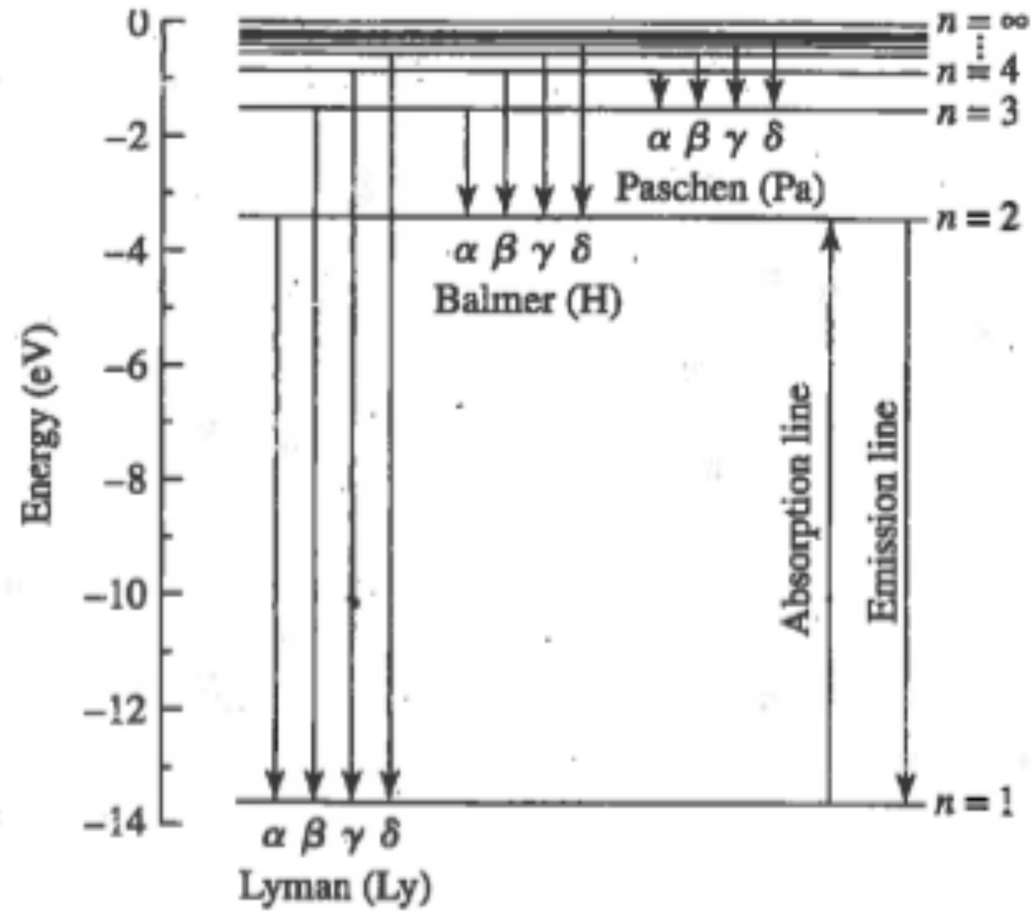
Binary Systems and Stellar Parameters

Reading assignment:
THURSDAY 10/1: Chapters 8.1(quite long)

Homework Assignment #2
Due by TUESDAY 10/6

$$E_{\rm photon} = h\nu = hc/\lambda = pc$$

$$E_{\rm photon} = \Delta E = E_{\rm high} - E_{\rm low} = -13.6eV\left(\frac{1}{n_{\rm high}^2} - \frac{1}{n_{\rm low}^2}\right)$$



