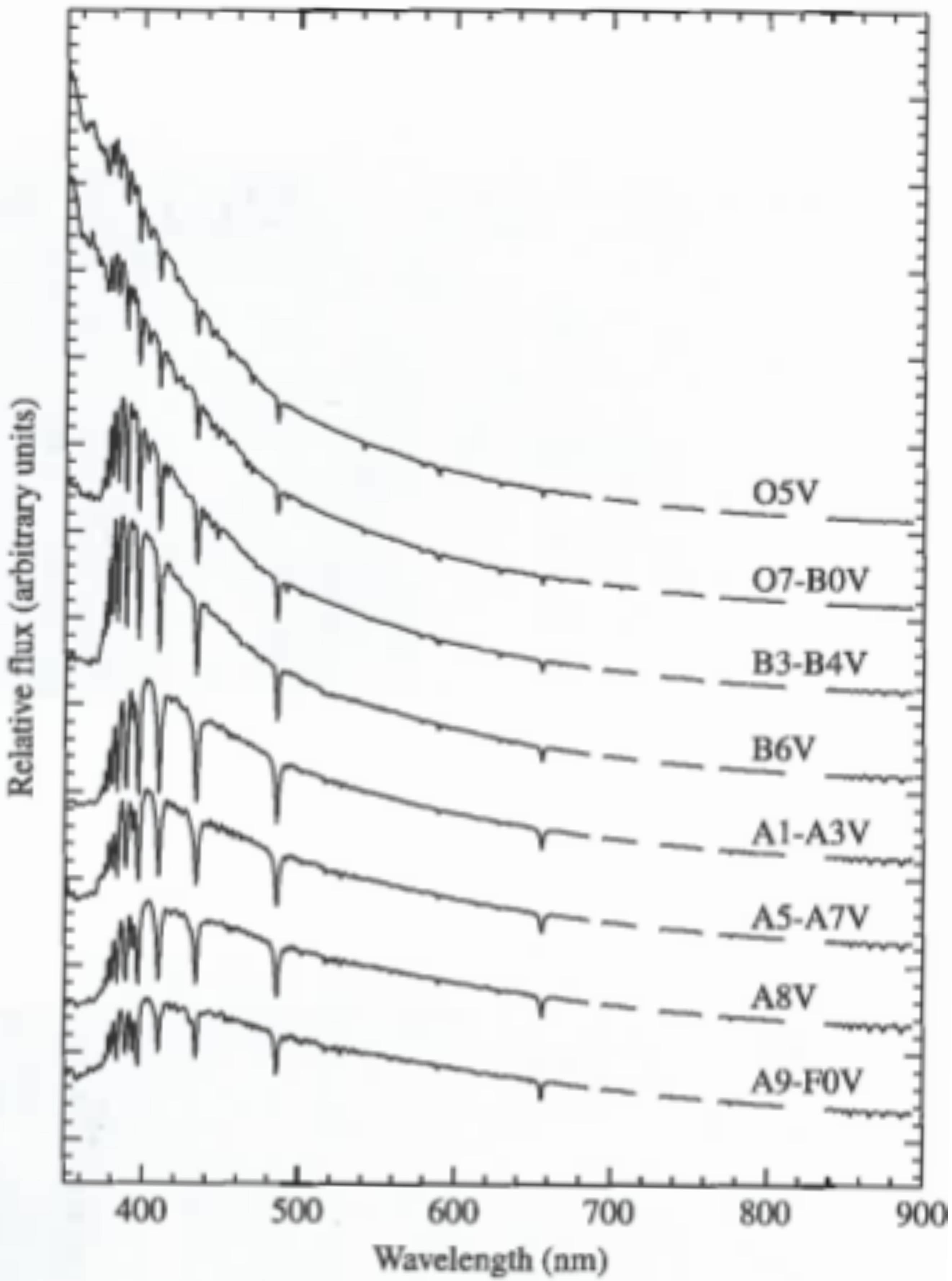
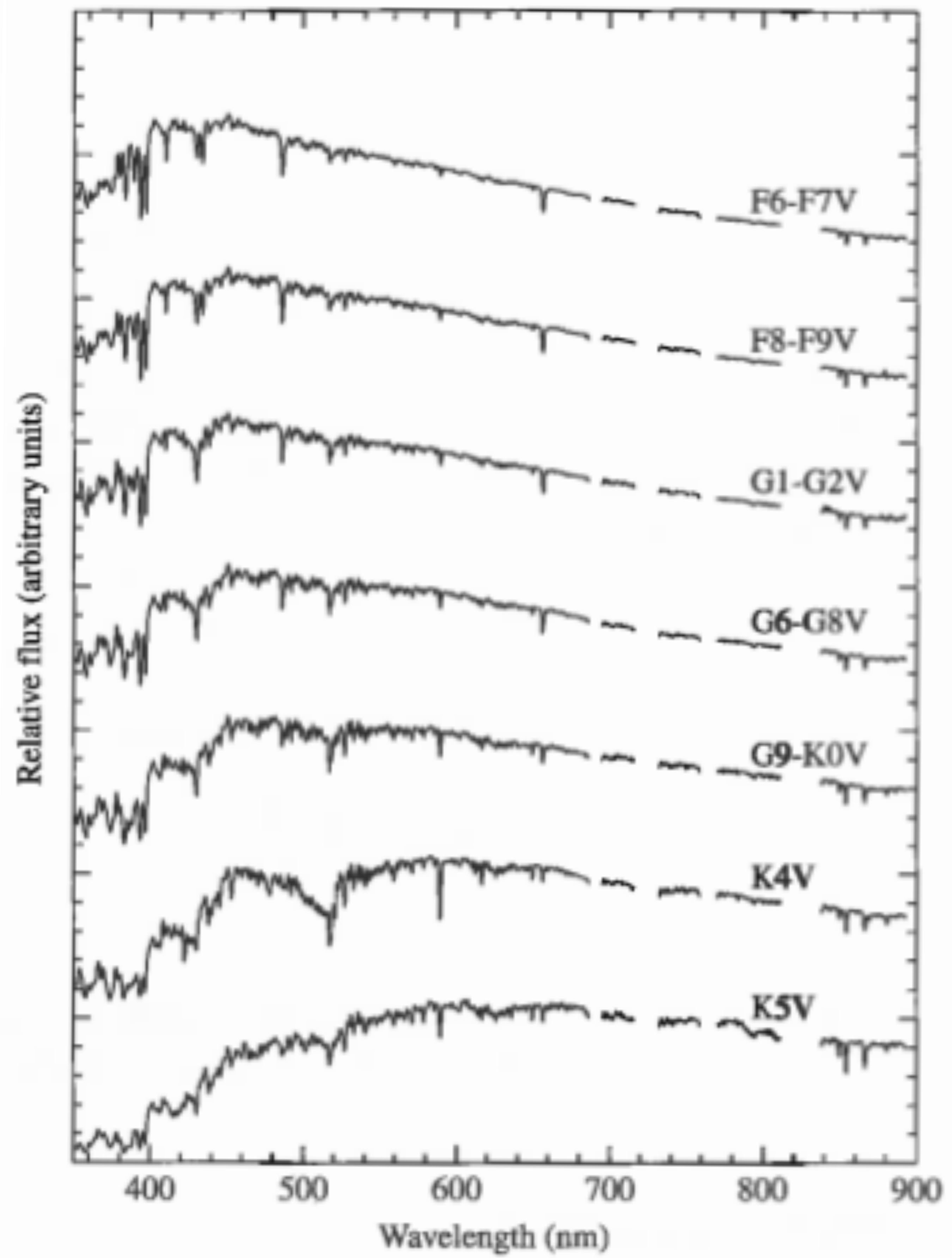
From $N_{i+1}/N_i \longrightarrow T, P_e$

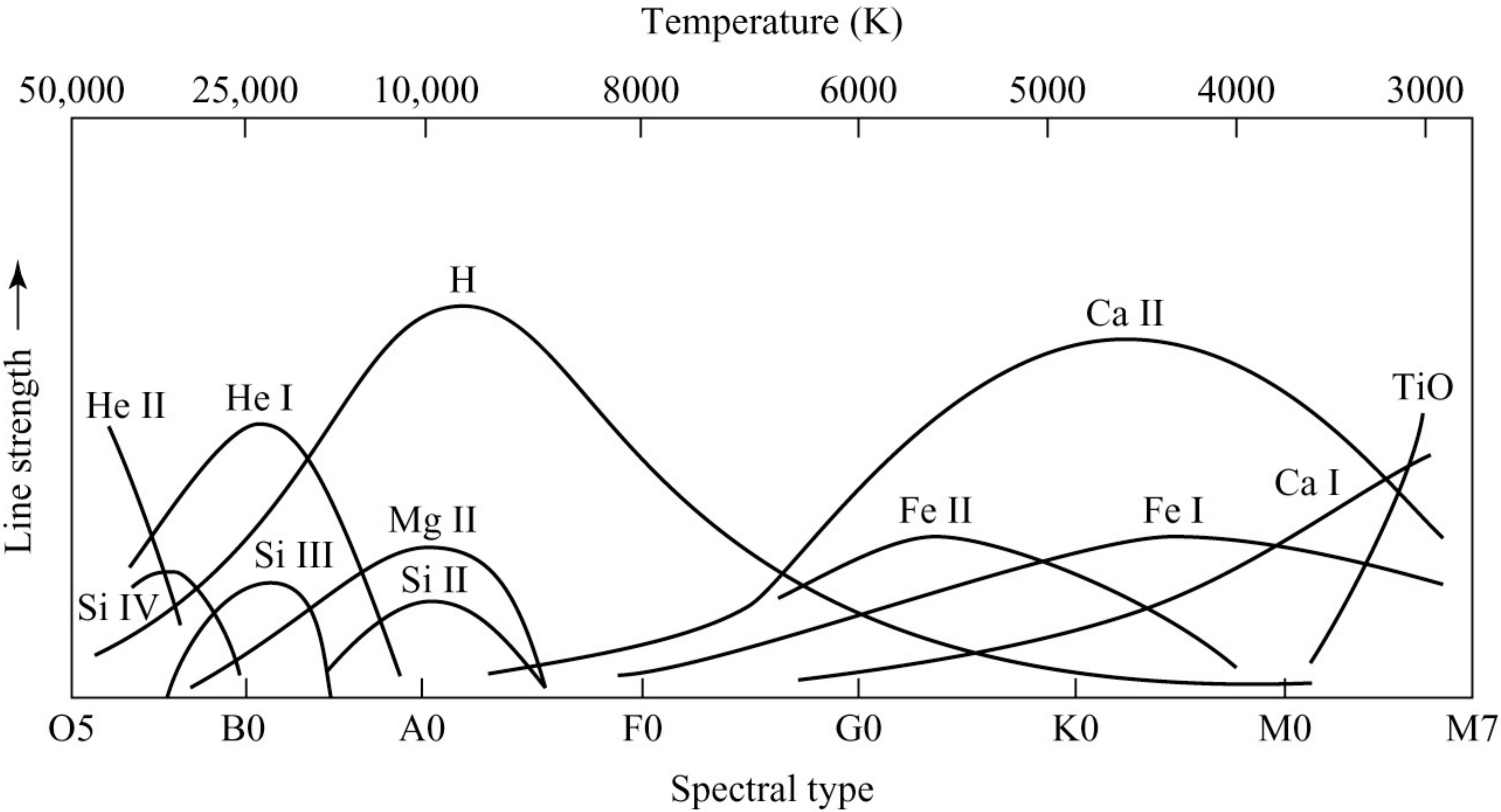
Vice versa, if I know T and P_e, I can estimate N_{i+1}/N_i, i.e., the abundance of elements.

Spectral Types of Stars





Strength of various spectral lines with temperature and spectral type



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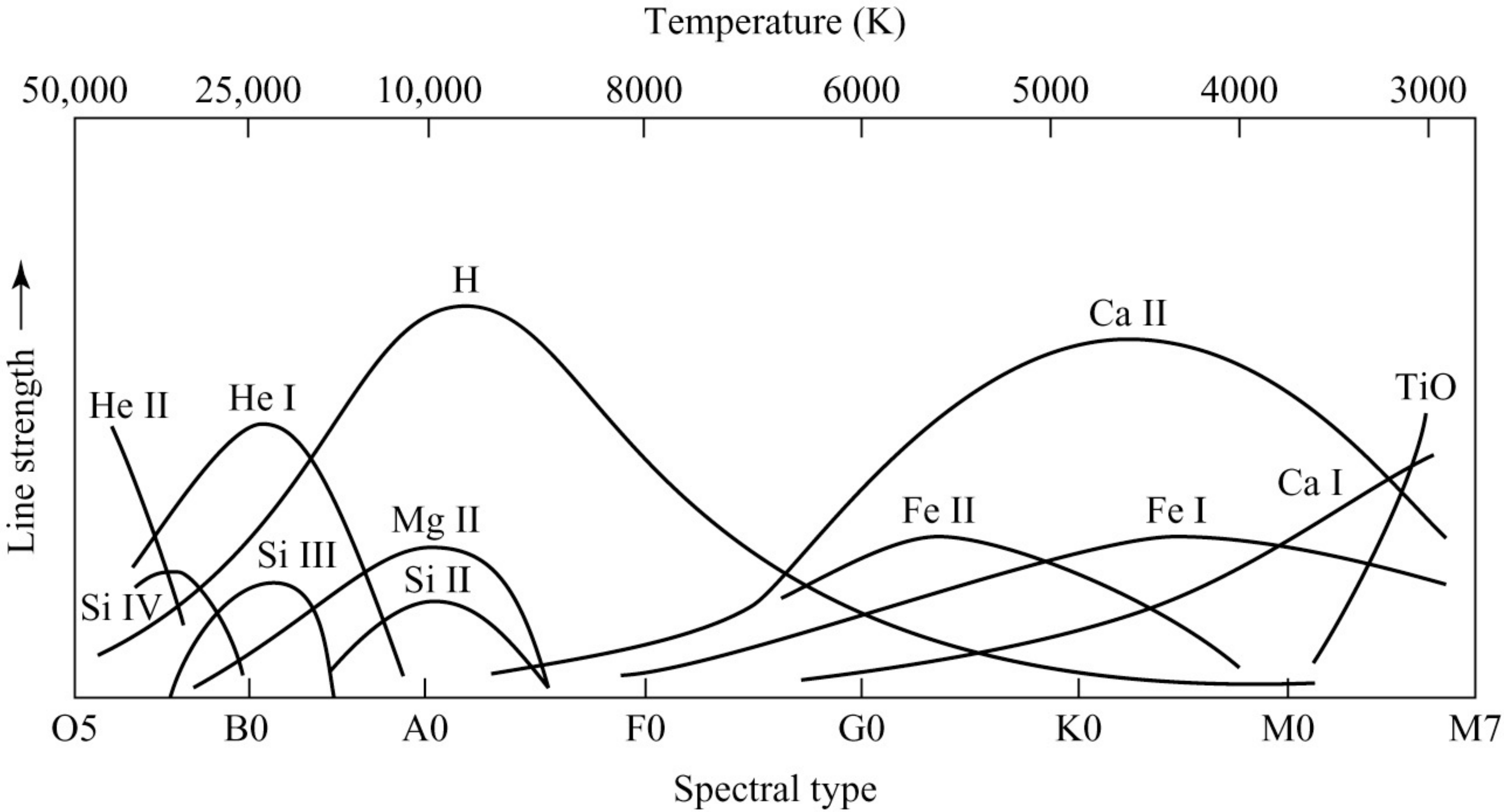

