The Classification of Stellar Spectra

Reading assignment:

TUESDAY 10/6: Chapters 8.2 (carefully) and 2.4 (don't worry about demonstration, focus on concept)

Homework Assignment #2 Due by TUESDAY 10/6 ECLIPSING BINARIES to determine radii and ratios of temperatures: if the smaller star is completely eclipsed by the larger one, a nearly constant minimum will occur



The dip in the light curve is deeper when the hotter star is behind its companion, from the Stefan-Boltzmann law and from the fact that the same total cross-sectional area is eclipsed.

 $F = O T_{eff}^{4}$ [erg s⁻¹ cm⁻²]



When both stars are visible:

If the smaller star is also the hotter, when the hotter star is entirely eclipsed:

When the cooler star is behind: f secondary = K (TR Range - TR Imall) Flarge + KTR Remail Finall dip Area of larger star minus

Area of larger star minus the area eclipsed by the smaller star

i.e., from the measurements of the light curve of the system, one can determine the ratio of temperatures of the two stars

 $L = 4\pi 7^{2} \sigma T_{eff}^{\dagger}$ From it, we can measure R once T_{eff} and L

are measured from independent techniques.

MEASURABLES: L, T_{surface}, R, M spectral features —> chemical composition at the stellar surface

==> theory of stellar structure and stellar evolution

Five concepts of stellar temperature for the photosphere (surface of star):

- 1. Effective temperature T_{eff}
- 2. Color temperature T_{col}
- **3.** Kinetic temperature T_{kin}
- 4. Excitation temperature T_{exc}
- 5. Ionization temperature T_{ion}

1. Effective temperature T_{eff}

Temperature of a blackbody with the same radiated power per unit of area



2. Color temperature T_{col}

Temperature of a blackbody with the same ratio of specific fluxes at two different wavelengths

$$\frac{F_{\lambda_{1}}}{F_{\lambda_{2}}} = \left(\frac{\lambda_{2}}{\lambda_{1}}\right)^{5} \frac{\frac{hc}{2}kr}{\frac{hc}{\lambda_{1}kr}} = \frac{f_{\lambda_{1}}}{f_{\lambda_{2}}}$$

$$\frac{F_{\lambda_{2}}}{\frac{hc}{\lambda_{1}kr}} = \frac{f_{\lambda_{1}}}{f_{\lambda_{2}}}$$

$$\frac{f_{\lambda_{2}}}{\frac{hc}{\lambda_{1}}}$$

$$\frac{f_{\lambda_{1}}}{\frac{hc}{\lambda_{2}}}$$

$$\frac{f_{\lambda_{1}}}{\frac{hc}{\lambda_{2}}}$$

$$\frac{f_{\lambda_{2}}}{\frac{hc}{\lambda_{2}}}$$

$$\frac{f_{\lambda_{1}}}{\frac{hc}{\lambda_{2}}}$$

$$\frac{f_{\lambda_{2}}}{\frac{hc}{\lambda_{2}}}$$

$$B-V = -0.586 + \frac{6850}{T_c EK} + f(T_c)$$

3. Kinetic temperature T_{kin}

Maxwell-Boltzmann distribution of velocities

Ratio between the kinetic energy mv²/2 and the thermal energy KT: low probability for particles to be much away from KT



OF PARTICLES IN THE GAS DER UNIT OF VOLUME W/ SPEEDS BETWEEN V & V4 dv

Maxwell-Boltzmann distribution of velocities





MAXWELL-BOLTZMANN EQ.





Peak of the distribution, i.e., most probable speed:

$$\frac{dn(v)}{dv} = 0 \quad \longrightarrow \quad V_{\text{peak}} = \sqrt{\frac{2KT}{M}}$$

Average speed:

$$V_{AVERAGE} := \frac{\int_{0}^{\infty} \sqrt{N(r)} dr}{N}$$

Average of v^2 : $\sqrt{2}$: = $\frac{1}{n} \int_{0}^{\infty} \sqrt{2} n(v) dv \Rightarrow \sqrt{ms} = \sqrt{\sqrt{2}}$

$$T_{KIN} = \frac{m V_{ms}^2}{3K} K$$

Kinetic temperature



MAXWELL-BOLTZMANN EQ.