$$\lambda_{\rm em,abs} = \frac{hc}{E_{\rm photon}}$$

Emission

Absorption

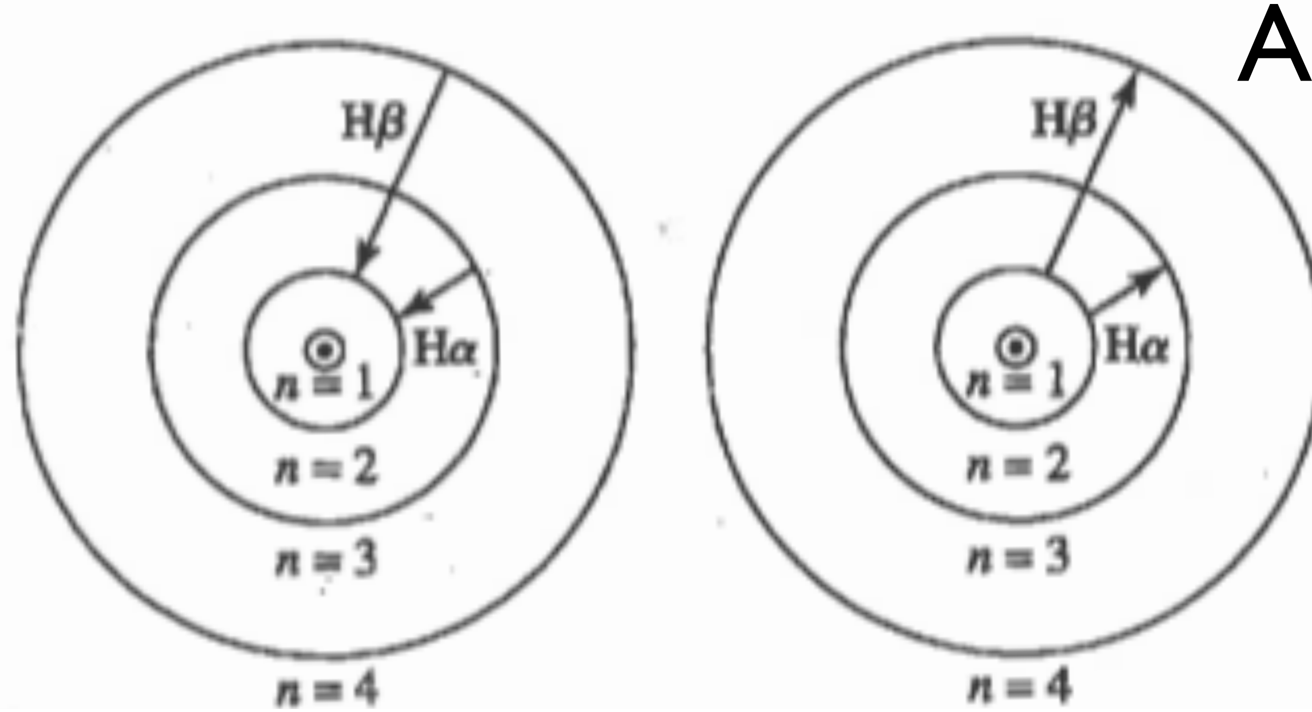


TABLE 5.2 The wavelengths of selected hydrogen spectral lines in air. (Based on Cox, (ed.), *Allen's Astrophysical Quantities*, Fourth Edition, Springer, New York, 2000.)

Series Name	Symbol	Transition	Wavelength (nm)	Medium
Lyman	Lyα	$2 \leftrightarrow 1$	121.567	vacuum
	Lyβ	$3 \leftrightarrow 1$	102.572	vacuum
	Lyγ	$4 \leftrightarrow 1$	97.254	vacuum
	Ly$_{\text{limit}}$	$\infty \leftrightarrow 1$	91.18	vacuum
Balmer	Hα	$3 \leftrightarrow 2$	656.281	air
	Hβ	$4 \leftrightarrow 2$	486.134	air
	Hγ	$5 \leftrightarrow 2$	434.048	air
	Hδ	$6 \leftrightarrow 2$	410.175	air
	Hϵ	$7 \leftrightarrow 2$	397.007	air
	H$_8$	$8 \leftrightarrow 2$	388.905	air
	H$_{\text{limit}}$	$\infty \leftrightarrow 2$	364.6	air
Paschen	Paα	$4 \leftrightarrow 3$	1875.10	air
	Paβ	$5 \leftrightarrow 3$	1281.81	air
	Paγ	$6 \leftrightarrow 3$	1093.81	air
	Pa$_{\text{limit}}$	$\infty \leftrightarrow 3$	820.4	air

TABLE 5.1 Wavelengths of some of the stronger Fraunhofer lines measured in air near sea level. The atomic notation is explained in Section 8.1, and the equivalent width of a spectral line is defined in Section 9.5. The difference in wavelengths of spectral lines when measured in air versus in vacuum are discussed in Example 5.3.1. (Data from Lang, *Astrophysical Formulae*, Third Edition, Springer, New York, 1999.)