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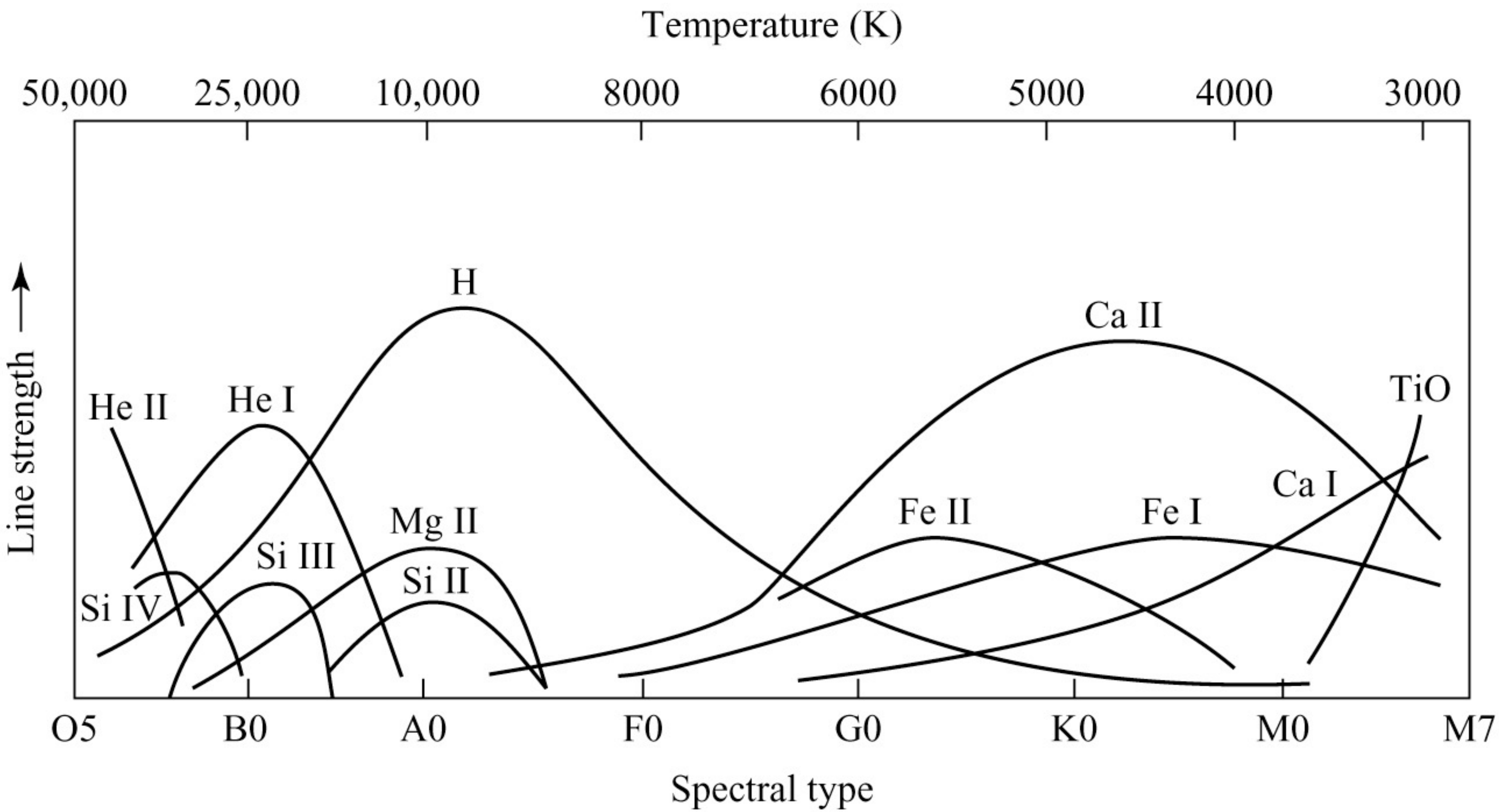

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OBAFGKM (from hotter to cooler): **O**h **B**eautiful **A**nd **F**ine **G**irl/Guy **K**iss **M**e

Example of spectra of stars

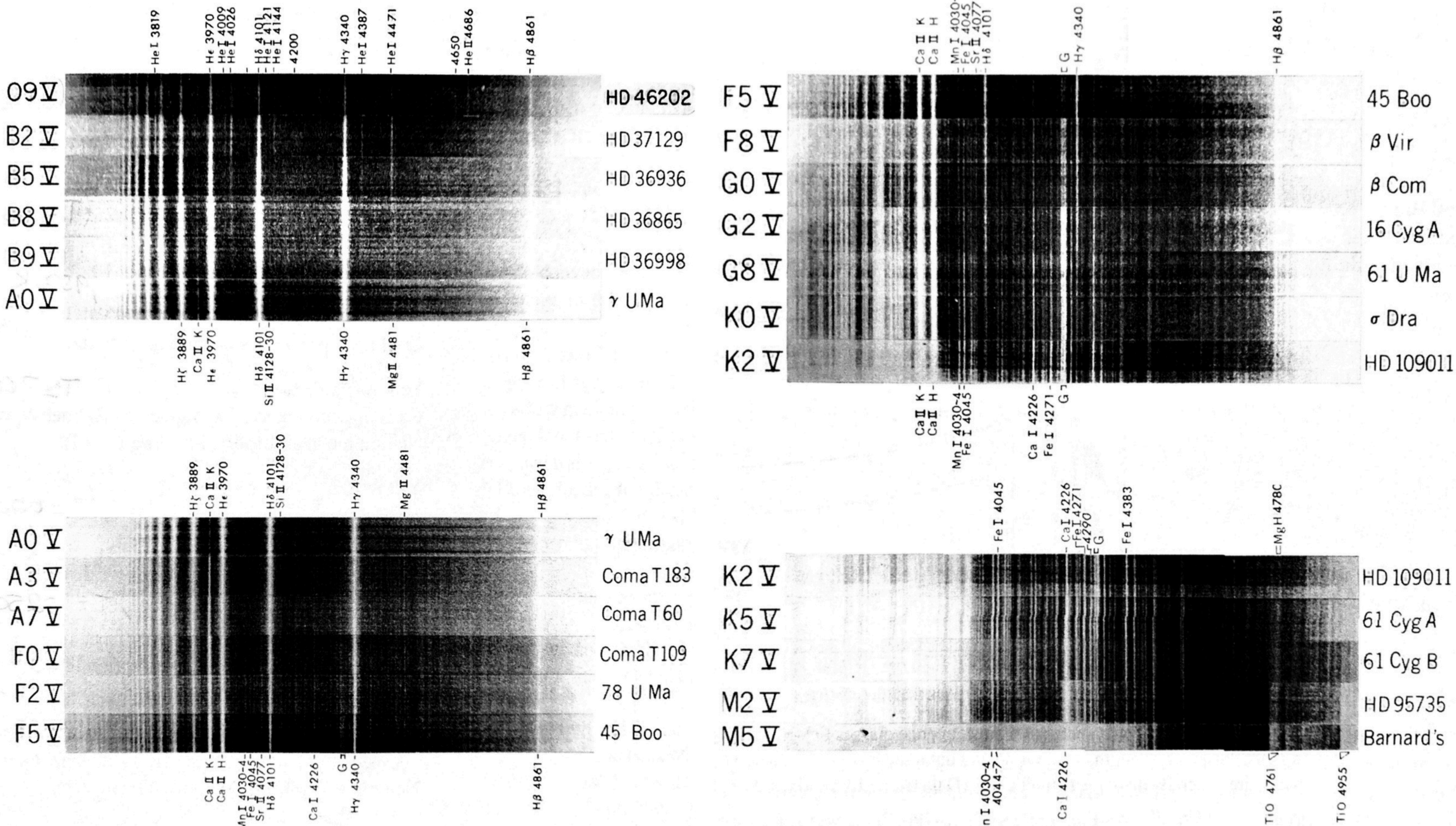


TABLE 8.1 Harvard Spectral Classification.

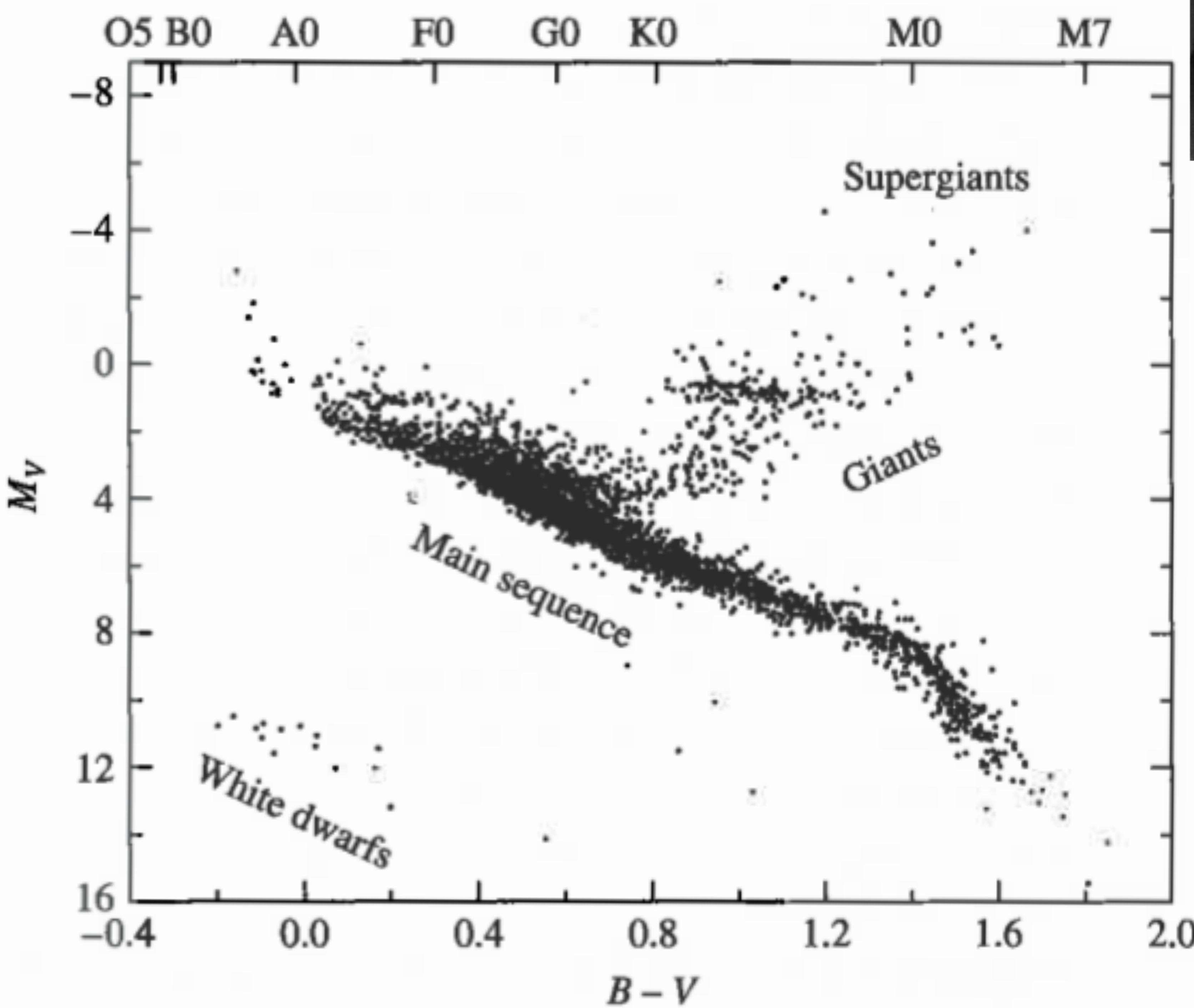
Spectral Type	Characteristics	
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.	T≈50000K
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.	(T= 22000 K)
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.	9500K T≃11000K
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).	T≃7600 K
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.	T≃6000K
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.	T= 5250K
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.	T≃3600 K
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.	T= 1300 - 2500 K
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.	T<1300K

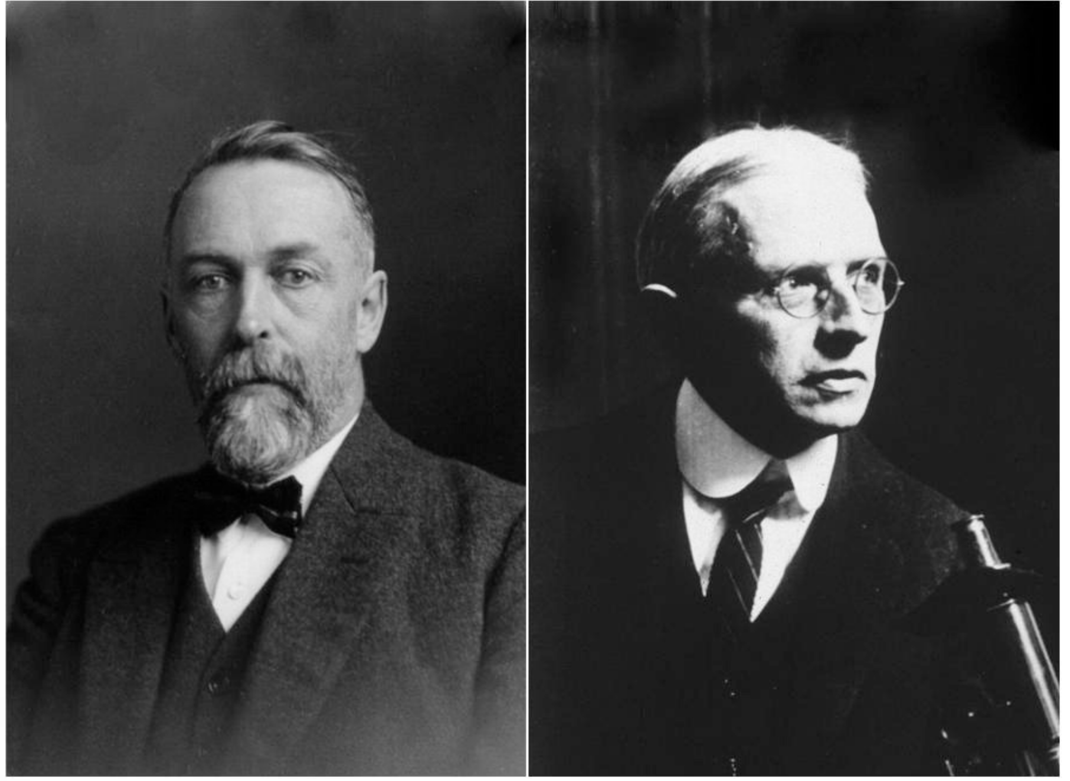
BROWN DWARFS

S and C spectral types for evolved giant stars are discussed on page 466.