 $I_{n} = \int_{0}^{\infty} x^{n} e^{-\frac{2}{3}x^{2}} dx$ $I_{n} = \int_{0}^{\infty} x^{n} e^{-\frac{2}{3}x^{2}} dx = \int_{0}^{2} 2I_{n} \quad \text{if nevel}$ $\int_{0}^{+\infty} x^{n} e^{-\frac{2}{3}x^{2}} dx = \int_{0}^{2} 0 \quad \text{if nod}$ The 121 1/1/33 n even 33 1/25 4 6



4. Excitation temperature T_{exc}

The atoms of a gas fain and loose energy as they collide

$\frac{N_b}{N_o} = \frac{g_b}{g_a} = \frac{-(E_b - E_a)/kT}{g_a}$	Boltzmann's Equation for the atoms of a given element in a specific state of ionization
No # OF ATOMS W/ ENERGY ED No # OF ATOMS W/ ENERGY ED Do # OF STRATES W/ ENERGY ED gb # OF STRATES W/ ENERGY ED gb	A STATISTICAL WEIGHTS b JOE THE ENERGY LEVELS

For Hydrogen: g_n=2n²

T_{exc} defined using the Boltzmann's Eq.

Example: given a gas of neutral H atoms (HI), find the temperature for which the number of atoms having electrons in the ground state (n=1) and the number of atoms having electrons in the first excited state (n=2) are the same.

For Hydrogen: g_n=2n²

 $N_1 \iff N=1$ $N_2 \iff N=2$ $\Rightarrow N_1 = N_2$ $\equiv N_1 = -\frac{13.6 \text{ eV}}{N^2}$ $N_2 \iff N=2$ $\frac{N_2}{N_1} = 1 = \frac{2(2)^2}{2(1)^2} - \left[\frac{(-13.6 eV}{2^8} - (-13.6 eV/1) \right] / KT$ K = 8.61 73324 × 10-5 eV/K Boltzmann constant $e_{\mu}(\frac{1}{4}) = \frac{-10.2 \text{ eV}}{\text{KT}} \rightarrow T = \frac{-10.2 \text{ eV}}{9.673324 \times 10^{-5} \text{ eV/k}} \frac{1}{9.44}$ T = 85383°K ~ 8.5 × 104°K

> High temperatures are required for a significant number of H atoms to have electrons in the first excited state



5. Ionization temperature T_{ion}

One must also consider the relative number of atoms in different stages of ionization

$$\frac{N_{i+4}}{N_{i}} = \frac{2Z_{i+1}}{N_{e}Z_{i}} \left(\frac{2\pi M_{e}KT}{h^{2}}\right)^{3/2} - \chi_{i}/KT$$

Saha's Equation



Meghanand Saha and his equation below: $\frac{N_{II}}{N_{I}} = \frac{2Z_{II}}{n_{e}Z_{I}} \left(\frac{2\pi m_{e}kT}{h^{2}}\right)^{3/2} e^{-\chi/kT}$

N: # OF ATOMS IN STAGE iN: # OF ATOMS IN STAGE i+1N: # OF ATOMS IN STAGE i+1N: # OF ATOMS IN STAGE i+1STAGES

recombine

$$\begin{array}{l} P_{e} \text{ pressure of free electrons} \\ \text{(ideal gas law)} \\ P_{e} = \mathcal{N}_{e} | \mathcal{LT} \\ P_{e} = \mathcal{N}_{e} | \mathcal{LT} \\ \hline \mathcal{N}_{e} = \mathcal{N}_{e} | \mathcal{LT} \\ \hline \mathcal{N}_{e} = \frac{2 \mathcal{KT} \mathcal{Z}_{i+1}}{\mathcal{N}_{i}} \left(\frac{2 \mathcal{TT} \mathcal{M}_{e} | \mathcal{LT}}{\mathcal{N}_{e}} \right)^{3/2} - \mathcal{X}_{i} / \mathcal{KT} \\ \hline \mathcal{N}_{i} = \frac{2 \mathcal{KT} \mathcal{Z}_{i+1}}{\mathcal{P}_{e} \mathcal{Z}_{i}} \left(\frac{2 \mathcal{TT} \mathcal{M}_{e} | \mathcal{LT}}{\mathcal{N}_{e}} \right)^{2/2} e^{-\mathcal{X}_{i} / \mathcal{KT}} \end{array}$$

Example: Let's combine the effect of Boltzmann's and Saha's equations. Let's consider a star whose atmosphere is composed purely by H and with $P_e=20 \text{ N/m}^2$. We can derive the fraction of atoms that are ionized, i.e.: $N_{II}/N_{tot} = N_{II}/(N_I+N_{II})$ as a function of temperature in [5000,25000] K.

$$\begin{aligned} \Xi_{II} = 1 \\ \Xi_{2} - \Xi_{1} = 10. \ \& eV \implies KT \quad for T < 25, \infty \ \& \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{2} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{1} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{1} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{1} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{1} - \Xi_{1})/KT \qquad ==> \text{ from Boltzmann's Eq., nearly} \\ \Rightarrow \quad = (\Xi_{1} - \Xi_{1})/KT \qquad = (\Xi_{1}$$