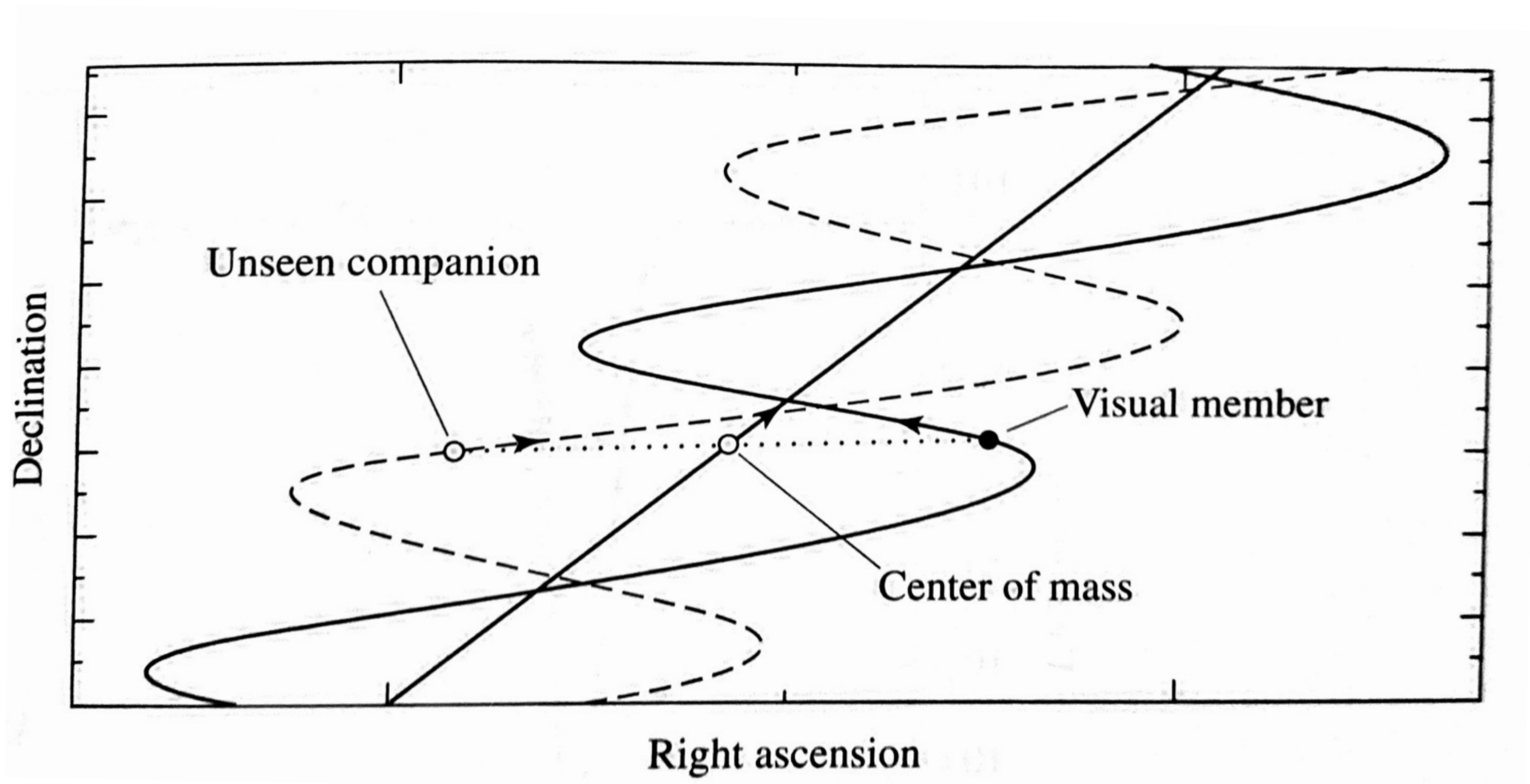
Wavelength (nm)	Name	Atom	Equivalent Width (nm)
385.992		Fe I	0.155
388.905		H_8	0.235
393.368	K	Ca II	2.025
396.849	H	Ca II	1.547
404.582		Fe I	0.117
410.175	h, Hδ	H I	0.313
422.674	g	Ca I	0.148
434.048	G$'$, Hγ	H I	0.286
438.356	d	Fe I	0.101
486.134	F, Hβ	H I	0.368
516.733	b_4	Mg I	0.065
517.270	b_2	Mg I	0.126
518.362	b_1	Mg I	0.158
588.997	D_2	Na I	0.075
589.594	D_1	Na I	0.056
656.281	C, Hα	H I	0.402

Binary Systems and Stellar Parameters

So far, we learned how to measure distance, luminosity, temperature (more to come), and radius. How to we measure the mass of a star?
Half of the stars are in multiple systems, orbiting their common centre of mass.

BINARY STAR SYSTEMS are classified based on observational characteristics.

1. Optical double: not binaries, but two stars lying along the same line of sight - not gravitationally bound
2. Visual binary: both stars in the binary can be resolved and if the orbital period is not too long, the motion of each member can be monitored
3. Astrometric binary: of one member is much brighter, it may outshine the other. The existence of the unseen member is deduced by observing the oscillatory motion of the visible component



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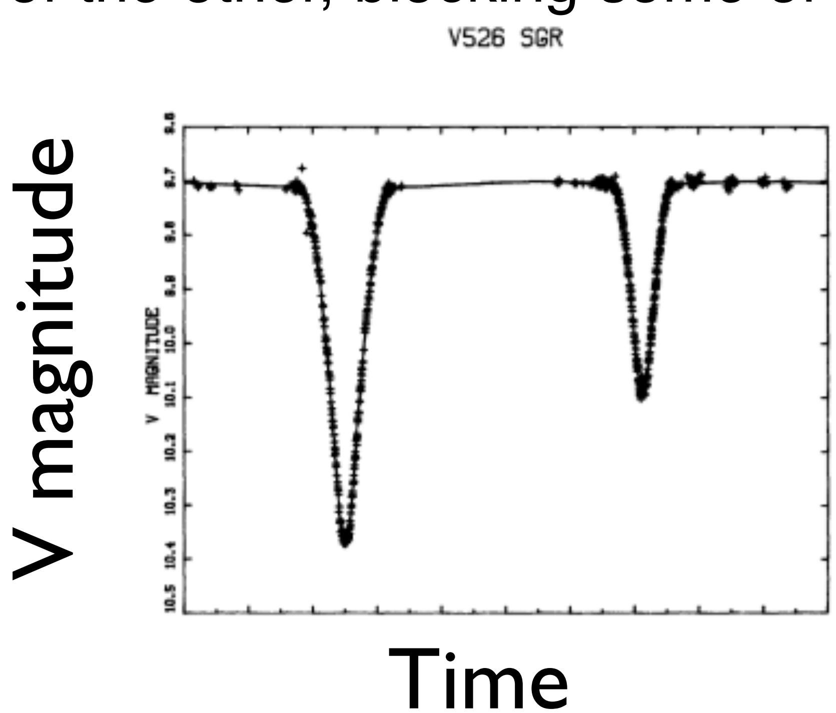
4. Eclipsing binary: when the orbital plane is roughly along the line of sight, one star may periodically pass in front of the other, blocking some of the light resulting in regular variations in the light received.