Hertzsprung-Russell (HR) diagram

snapshot of stellar structure
and stellar evolution

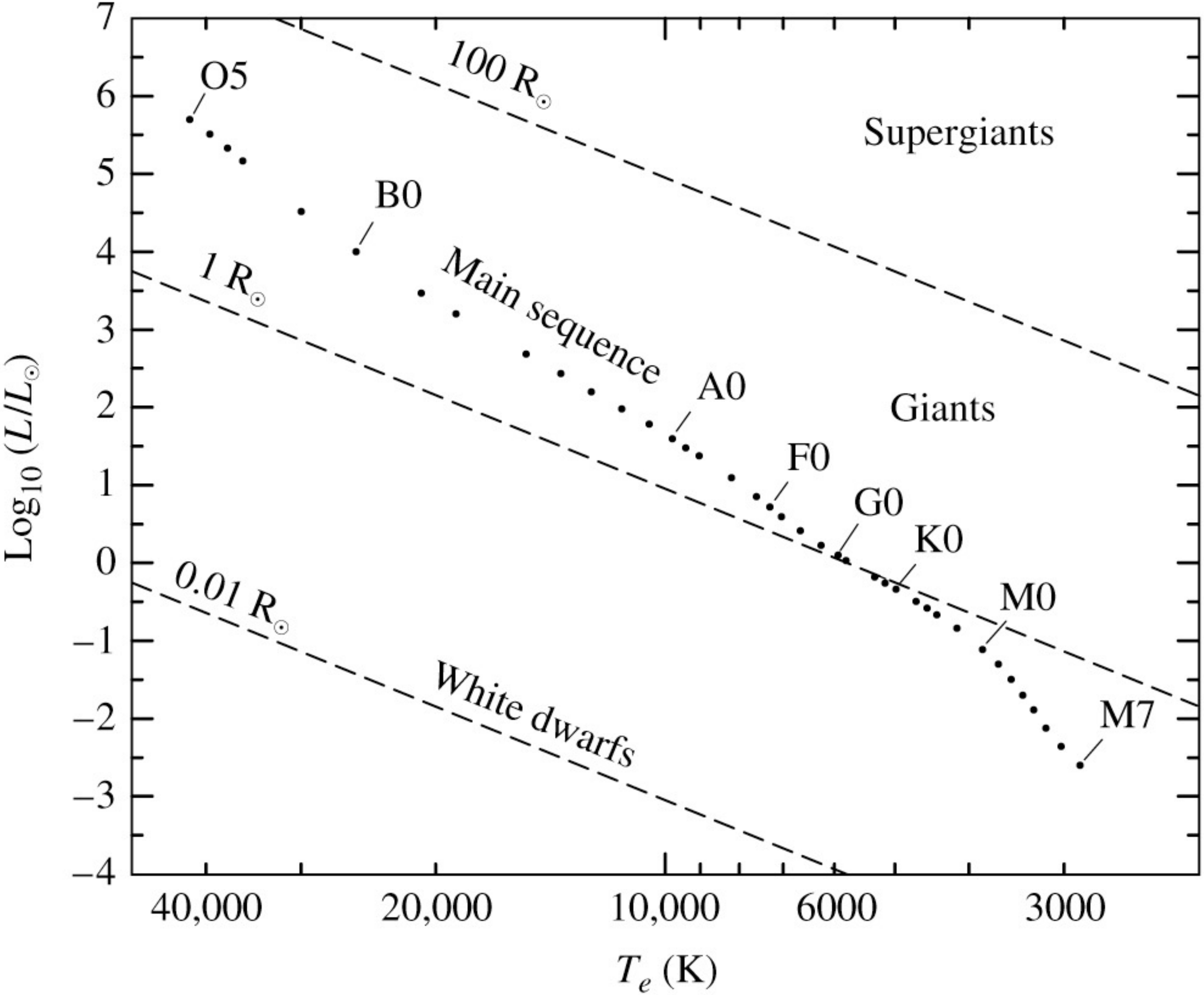




$$L = 4\pi R^2 \sigma T_{eff}^4$$

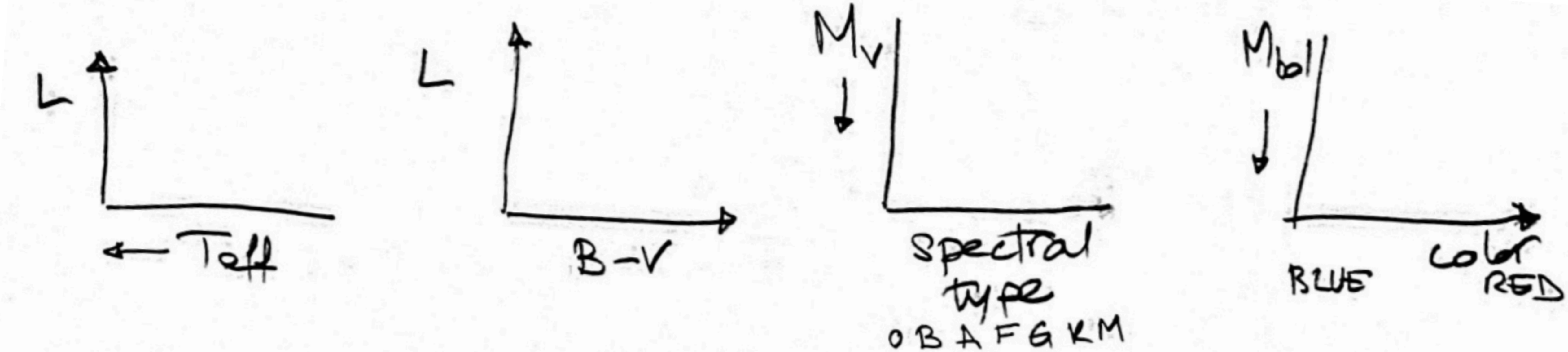
$$\rightarrow \quad R = \frac{1}{T_{eff}^2}\sqrt{\frac{L}{4\pi\sigma}}$$

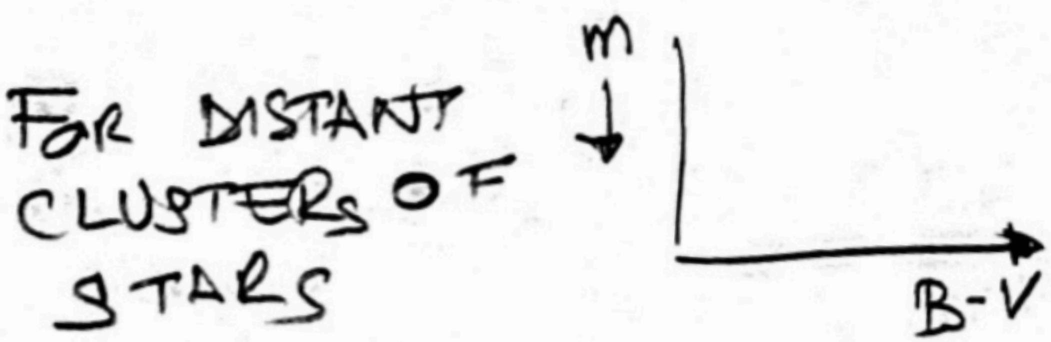
If two stars gave the same T_{eff}, or the same spectral type or color, then the more luminous star must be larger (bigger R)



7
6
5
4
3
2
1
0
−1
−2
−3
−4
Log10 (L/L⊙)
40,000
20,000
10,000
6000
3000
Te (K)
100 R⊙
1 R⊙
0.01 R⊙
O5
B0
A0
F0
G0
K0
M0
M7
Supergiants
Giants
Main sequence
White dwarfs

Examples of HR diagrams



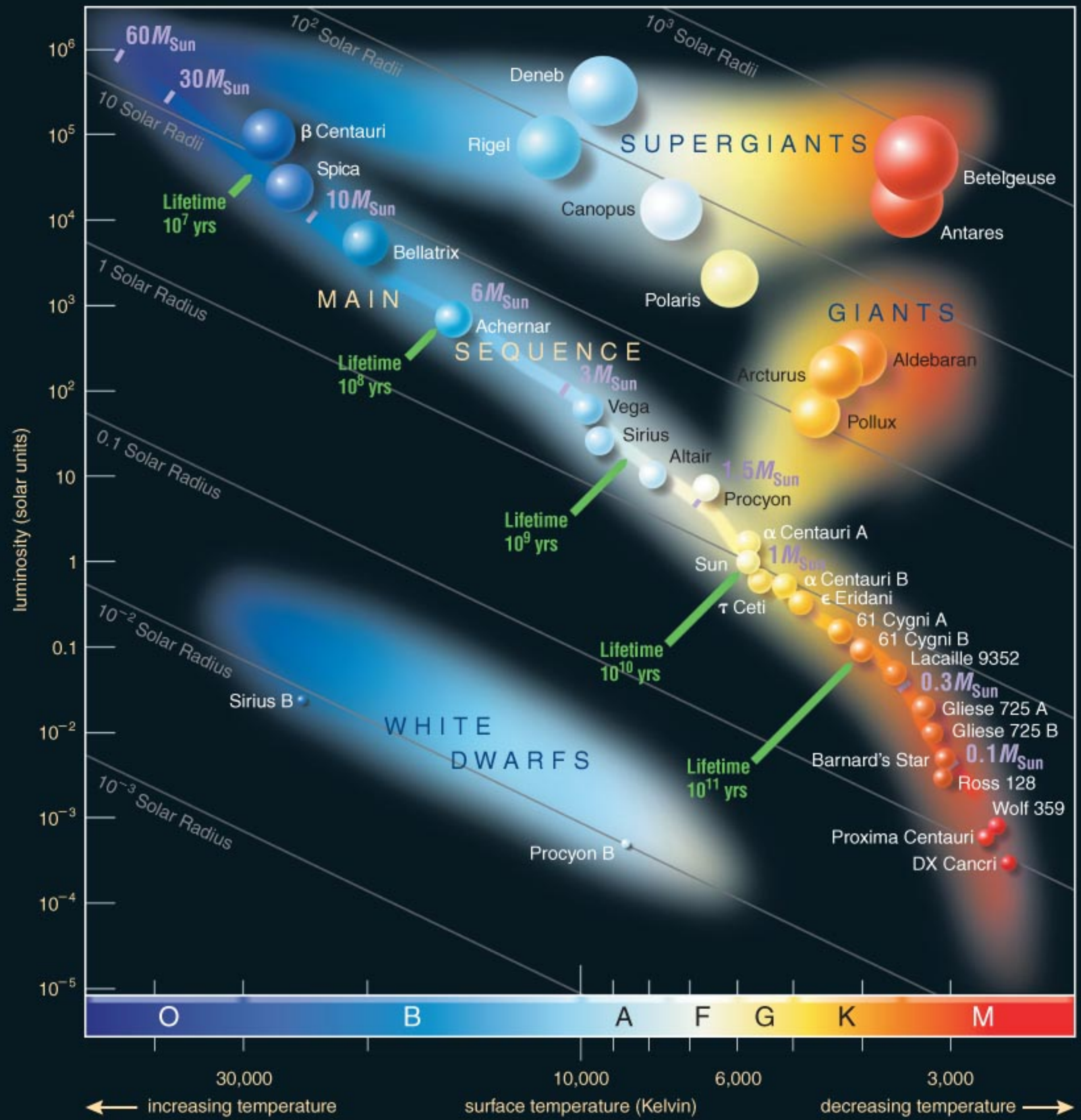


In a cluster of stars, all stars can be assumed to be at the same distance from us, hence m is proxy for M. Moreover, all stars in a cluster of stars can be assumed to have all the same age

The Classification of Stellar Spectra

Reading assignment

THURSDAY 10/15: Chapter 9 - refresh…



$60M_{Sun}$
$30M_{Sun}$
$10M_{Sun}$
$6M_{Sun}$
$3M_{Sun}$
$1.5M_{Sun}$
$1M_{Sun}$
$0.3M_{Sun}$
$0.1M_{Sun}$
10^2 Solar Radii
10^3 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^{-2} Solar Radius
10^{-3} Solar Radius
Deneb
Rigel
β Centauri
Spica
Canopus
Betelgeuse
Antares
Bellatrix
Polaris
SUPERGIANTS
GIANTS
MAIN
SEQUENCE
Achernar
Arcturus
Aldebaran
Pollux
Vega
Sirius
Altair
Procyon
α Centauri A
Sun
α Centauri B
ε Eridani
τ Ceti
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
WHITE
DWARFS
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^{10} yrs
Lifetime 10^{11} yrs
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature

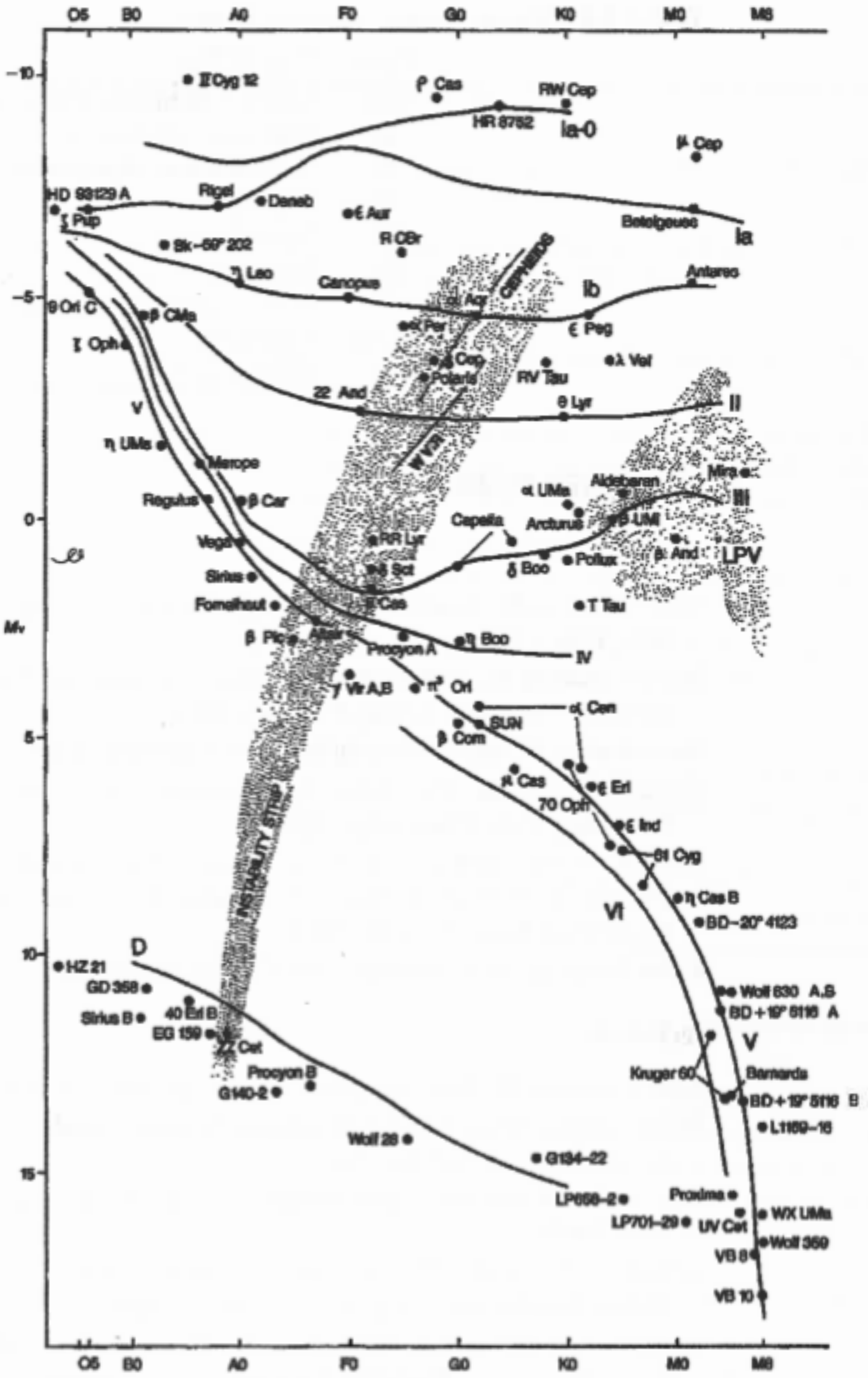
The mass of a star is the most important parameter:
O star —> most massive, up to 100 M_{Sun}
M star —> least massive, down to 0.08 M_{Sun}

$$\bar{\rho}_\odot = \frac{M_\odot}{\frac{4}{3}\pi R_\odot^3} \simeq 1410 \ Kg/m^3 \simeq \text{DENSITY OF WATER}$$