Let's look at the Hydrogen Balmer lines (m=2, n=3, 4, 5, 6, ...): the strength of the Balmer lines in absorption depends on N_2/N_{tot} , with N_2 the number of H atoms in the first excited state. This is obtained with the combination of the Boltzmann's and Saha's equations. Because wirtually all HI atoms are in either the ground or the first excited states, $N_1+N_2\sim N_1$.

$$= \frac{N_2}{N_{TOT}} = \frac{N_2}{N_1 + N_2} \xrightarrow{N_T}_{N_T + N_T} = \underbrace{\begin{pmatrix} N_a / N_1 \\ I + \frac{N_a}{N_1} \end{pmatrix}}_{N_{T+} + \frac{N_T}{N_T}} \cdot \underbrace{\begin{pmatrix} I \\ I + \frac{N_T}{N_T} \end{pmatrix}}_{FROM} \cdot \underbrace{FROM}_{FROM} \cdot FROM$$



NOTE: Stellar atmospheres are not composed of pure H, and the results depend on P_e . In stellar atmospheres, there is He for every ten H atoms. The presence of ionized He provides more electrons with which H ions can recombine. With He, it takes a higher T to achieve the same degree of H ionization.



to make upper transitions => no Balmer lines

excited on the 1st excited level => Balmer lines

ionized, so no electrons => no Balmer lines

Strength of various spectral lines with temperature



Tion: the ionization temperature is obtained from the distribution of atoms in the ionized levels from the Saha's equation. By comparing the strengths of the absorption lines of two different states of ionization of the same atom, it is possible to determine the relative number densities of the two stages of ionization of the atom (e.g., H and K lines of Call with the line of neutral Ca @ 422.6 nm).

Taking the log of the Saha's equation:

$$\frac{\log N_{i+1}}{N_i} = -\frac{5040}{T(42)} \chi_i(eV) + 2.5 \log T - 0.18 + \log \frac{4i+1}{2i} - \log Pe$$

Most important term as a function of temperature, i.e., proportional to T^{2.5}

The one-parameter Harvard classification of spectral type is directly explained by the above, since the dominant dependence is with temperature. However, P_e is also to be considered to properly characterize stars with ionized atoms in their atmospheres. P_e is smaller in cooler stars than in hotter stars, but I can consider it a constant at first approximation for main-sequence stars on the H-R diagram.

From $N_{i+1}/N_i \longrightarrow T$, P_e Vice versa, if I know T and P_e , I can estimate N_{i+1}/N_i , i.e., the abundance of elements.

Spectral Types of Stars



Relative flux (arbitrary units)



Strength of various spectral lines with temperature and spectral type

As T changes, a smooth variation from one spectral type to the next occurs, indicating that there are only minor differences in the chemical composition of stars

OBAFGKM (from hotter to cooler): **Oh B**eautiful **And Fine Girl/Guy Kiss Me**

	TABLE 8.1 H	larvard Spectral Classification.
	Spectral Type	Characteristics
	0	Hottest blue-white stars with few lines
		Strong He II absorption (sometimes emission) lines. TE 50000 K
		He I absorption lines becoming stronger.
	в	Hot blue-white
		He I absorption lines strongest at B2. (T= 22000 K)
		H I (Balmer) absorption lines becoming stronger.
	A	White ,9500K T211000K
		Balmer absorption lines strongest at A0, becoming weaker later.
		Ca II absorption lines becoming stronger.
	F	Yellow-white Ty 7600 K
	-	Ca II lines continue to strengthen as Balmer lines continue to weaken.
		Neutral metal absorption lines (Fe I, Cr I).
	G	Yellow
		Solar-type spectra. $T \simeq 6000 \text{k}$
		Ca II lines continue becoming stronger.
		Fe I, other neutral metal lines becoming stronger.
	к	Cool orange , T= 5250k
		Ca II H and K lines strongest at K0, becoming weaker later.
		Spectra dominated by metal absorption lines.
	м	Cool red
		Spectra dominated by molecular absorption bands.
		especially titanium oxide (TiO) and vanadium oxide (VO).
		Neutral metal absorption lines remain strong.
	ΓL	Very cool, dark red T- 1300 - 2500 K
0		Stronger in infrared than visible.
BROWN		Strong molecular absorption bands of metal hydrides (CrH, FeH), water
DULARES	L	(H ₂ O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs).
UMANO		TiO and VO are weakening.
	Т	Coolest, Infrared TK1300 K
		Strong methane (CH ₄) bands but weakening CO bands.

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