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(Visual binaries + parallax) OR (Visual binaries for which radial velocities are available over a complete orbit) OR (eclipsing, double-line, spectroscopic binaries) —> MASS

VISUAL BINARIES:
From orbital data —> orientation of the orbits and center of mass can be determined —> ratio of m_1 and m_2
From distance —> linear separation of the stars can be determined —> individual masses

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Consider two stars in orbit about center of mass.

From definition of center of mass, and adopting the center of mass reference system:

$$\mathbf{R} = \frac{m_1\mathbf{r_1} + m2\mathbf{r_2}}{m_1 + m_2} = 0 \qquad \mathbf{r_1} = -\frac{\mu}{m_1}\mathrm{r} \qquad \mathbf{r_2} = \frac{\mu}{m_2}\mathrm{r}$$

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Assuming orbital plane perpendicular to the observer's line of sight, and considering only the lengths of $\mathbf{r_1}$ and $\mathbf{r_2}$:

$$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$$

With a_1 and a_2 semi-major axis of the ellipses

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With angles here can also be in arcsec.

Mass ratio can be determined even if the distance d is not known

From revised Kepler's 3rd law: $$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$a = a_1 + a_2$

SEMI-MAJOR AXIS OF THE ORBIT OF THE REDUCED MASS

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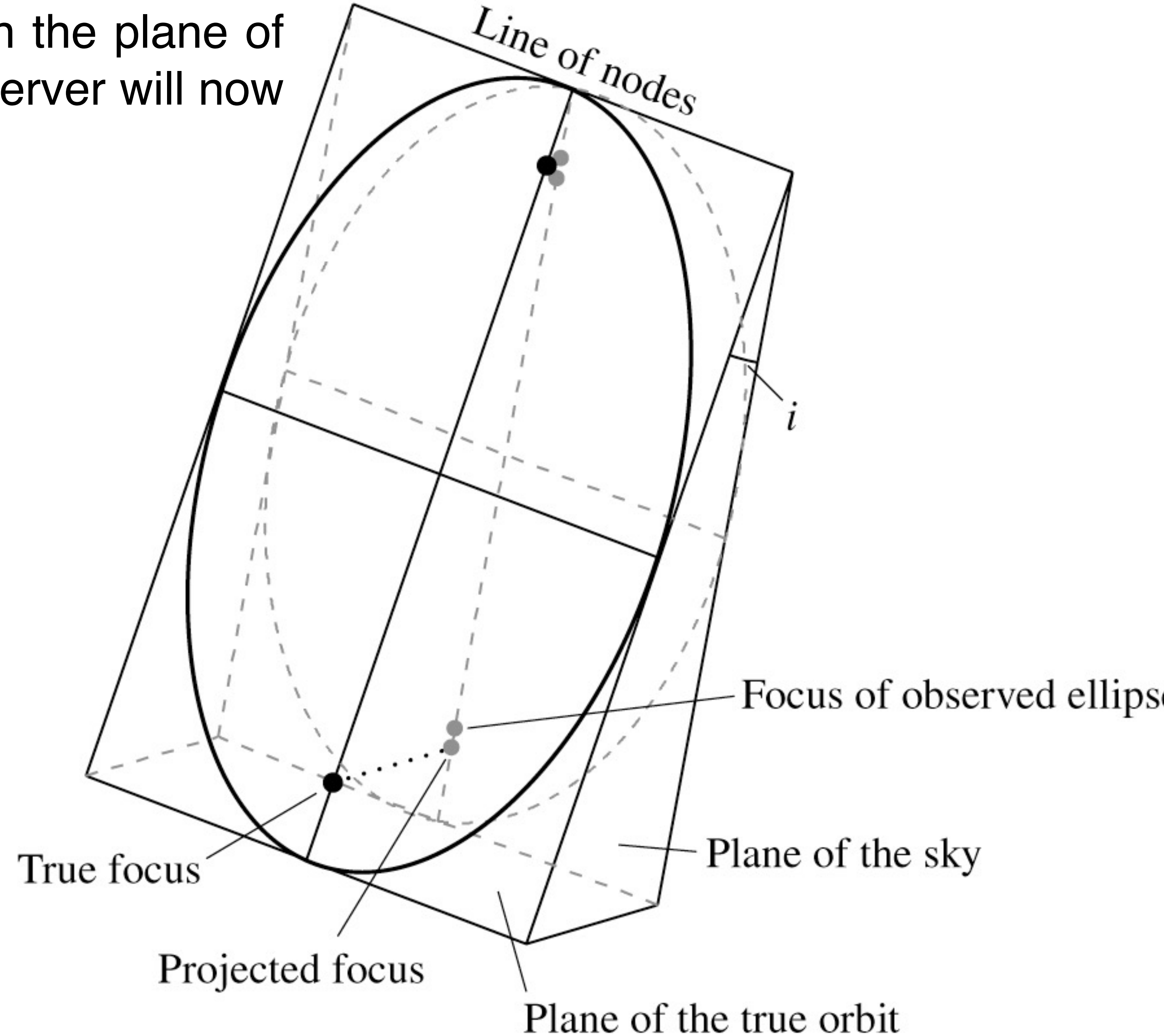
This is complicated by:

1. The proper motion of the center of mass, which moves at a constant velocity
2. The system may not be perpendicular to the line of sight