$$\bar{\rho}_{SIRIUS} \simeq 0.5\, \bar{\rho}_\odot$$

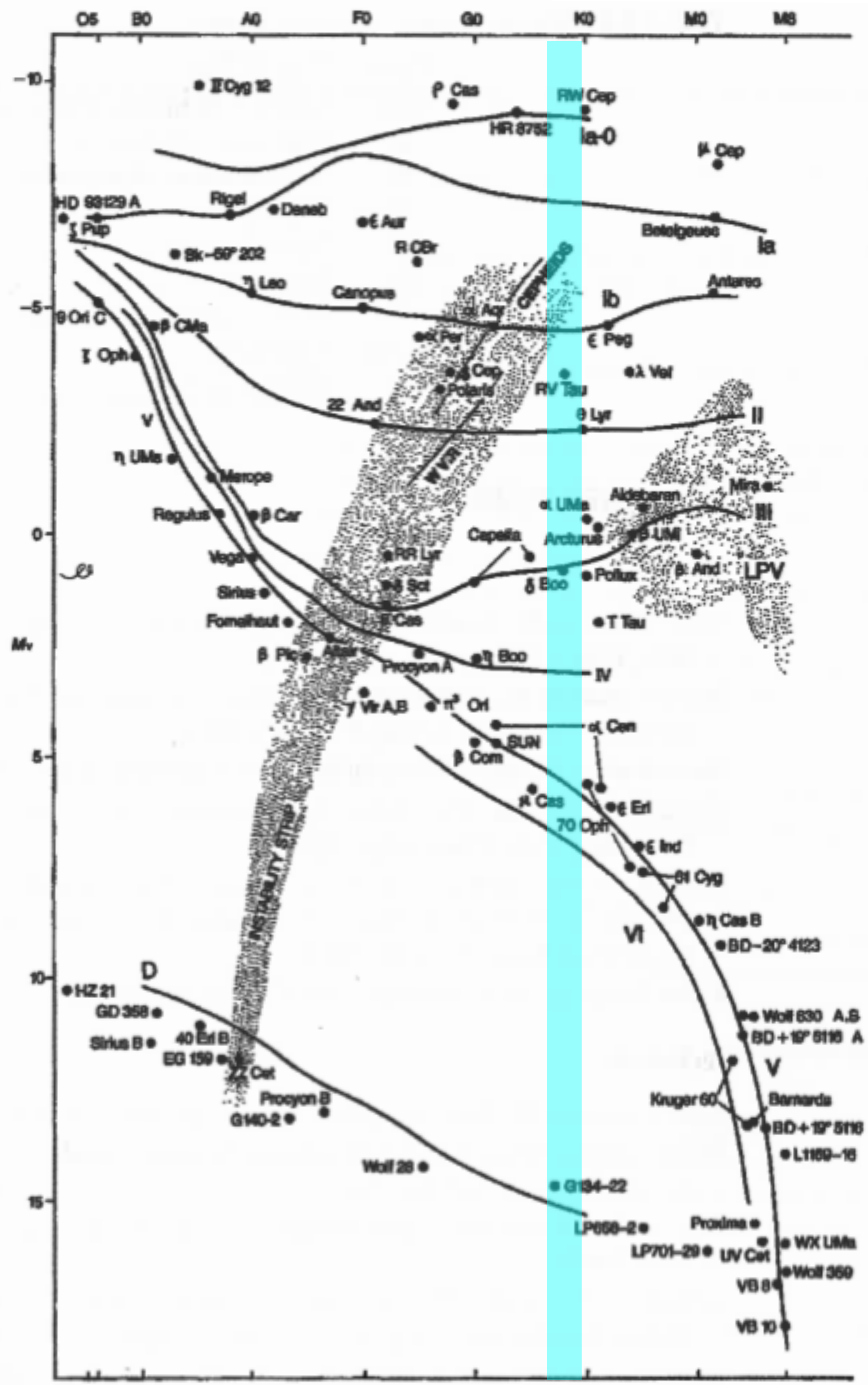
$$\bar{\rho}_{Betelgeuse} \simeq 10^{-8}\, \bar{\rho}_\odot$$

Very tenuous ghostly object!!, a hundred thousand times less dense than the air be breath!



Morgan–Keenan Luminosity Classes.

Class	Type of Star
Ia-O	Extreme, luminous supergiants
Ia	Luminous supergiants
Ib	Less luminous supergiants
II	Bright giants
III	Normal giants
IV	Subgiants
V	Main-sequence (dwarf) stars
VI, sd	Subdwarfs
D	White dwarfs



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Spectroscopic Parallax

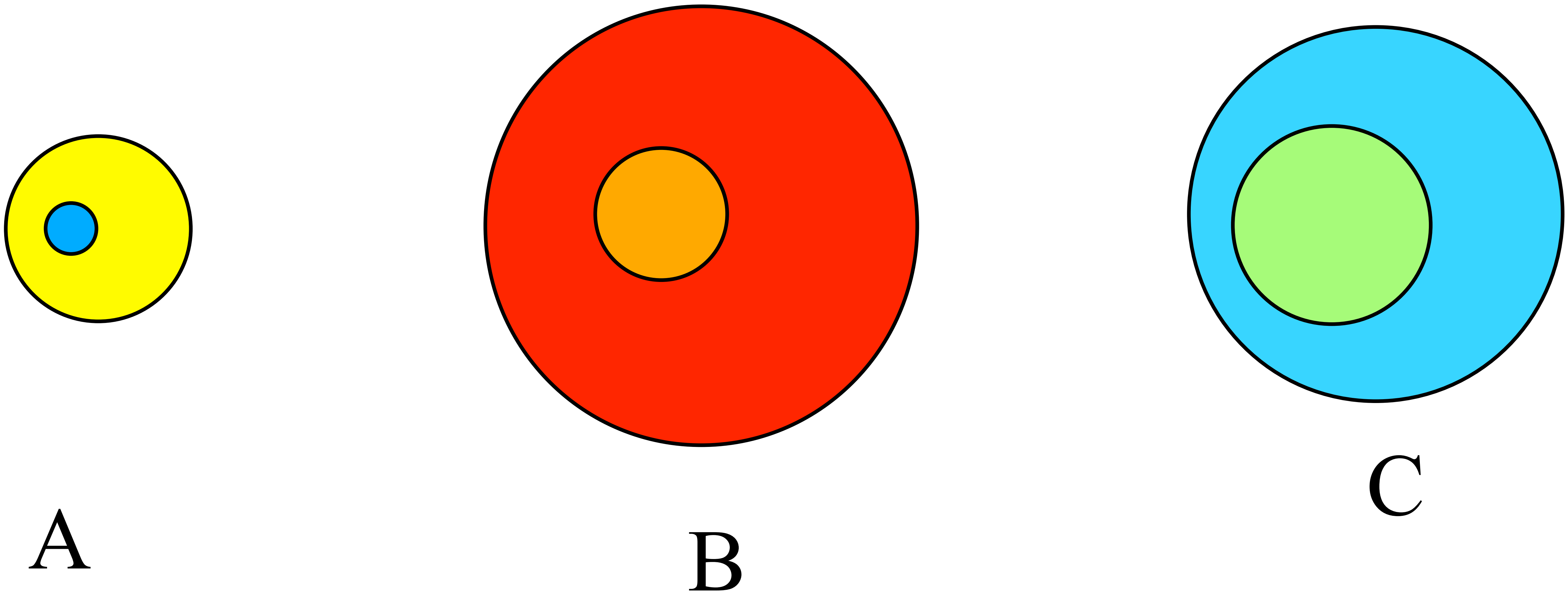
KNOWING THE SPECTRAL TYPE + LUMINOSITY CLASS $\rightarrow$ M $\rightarrow$ DISTANCE d AFTER MEASURING m

$$d = 10^{(m-M+5)/5}$$

w/ $[d] = pc$

(uncertainty ~60%)

Which of these is physically wrong if they are all main sequence stars?



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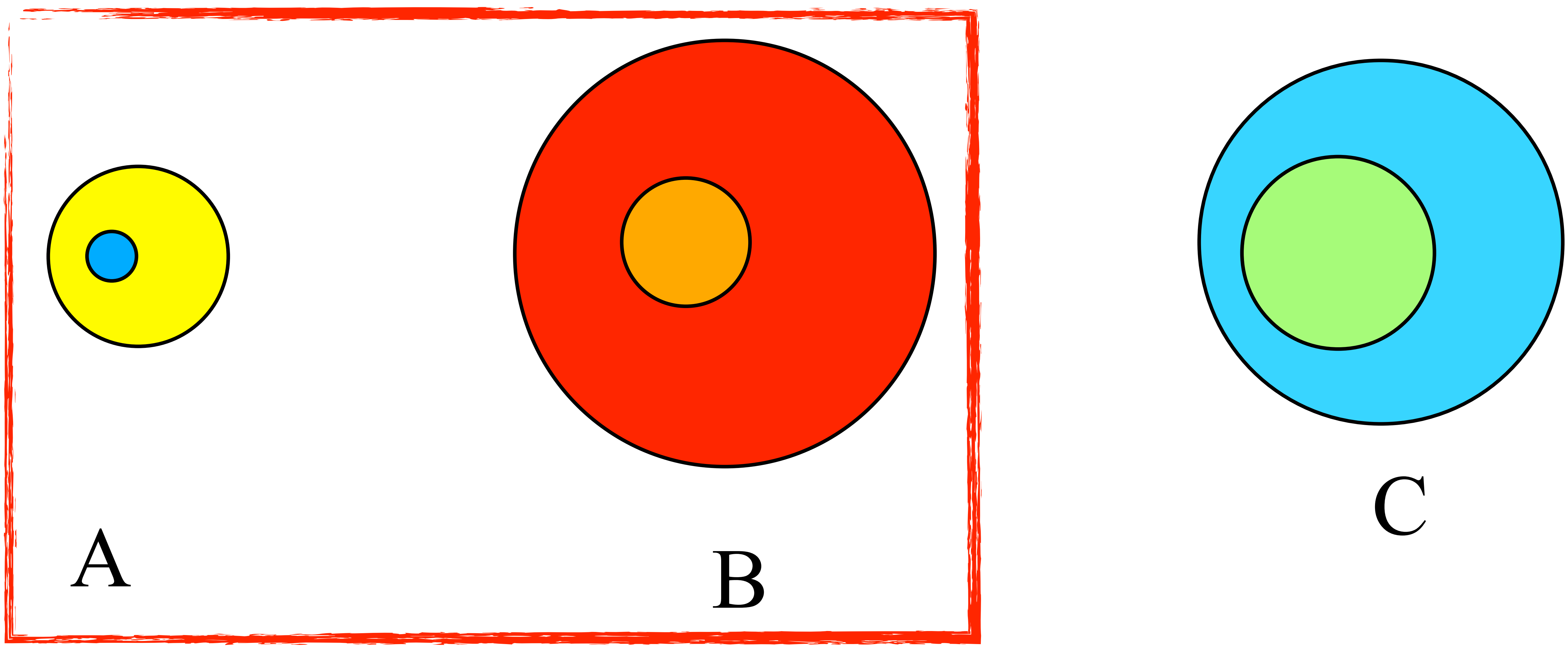

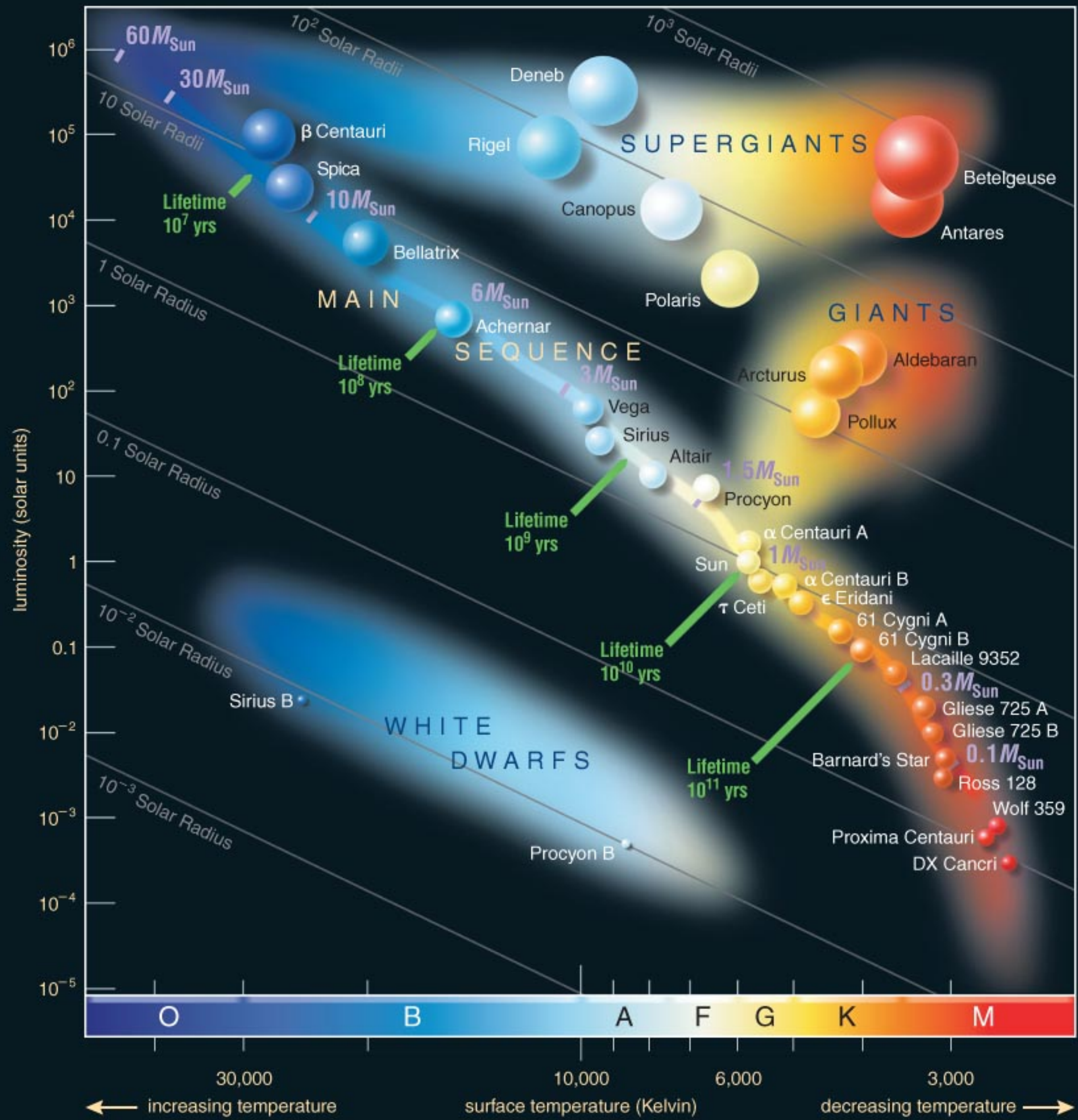

luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
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Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^{10} yrs
Lifetime 10^{11} yrs
SUPERGIANTS
GIANTS
MAIN SEQUENCE
WHITE DWARFS
Deneb
Rigel
Canopus
Polaris
Betelgeuse
Antares
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Altair
Procyon
Arcturus
Aldebaran
Pollux
α Centauri A
Sun
α Centauri B
ε Eridani
τ Ceti
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature

Sources of Energy

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1. Work done by gravitational contraction:

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Total gravitational binding energy of the Sun:

$$\Omega_\odot = -\frac{GM^2}{R} = -\frac{\left(7\times10^{-8}\ \frac{cm^3}{g\,s^2}\right)\left(2\times10^{33}\ g\right)^2}{7\times10^{10}\ cm} = -4\times10^{48}\ erg$$

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If energy is radiated at the present rate, i.e., the luminosity of the Sun:

$$t_\odot L_\odot = \Omega_\odot \quad\Rightarrow\quad t_\odot \simeq 10^{15}\ sec \simeq 3\times10^{7}\ yr$$

$$L_\odot = 3.9\times10^{33}\ \frac{erg}{s} \qquad\qquad <<< 4.6\times10^{9}\ yr\ (t_{EARTH})$$

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Stars are predominantly H, about 90% of all nuclei. The most abundant source of energy in stellar interior is the fusion of 4H —> He.

$$(4M_H - M_{He})c^2 = 26.73 \text{ MeV}$$
$$4M_H \simeq 4 \times 931 \text{ MeV}$$

$\Rightarrow$ fraction converted $= \dfrac{26.73 \text{ MeV}}{4 \times 931 \text{ MeV}} = 0{,}007$

ie: 7‰

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$$\Rightarrow \frac{2 \times 10^{54}\ \frac{\mathrm{erg}}{\mathrm{s}} \cdot 0{,}007}{4 \times 10^{33}\ \frac{\mathrm{erg}}{\mathrm{s}}} = 3 \times 10^{18}\ \mathrm{s} \simeq 100\ \mathrm{Gyr}\ \text{lifetime}$$