Let “i” be the angle of inclination between the plane of an orbit and the plane of the sky. The observer will now measure:

$$\tilde{\alpha}_1 = \alpha_1 \cos i$$
$$\tilde{\alpha}_2 = \alpha_2 \cos i$$



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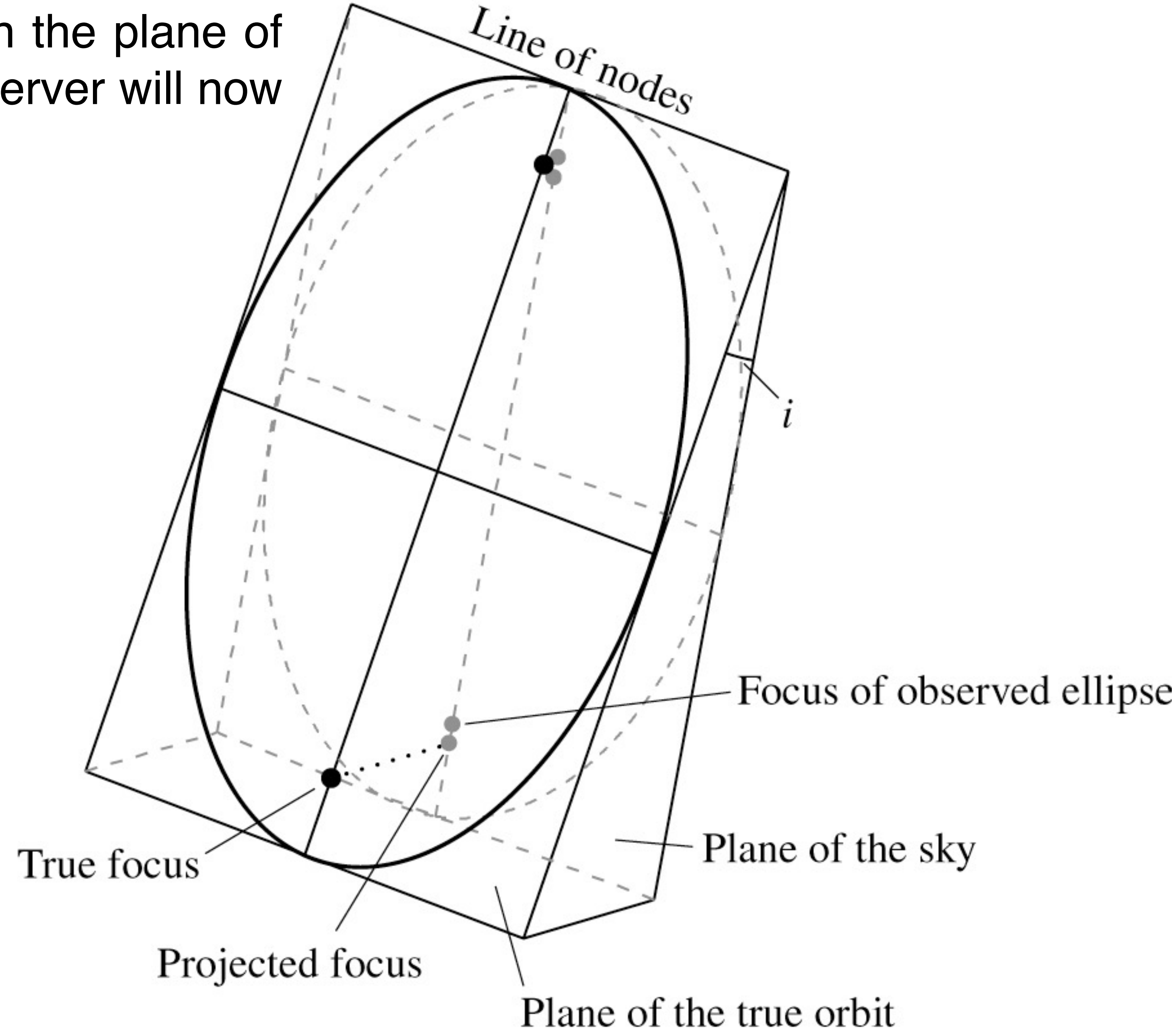

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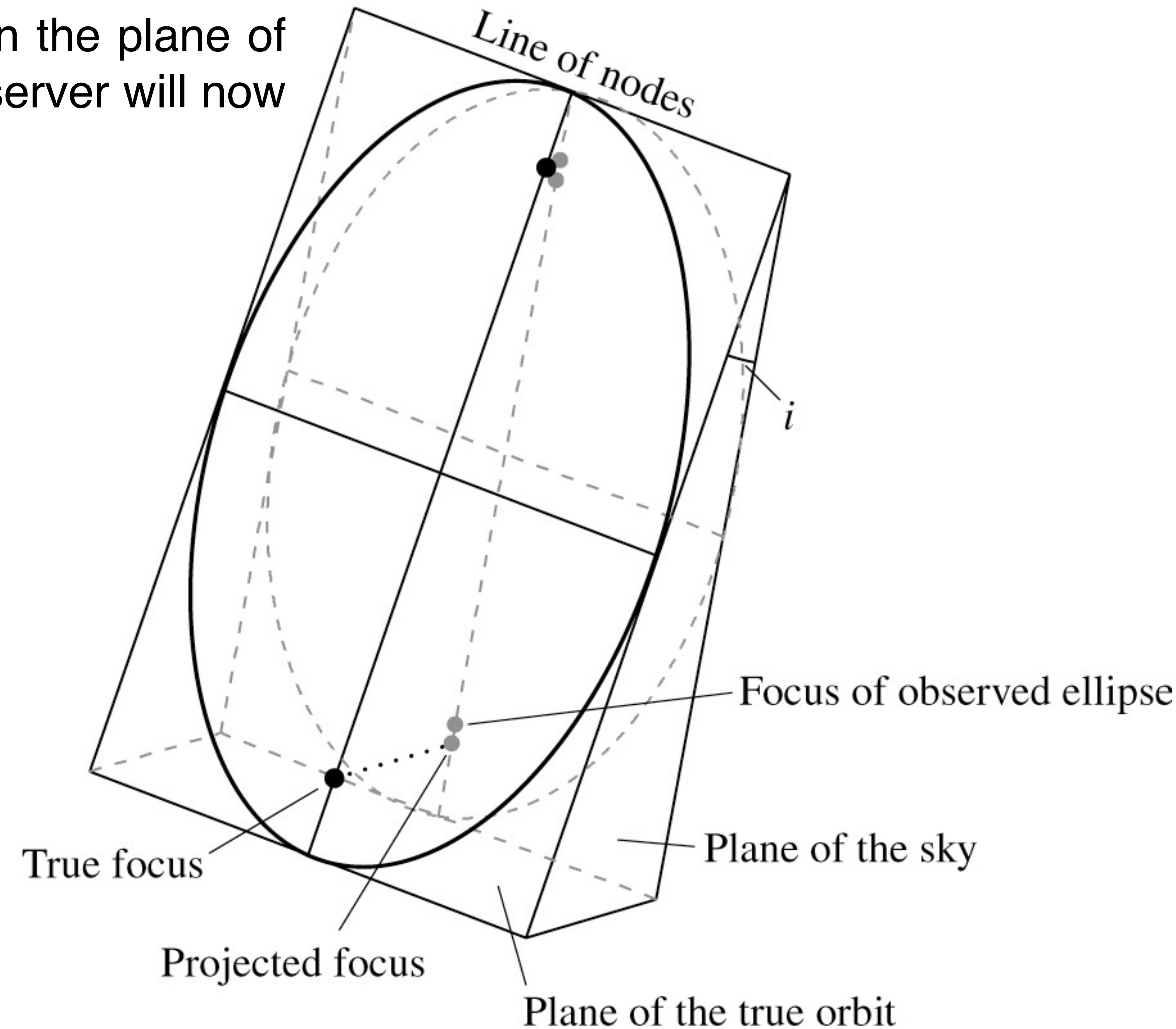

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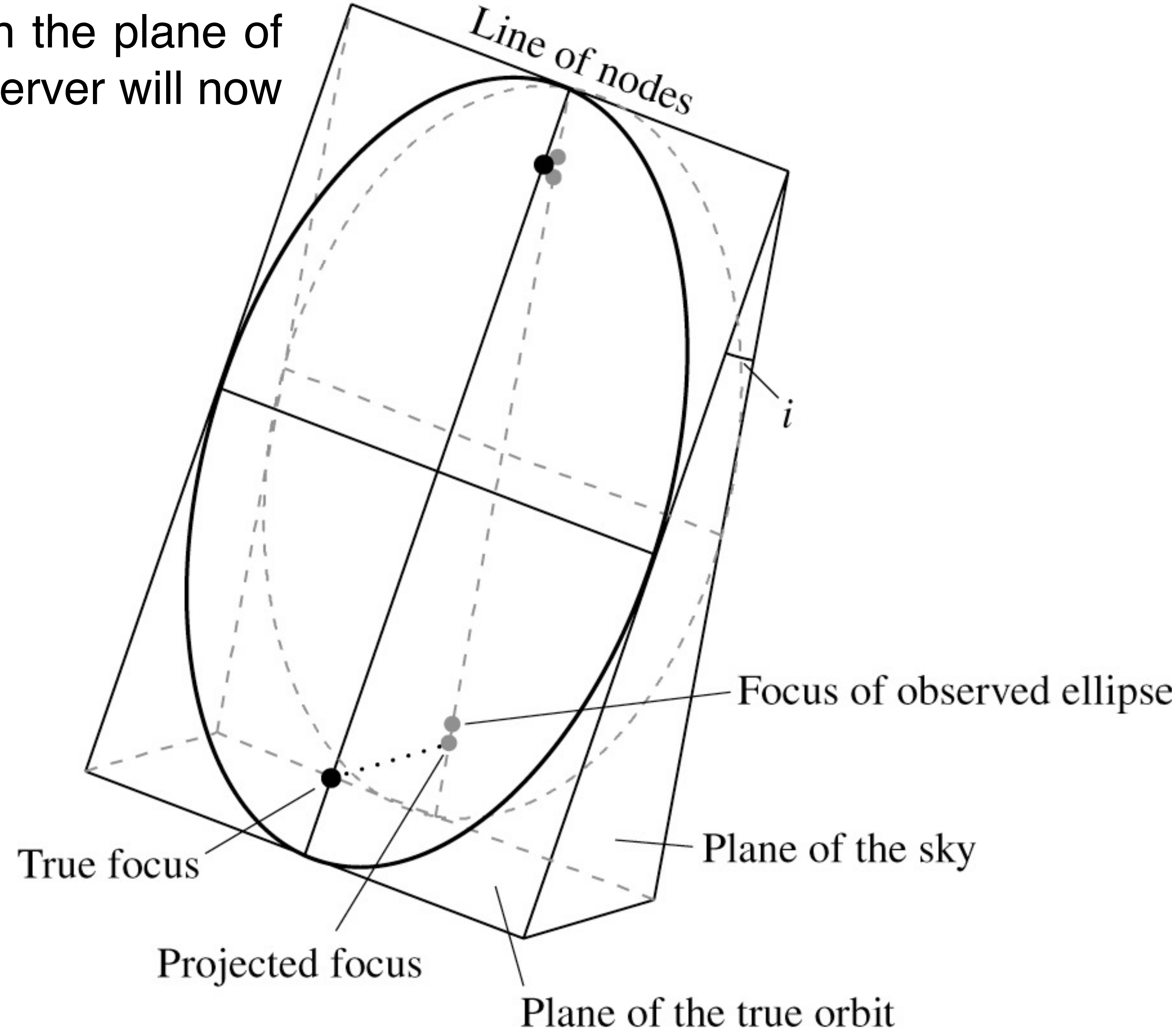