Star Clusters

Star Clusters

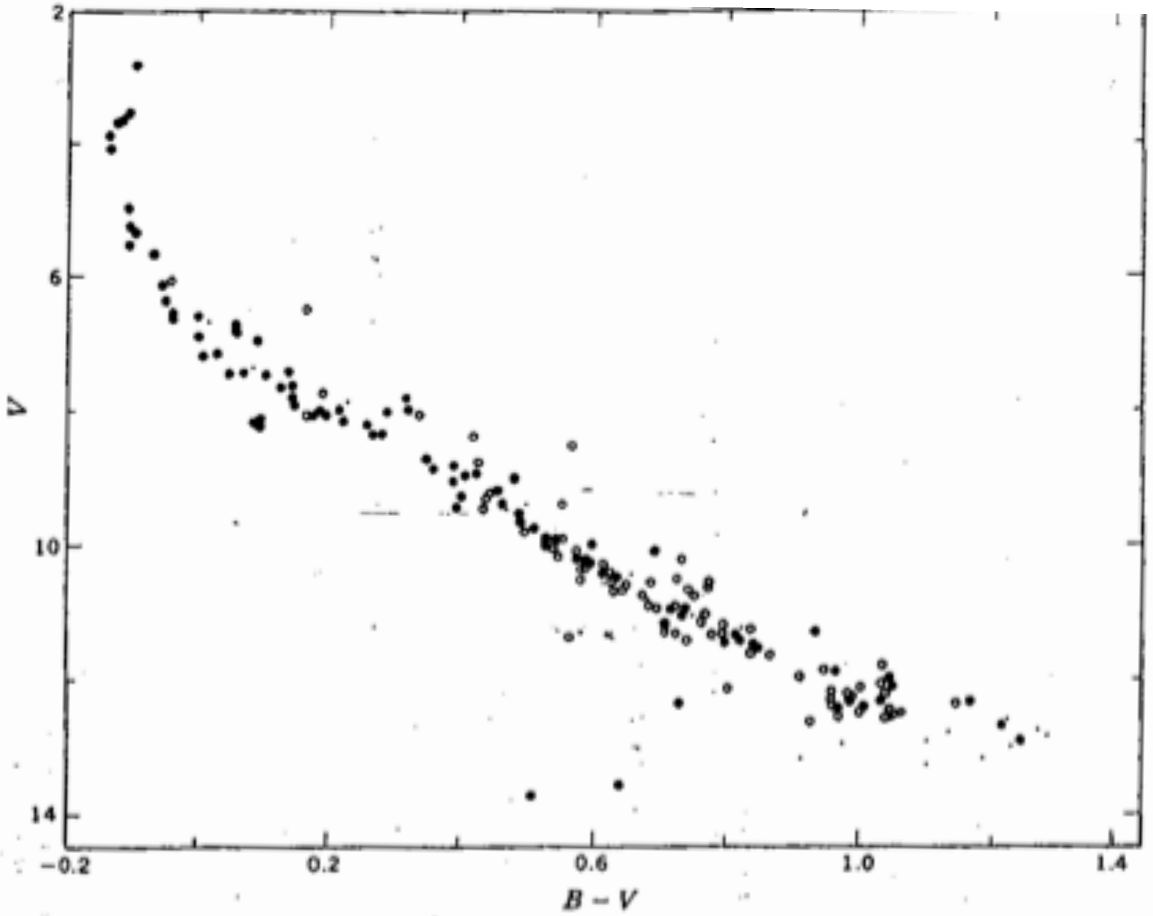
Group of stars with a much stronger gravitational attraction to each other than to the general field stars. The number of stars varies from 10^5 to loose associations of a few stars.

The richest clusters are massive, spheroidal ones with 10^5 stars (**globular clusters**). We measure their distances using apparent magnitudes of RR Lyrae variable stars ($M^{RRLyrae}{}_V=0$). Typical diameters of the high star-density regions is tens of pc, with central density as high as 10^3 stars per pc^3!!

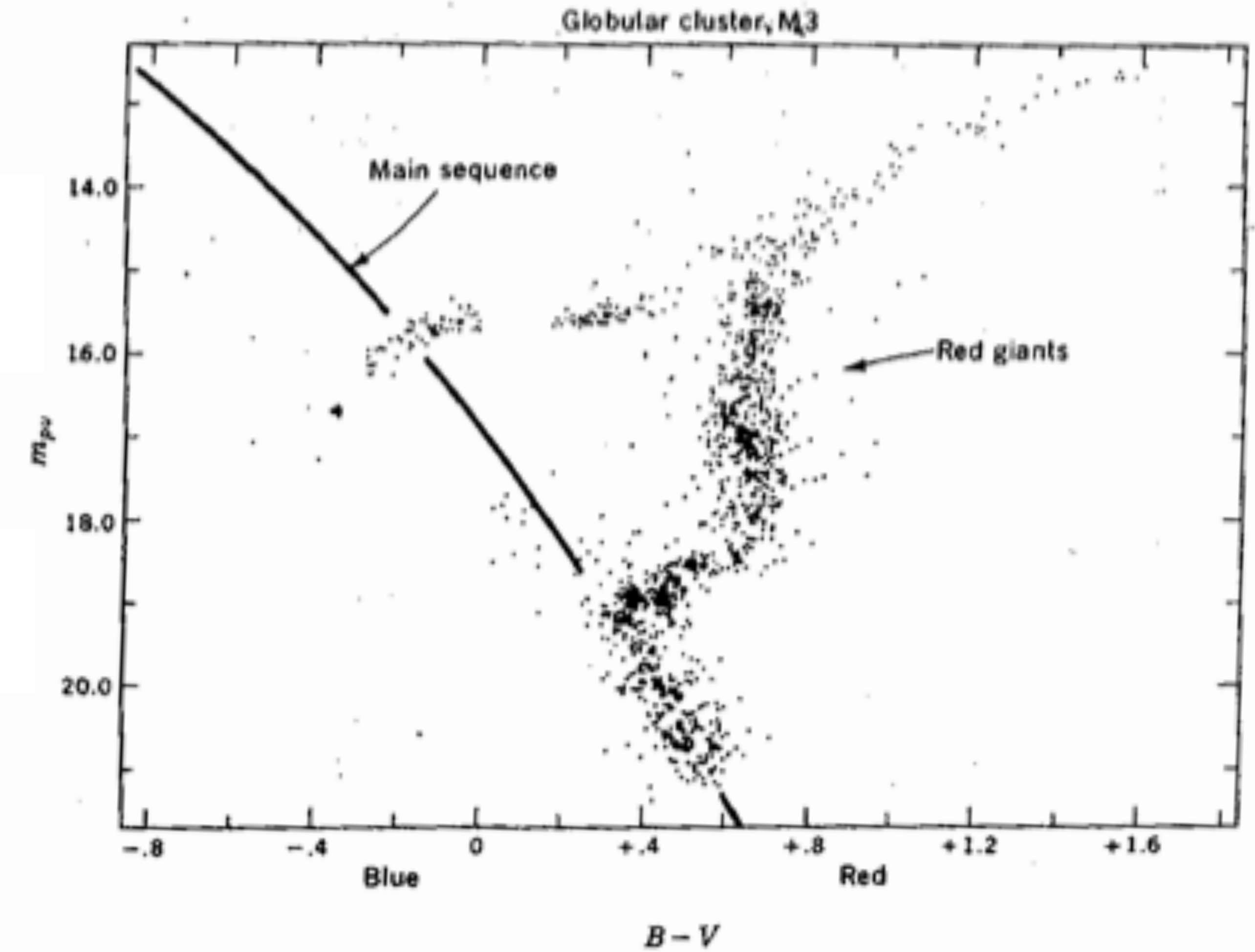
Open clusters are more irregular groupings of a few to a few hundred stars. They are found in the disks of galaxies. **Association**: special type of open cluster with most luminous main sequence being O and B stars (and Wolf Rayet stars) over dimension of ~100 pc.

Pop I stars: relative young stars with blue giants as most luminous member
Pop II stars: old stars with red giants as most luminous member
Pop III stars: very first generation of stars

Pleiades - open cluster



M3 - globular cluster



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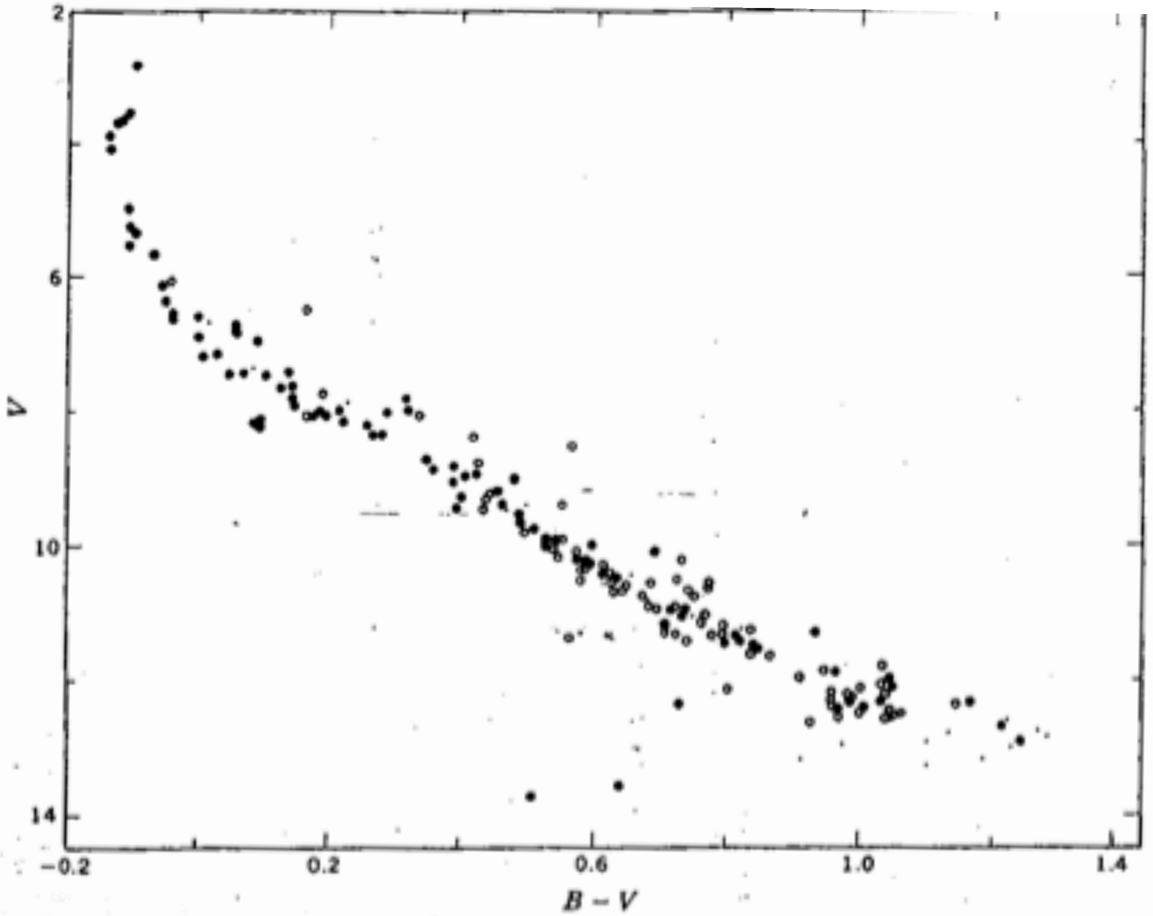

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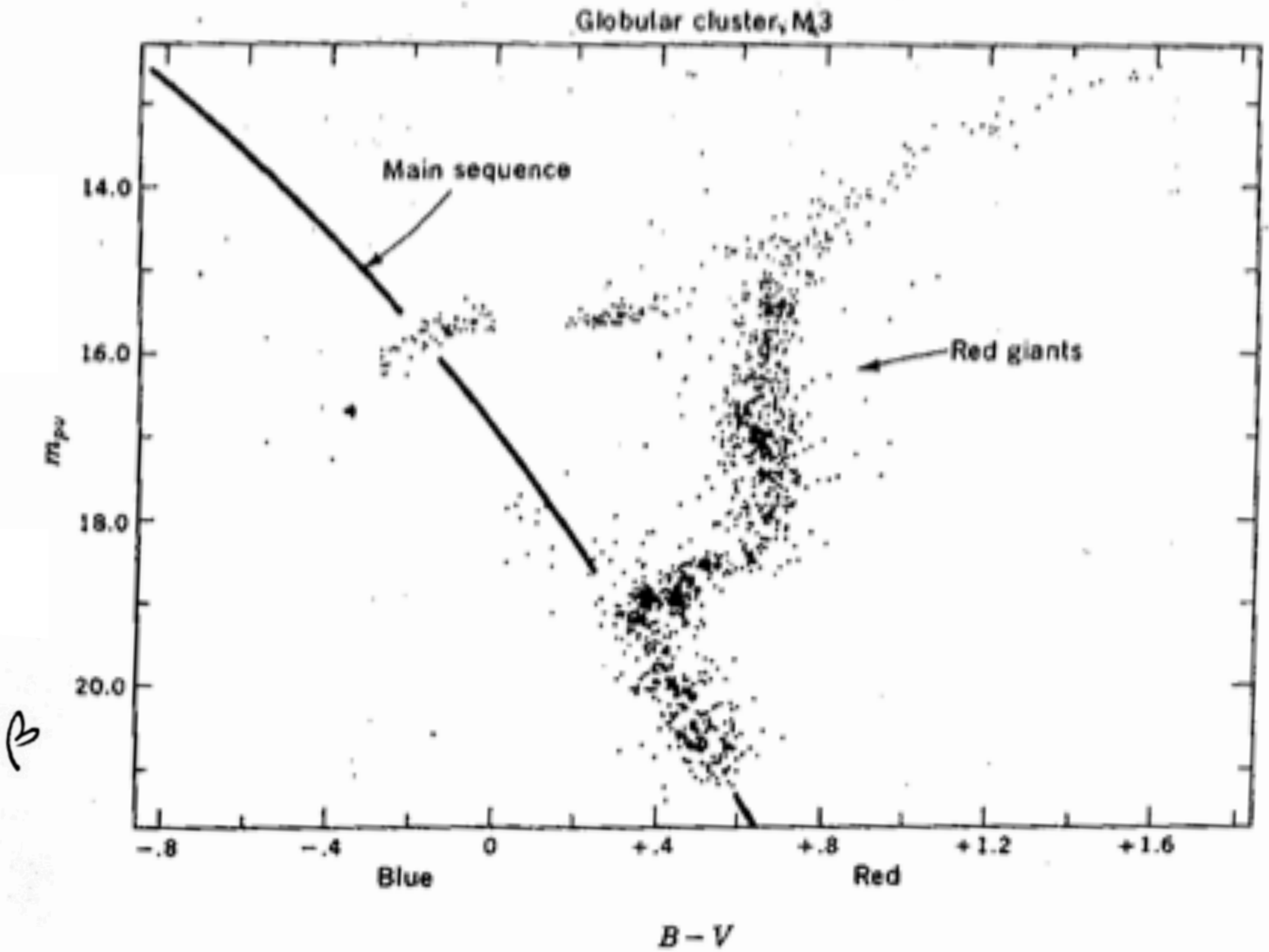
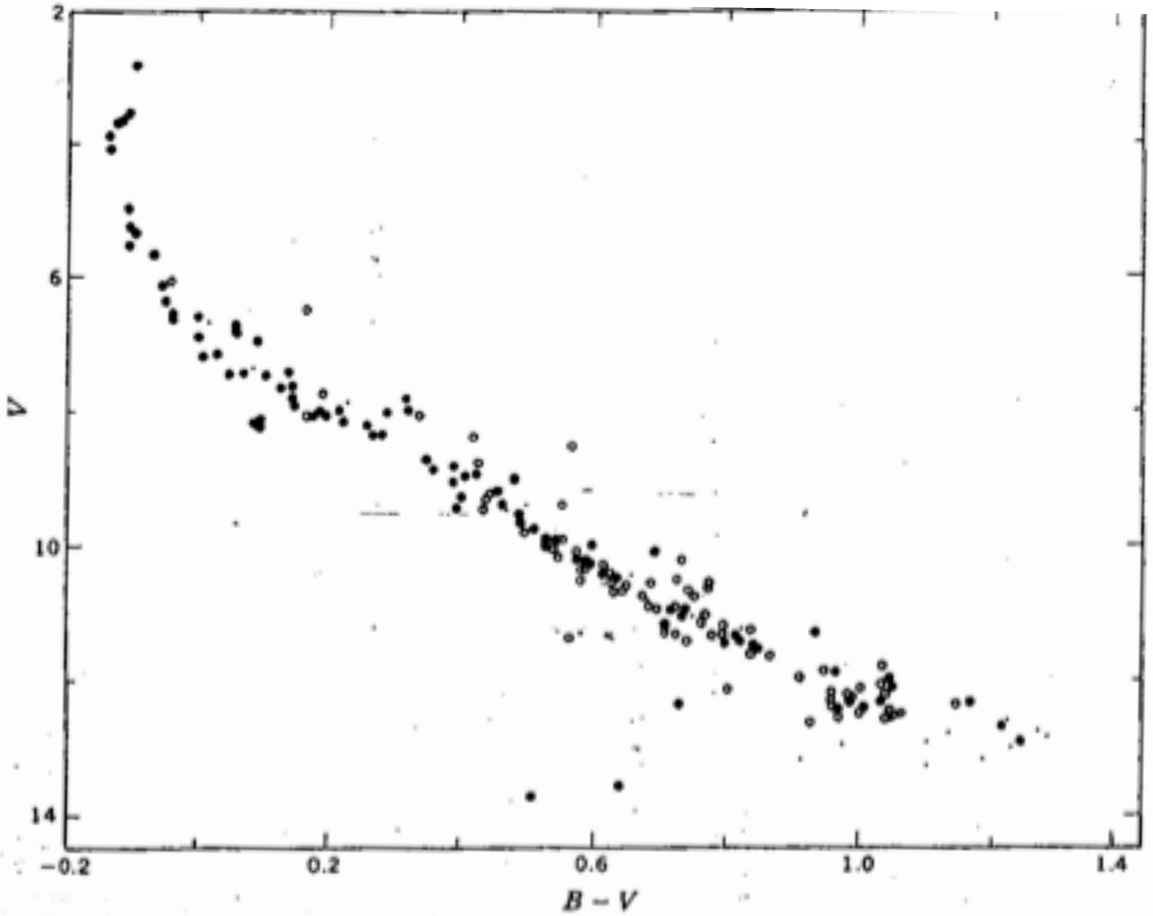