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Since: $\alpha = a/d$

$$m_1 + m_2 = \frac{4\pi^2}{G}\frac{(\alpha d)^3}{P^2} = \frac{4\pi^2}{G}\left(\frac{d}{\cos i}\right)^3 \frac{\tilde{\alpha}^3}{P^2}$$

$$\tilde{\alpha} = \tilde{\alpha}_1 + \tilde{\alpha}_2$$

NOTE: we need to know the inclination "i"

NOTE: Because of the inclination, the observed ellipse will have a different eccentricity. The center of mass will not be located at one of the foci of the projection. The geometry of the true ellipse may be determined by comparing the observed stellar positions with mathematical projections of various ellipses onto the plane of the sky.

If the distance is not known of the visual binary system, but detailed radial velocities are, the projection of the velocity vectors onto the line of sight, combined with info about the stars' positions and the orientation of their orbits, provide a way for determining the semi-major axis of the ellipses. Consequently, the stellar masses of the individual members can be determined.

ECLIPSING, SPECTROSCOPIC BINARIES:

Consider a spectroscopic binary star system for which the spectra of both stars are seen (double-line, spectroscopic binary).

v_1 = velocity of star m_1 at some instant

v_2 = velocity of star m_2 at some instant

Observed radial velocities cannot exceed: $\max(v_{1r})=v_1 \sin(i)$, $\max(v_{2r})=v_2 \sin(i)$

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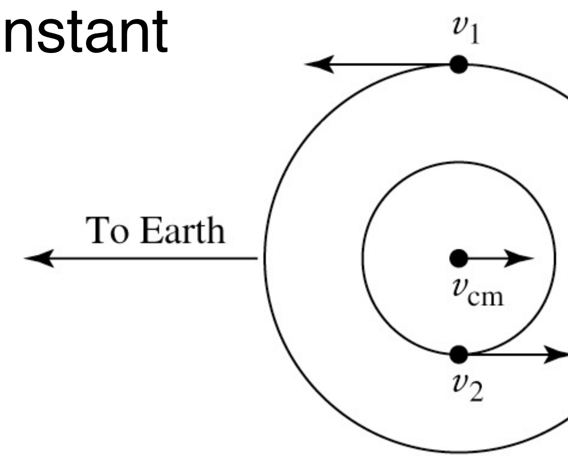
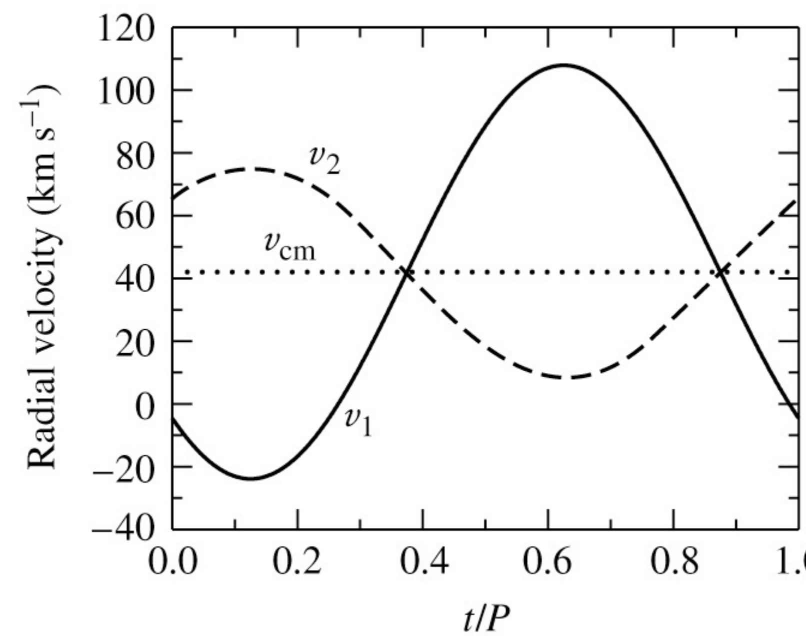
For circular orbits: v_1=constant, v_2=constant

Changing the inclination does not change the shape of the velocity curves (sinusoidal), it only changes the amplitudes by a factor of sin(i).