$L \propto M^{\beta} \qquad \beta \in (3.5, 4)$

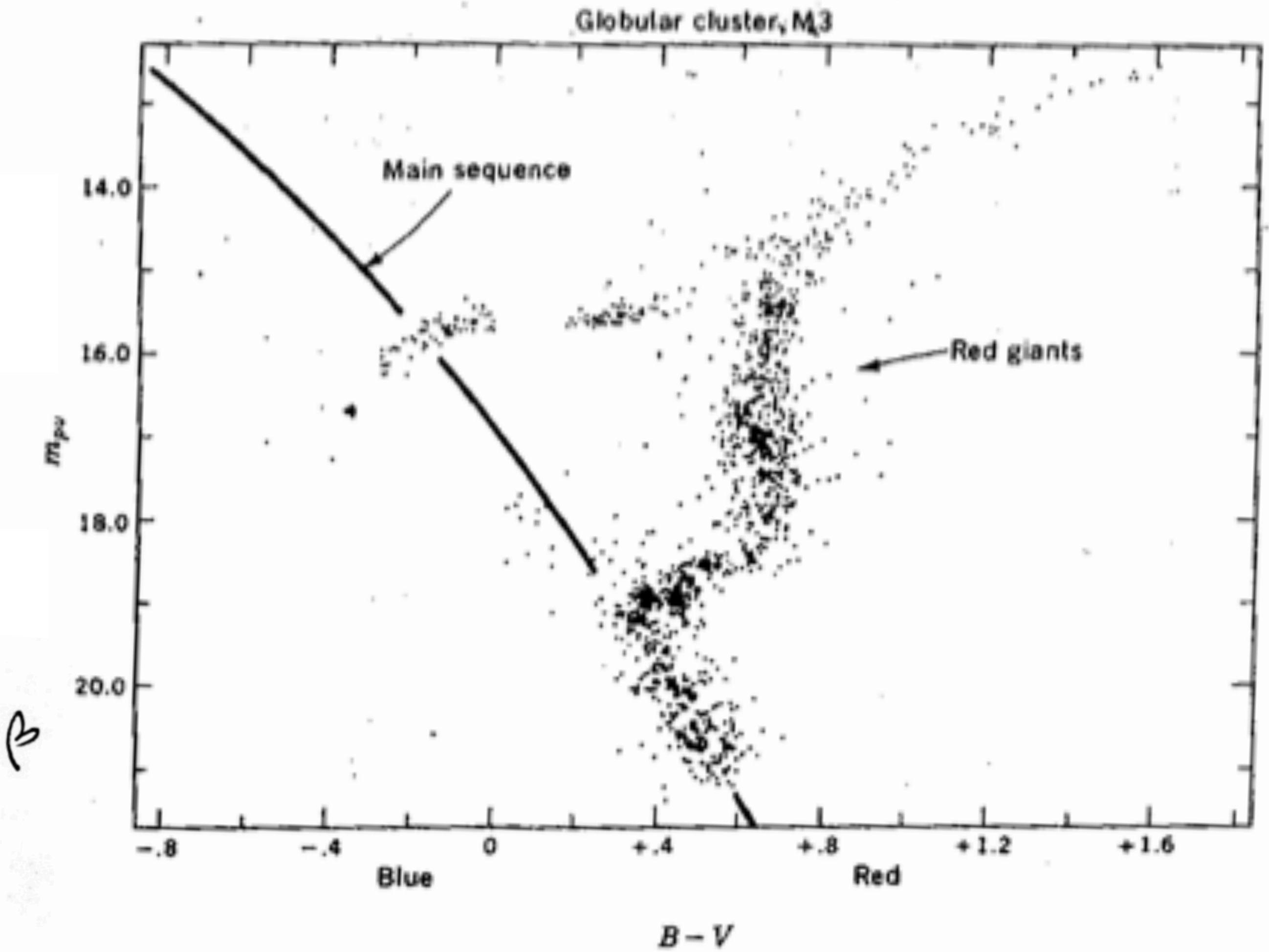
lifetime of star $\propto \frac{M_H}{L}$ const $\propto \frac{M}{L} \propto \frac{M}{M^{\beta}} \propto M^{1-\beta}$

$\Rightarrow \; t \simeq \underbrace{10^{10}\,\mathrm{Gyr}}_{t_\odot} \left(\frac{M}{M_\odot}\right)^{-3}$

Pleiades - open cluster



M3 - globular cluster



$L \propto M^{\beta} \qquad \beta \in (3.5, 4)$

$$\text{lifetime of star} \propto \frac{M_H}{L}\,\text{const} \propto \frac{M}{L} \propto \frac{M}{M^{\beta}} \propto M^{1-\beta}$$

$$\Rightarrow t \simeq \underbrace{10^{10}\ \text{Gyr}}_{t_\odot} \left(\frac{M}{M_\odot}\right)^{-3}$$

Stars form from gas in the interstellar medium (ISM), composed predominantly of H. They contract until T_{core} is high enough for H thermonuclear reactions to begin. At this point, they radiate energy at a rate equal to that liberated by the nuclear reactions. They remain static as long as there is H fuel in the core. When innermost 10-20% of H in the core is exhausted, the outer regions expand and the inner core contracts —> same L but lower $T_{surface}$, hence redder color —> the star evolves off the main sequence and into the giant region. **More massive stars evolve faster off the main sequence.**

Total energy radiated from the star in a lifetime on the main sequence:

$$E = f\, X_H\, M \left(6.4 \times 10^{18} \frac{\text{erg}}{\text{g}}\right) = 1.3 \times 10^{52} f X_H \frac{M}{M_\odot} \text{ erg}$$

Mass of the star

Fraction of H

Fractional mass before H fusion stops in the core (~15%)

Energy released by one reaction 4H—>He

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Fraction of H

Fractional mass before H fusion stops in the core (~15%)

Energy released by one reaction ^{4}H—>He

Lifetime on the main sequence:

$$t_E = \frac{E}{L} = 1.1 \times 10^{11} f X_H \frac{M/M_\odot}{L/L_\odot} \text{ yr}$$

$f \sim 15\%$

$$\Rightarrow \quad t_E = 12 \times 10^9 X_H \frac{M/M_\odot}{L/L_\odot} \text{ yr}$$

BUT $M/M_\odot \simeq (L/L_\odot)^{1/4}$

USING $X_H = 0.6$

$$\Rightarrow t_E \simeq 12 \left(\frac{L}{L_\odot}\right)^{-3/4} \times 10^9 \text{ yr}$$

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The crucial point for determining the age of the cluster lies in determining the absolute magnitude of the turn off point, or the surface temperature of the same point.

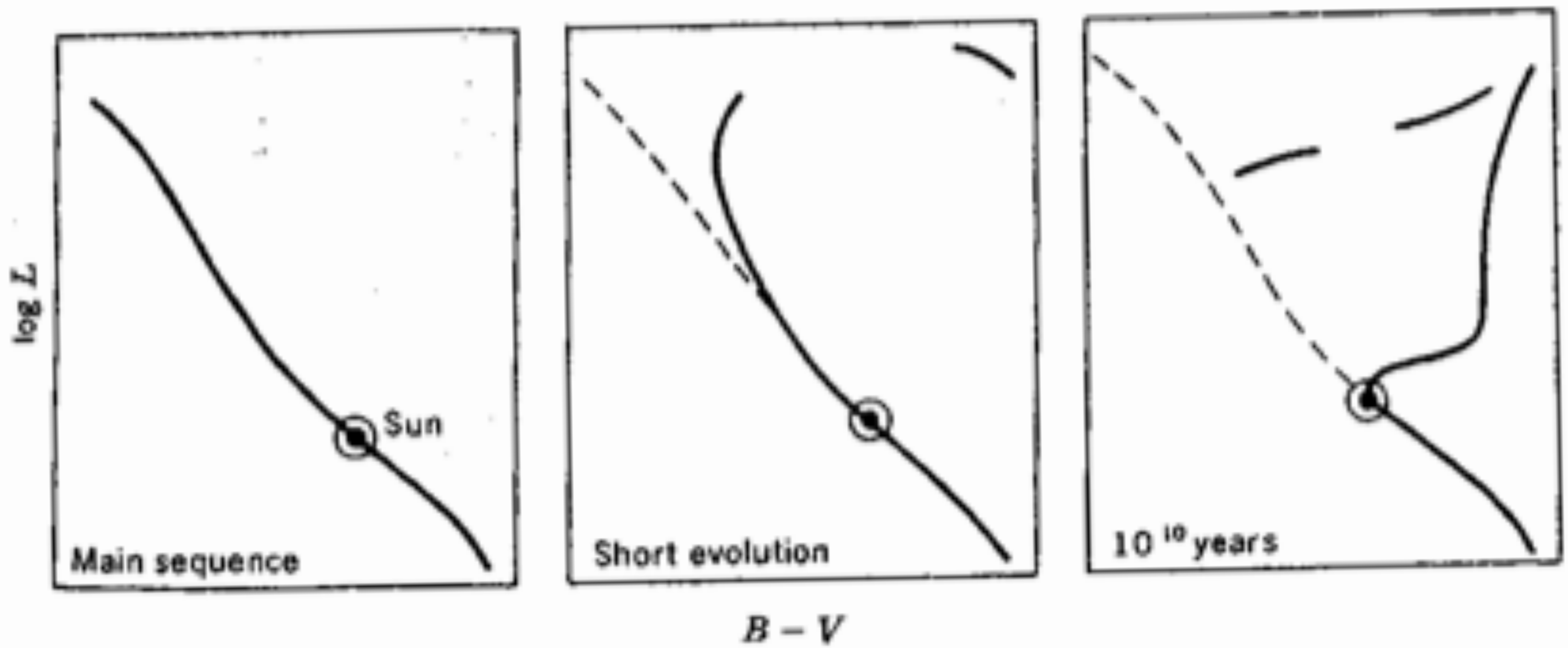

Fig. 1-20 Schematic representation of the H-R diagram of a cluster of stars at three different epochs in its history. After a short period of evolution, say about 10^8 years, the zero-age main sequence has become an evolved main sequence similar to the Pleiades. After a long period of evolution, say about 10^{10} years, the diagram resembles those of the globular clusters. Supergiants and white dwarfs have been omitted from this diagram because their participation is poorly understood.

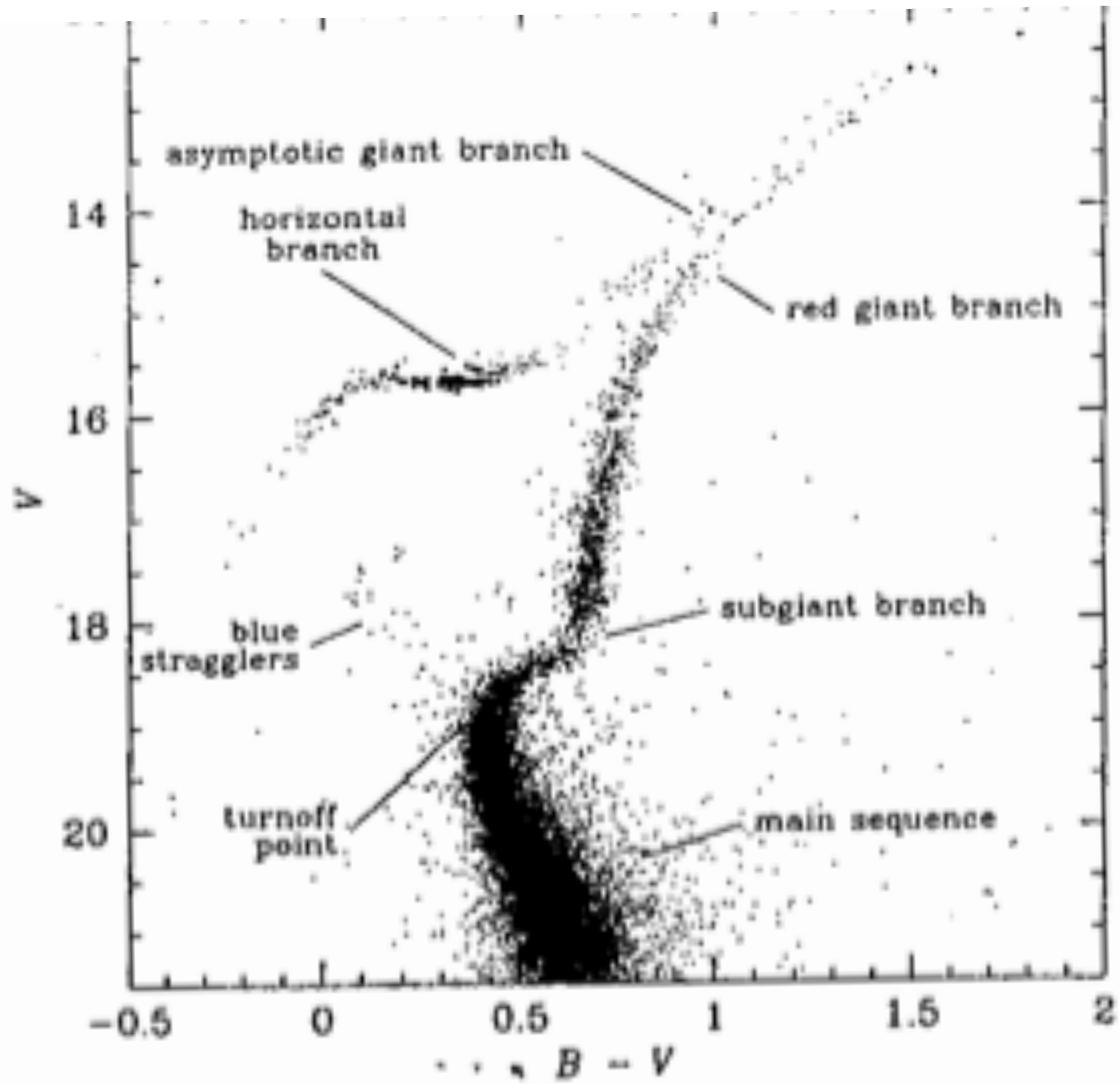

Figure 6.2 The color-magnitude diagram for the globular cluster M3. Known variable stars are shown as open circles, and the principal sequences are annotated. [From data published in Buonanno *et al.* (1994)]

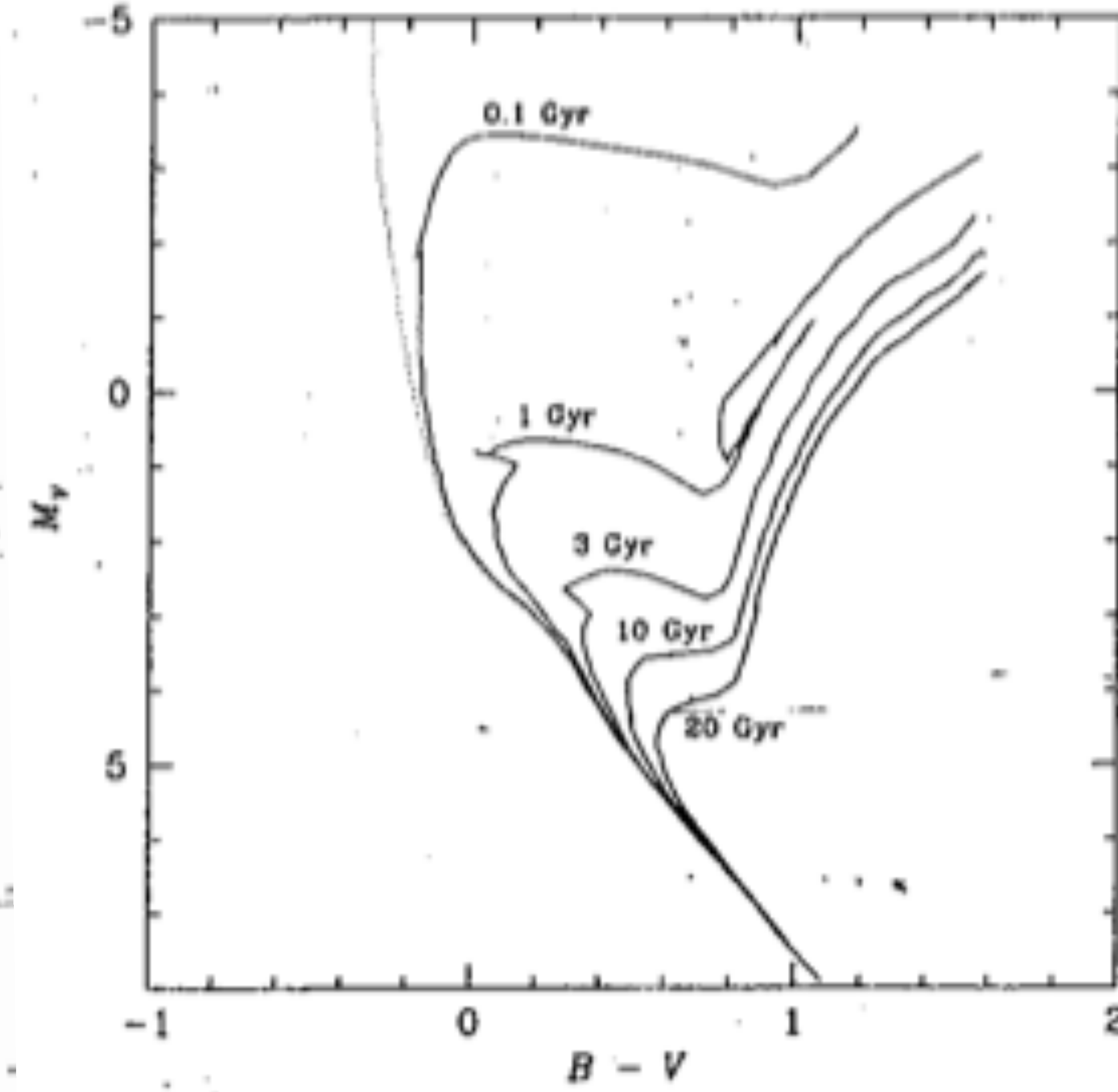

Figure 6.6 Theoretically calculated isochrones showing how a stellar population with $Z = 0.004$, $Y = 0.24$ evolves away from the ZAMS (dotted line) in the CM diagram. Each isochrones is labeled by its age. [From the calculations of Bertelli *et al.* (1994)]

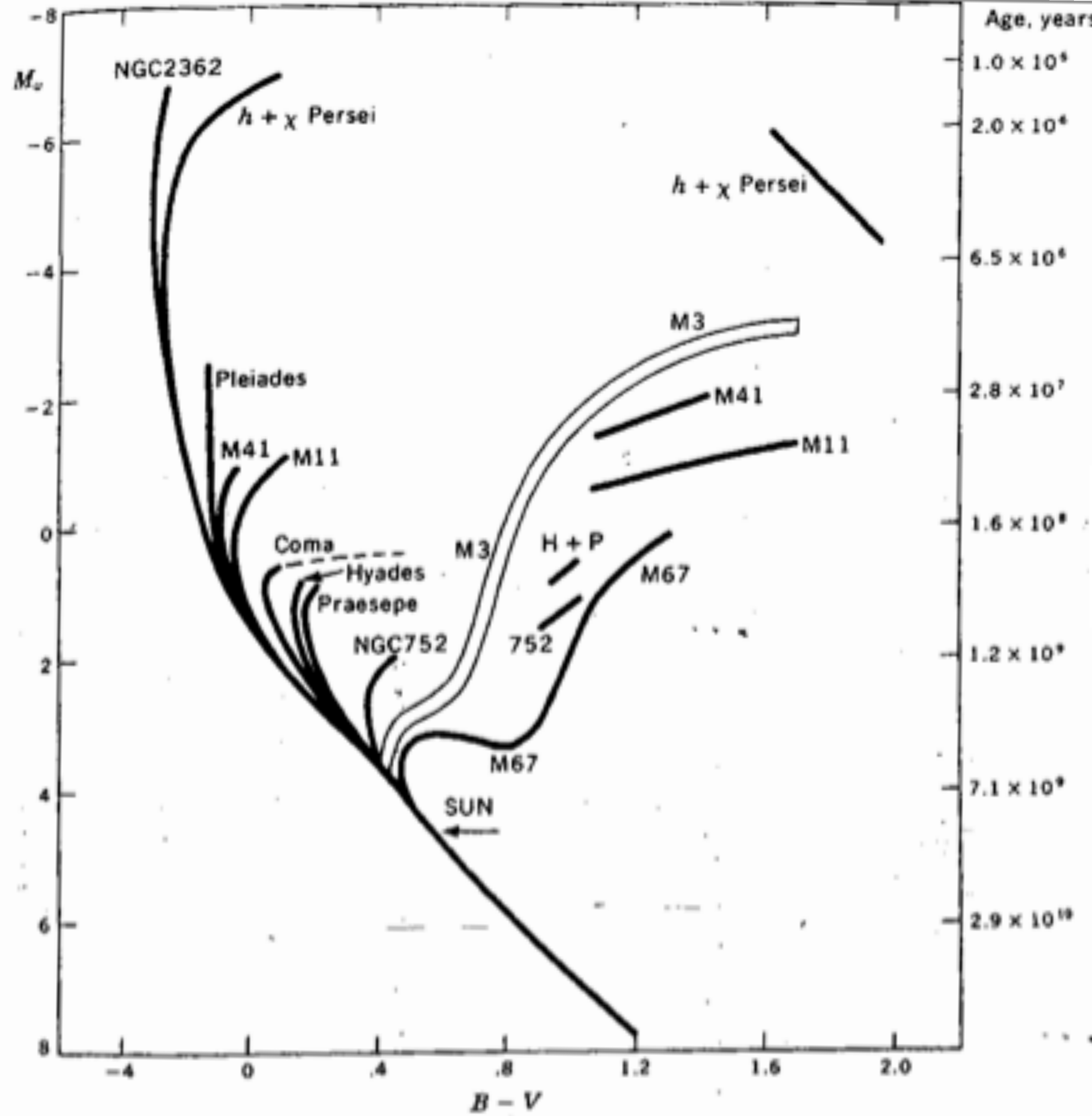

Fig. 1-21 A composite color-magnitude diagram of 10 galactic clusters and 1 globular cluster. Ages corresponding to the various main-sequence termination points are given along the right-hand ordinate. The zero-age main sequence is taken to be the blue envelope of the observed sets of main-sequence stars. Notice the rapidly evolved red giants in $h + \chi$ Persei, which are apparently no more than 2 million years old. Some white dwarfs are known in the Hyades, indicating that it is possible to form them in a few million years, either directly or as the end product of the evolution of upper-main-sequence stars. Curiously enough, the Hyades has no red giants. The oldest galactic cluster, M 67, is older than the sun and has scores of white dwarfs. Many fascinating problems are uncovered in the attempts to interpret the star densities in these diagrams quantitatively. [*After A. Sandage, Astrophys. J.*, **125**:435 (1957). *By permission of The University of Chicago Press. Copyright* 1957 *by The University of Chicago.*]

Question

When would you receive the least amount of light from a binary star system consisting of an M5 Red Giant and an M5 main sequence star?

(A) When the Red Giant is in front of the main sequence star

(B) When the Red Giant is behind the main sequence star

(C) You would receive the same amount of light from both situations described in choices “A” and “B”

(D) None of the above

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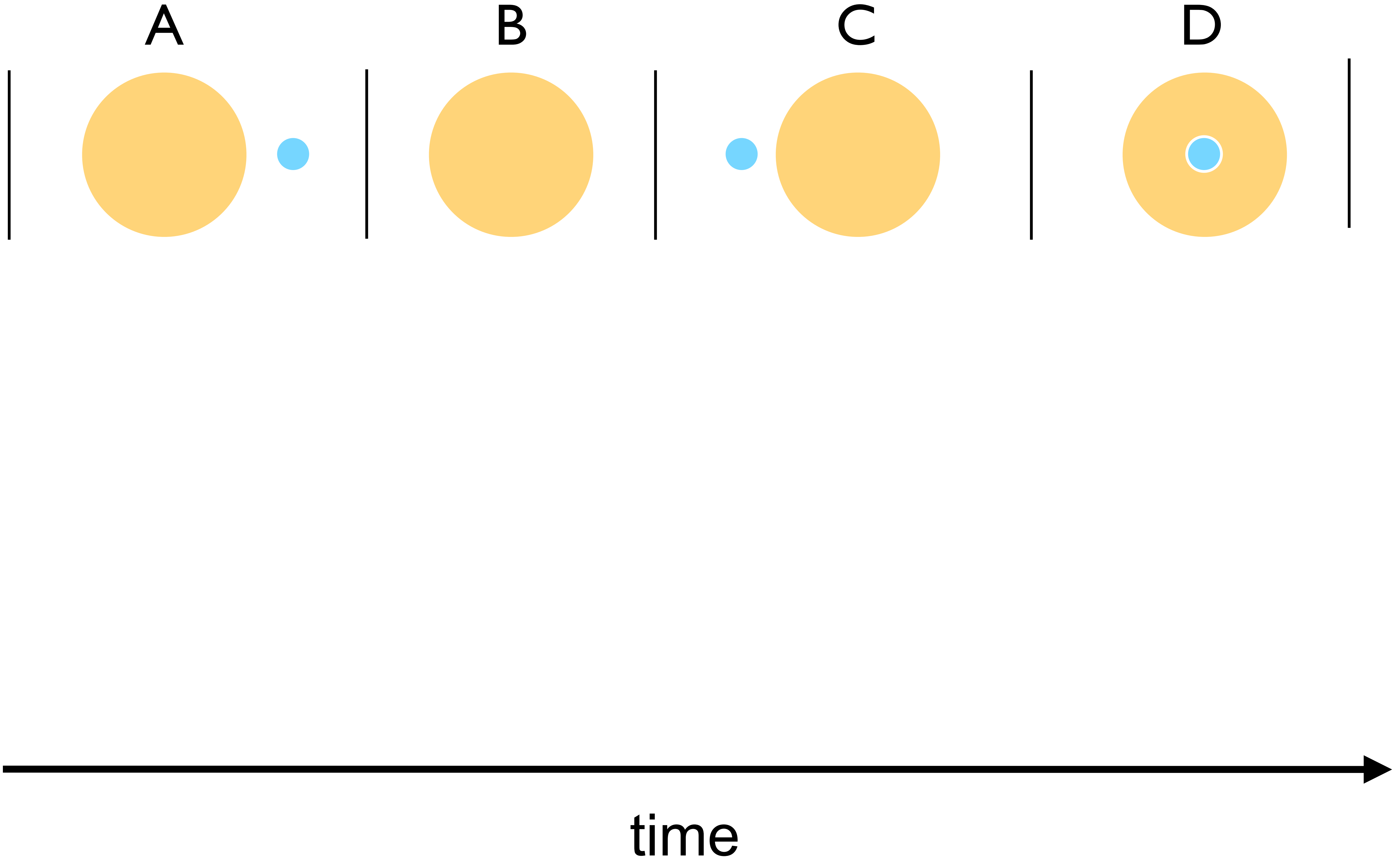
http://astro.unl.edu/classaction/animations/binaryvariablestars/eclipsingbinarysim.html



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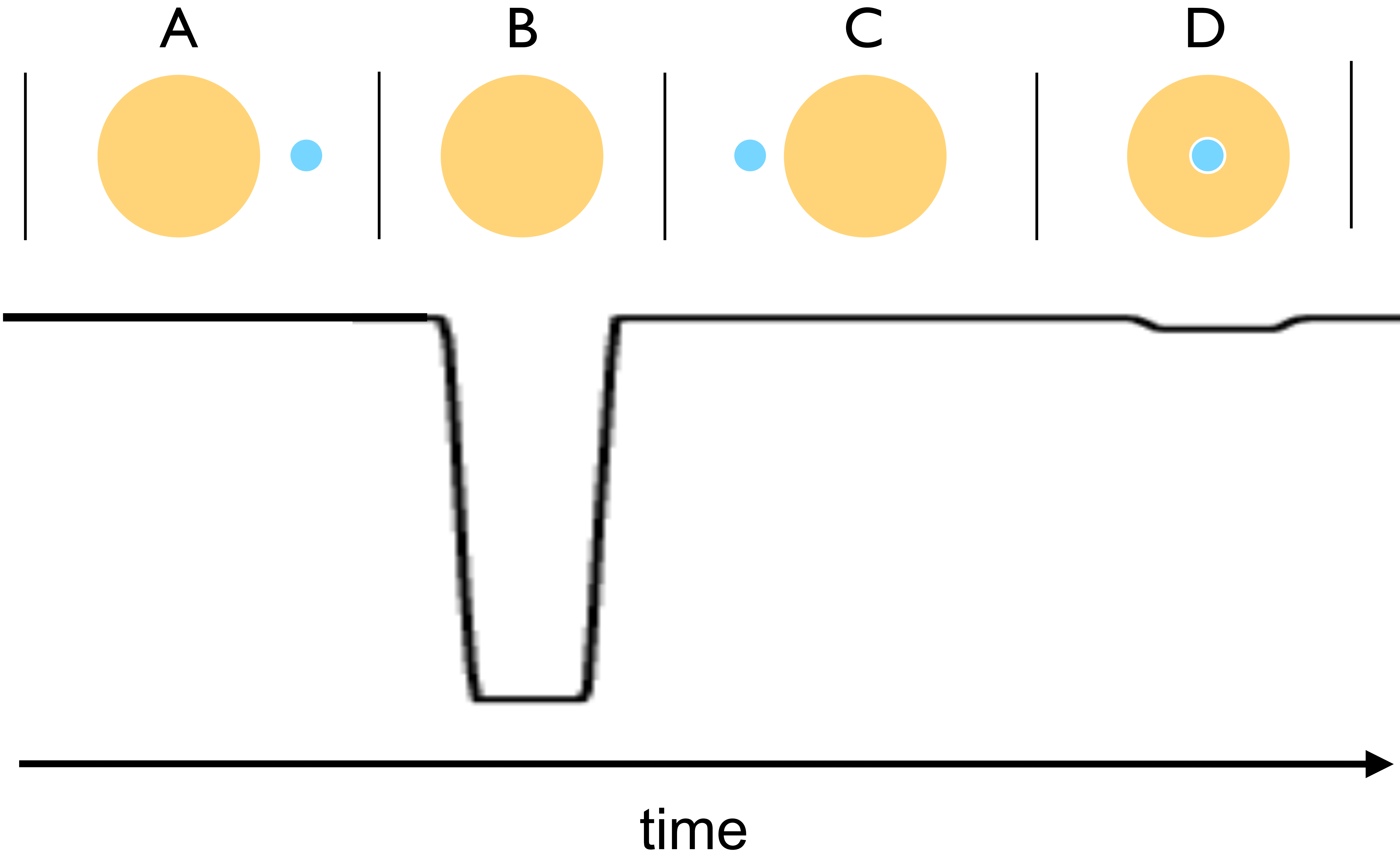

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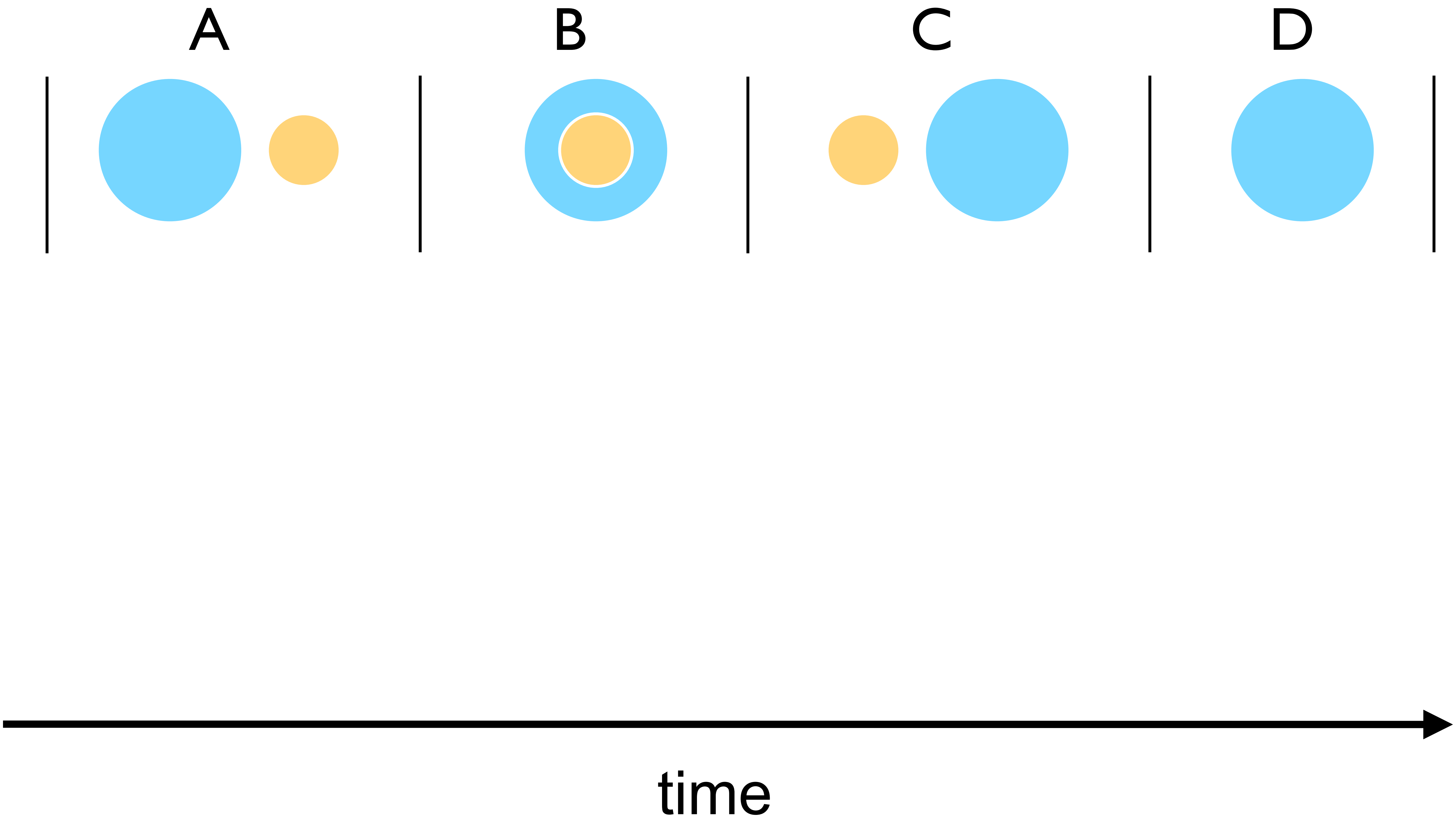

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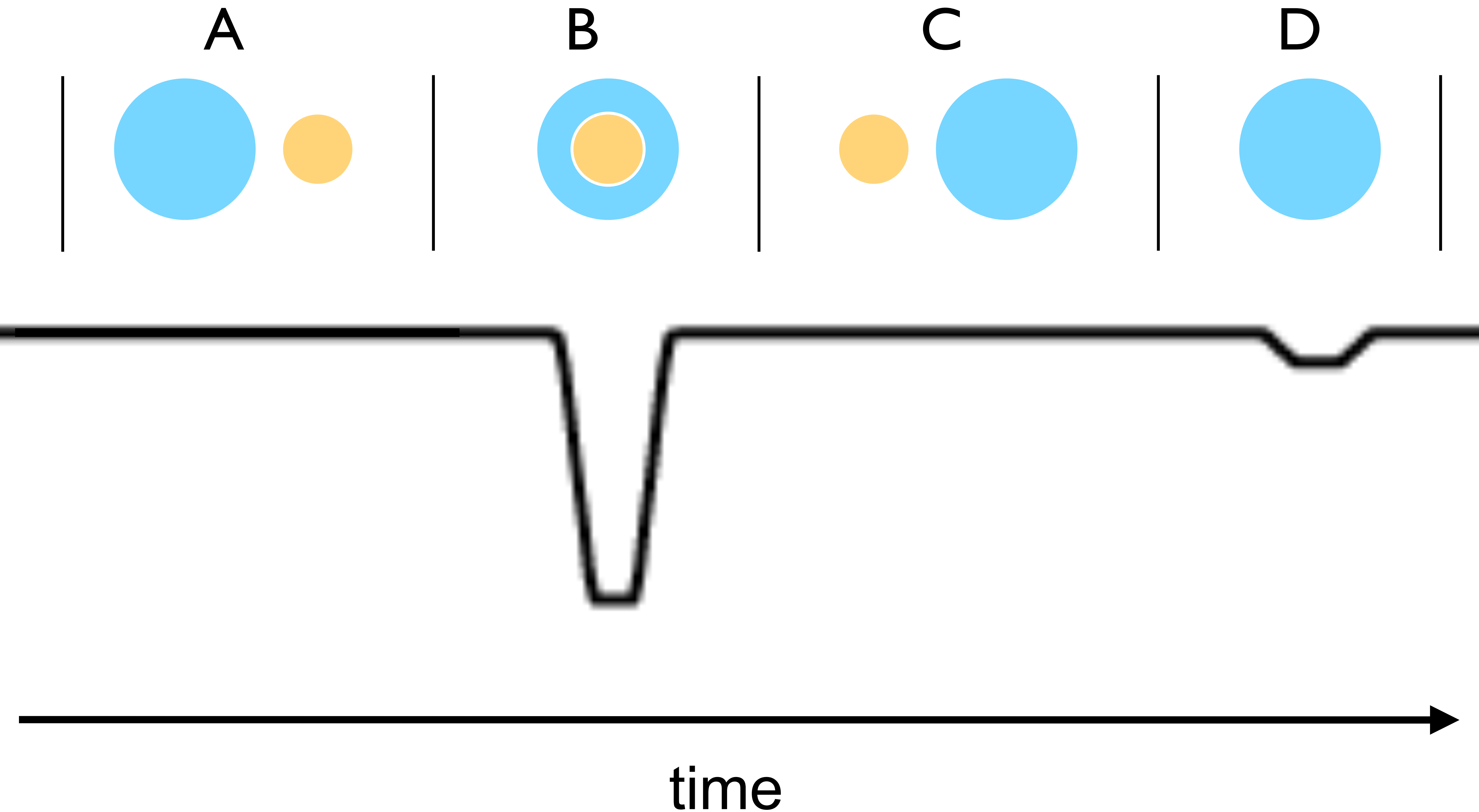

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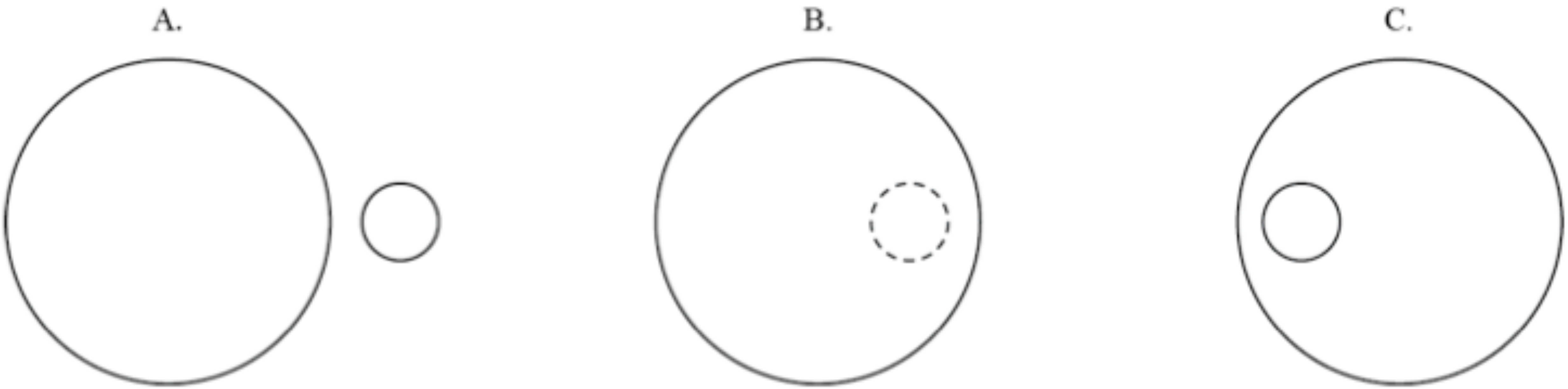

The sketches illustrate how two main sequence stars might look at three different times. In which case would the amount of light we would observe from Earth be the least?

(A) at time A
(B) at time B
(C) at time C
(D) at more than one of the times
(E) there is not enough information to determine this

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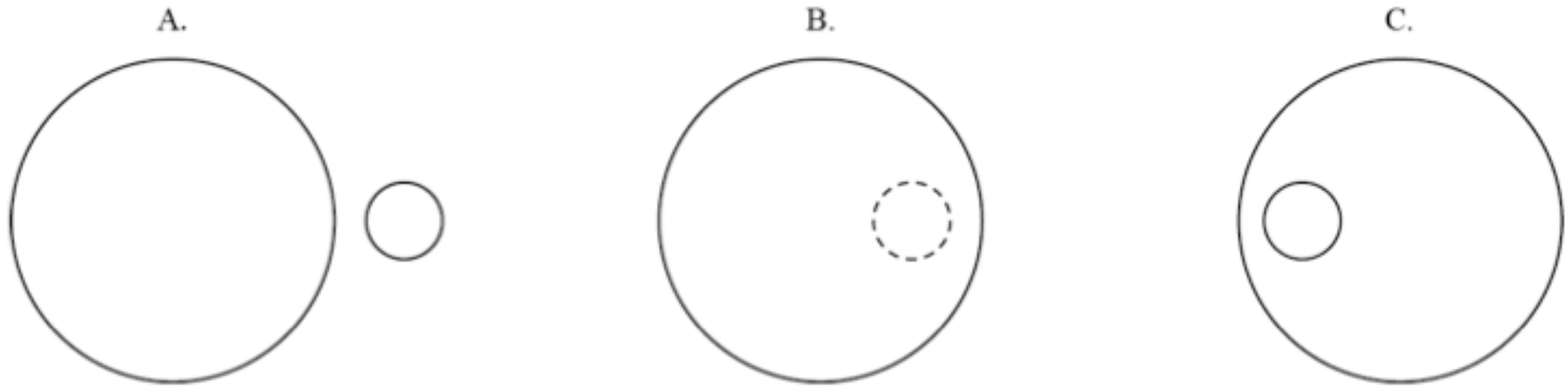

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VIRIAL THEOREM

Let's assume a system which is a gravitationally bound system in equilibrium; let's assume hydrostatic equilibrium (for a spherically symmetric gas).

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$$\frac{dP}{dr} = -\rho\frac{GM(r)}{r^2}$$

mass density

Equation of hydrostatic equilibrium, directly from mechanic equilibrium:

$$\rho\ddot{r} = -\frac{GM(r)\rho}{r^2} - \frac{dP}{dr} = 0$$

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$$V(r)\, dP = -\frac{1}{3} 4\pi r^2 \rho\, dr \frac{GM(r)}{r} = -\frac{1}{3} \frac{GM(r)}{r} dM$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

$$\rightarrow \underbrace{\int_0^R V(r)\, dP}_{} = -\frac{1}{3}\int_0^M \frac{GM_r}{r}\, dM$$

$$\underbrace{\int_0^R d(P.V)}_{PV\big|_0^R = 0 \text{ since}} - \int_0^R P\, dV \Rightarrow -\int_0^R 3P\, dV = \underbrace{-\int_0^M \frac{GM_r}{r}\, dM}_{\text{definition gravitational potential energy } U}$$

$P(R) = 0$

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For non-relativistic particles in a gas $p = \frac{2}{3}k$

w/ k internal kinetic energy density,

ie: $K = k \cdot V$

For relativistic particles, $p = \frac{1}{3}k$

$$\Rightarrow \begin{aligned} -3\int \frac{2}{3} k\, dV &= -2K \quad \text{for non-relativistic particles} \\ -3\int \frac{1}{3} k\, dV &= -K \quad \text{for relativistic} \quad " \end{aligned}$$

$$\Rightarrow \boxed{2K + U = 0}$$

VIRIAL THEOREM for non-relativistic particles

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Rewriting the virial theorem: $$K = -\frac{1}{2}U$$

i.e., only half of the change in the gravitational potential energy (i.e., of a contracting cloud of gas) goes into kinetic energy (which increases the internal temperature), whereas the other half has to be radiated away by the contracting cloud.