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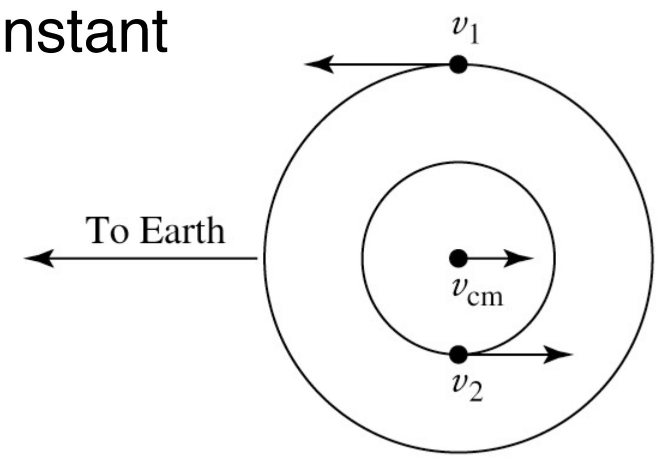

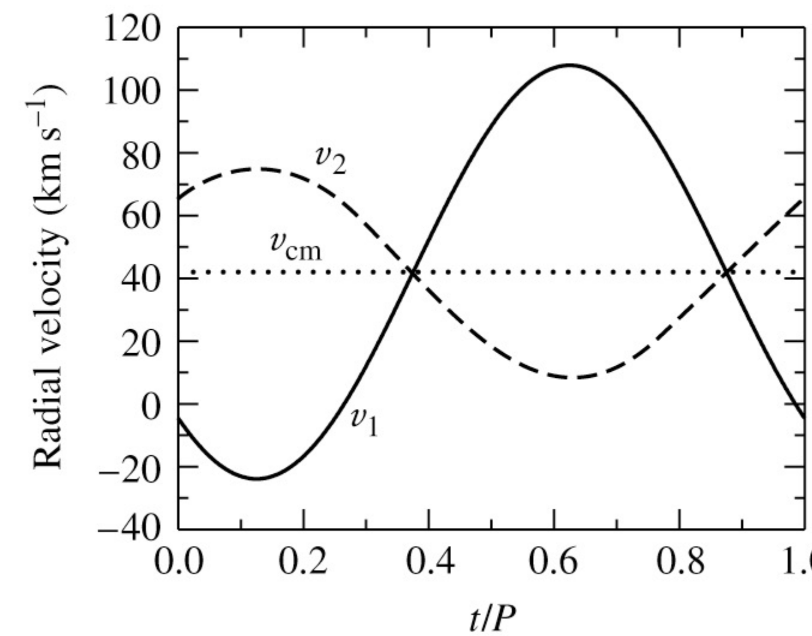

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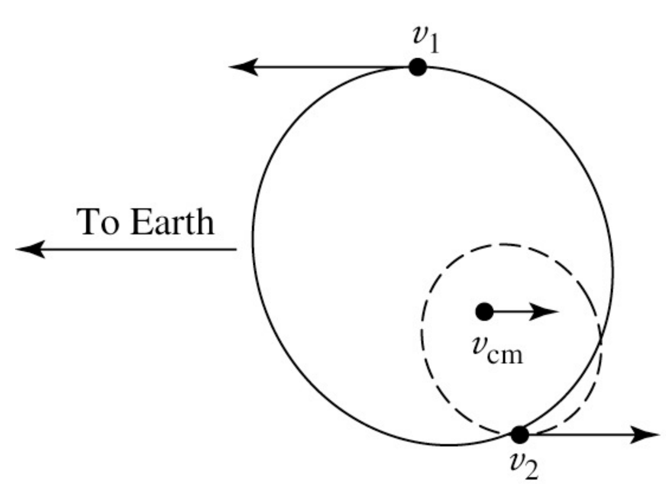
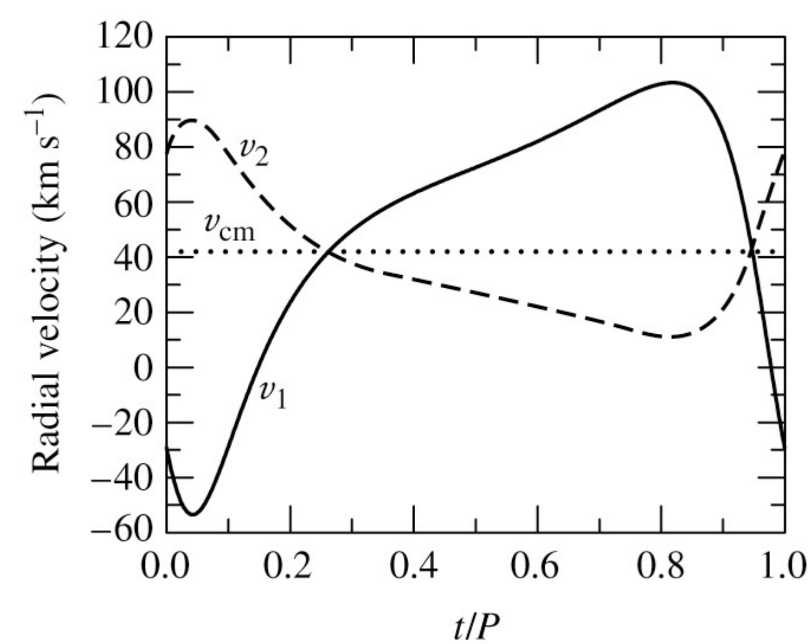
For non-circular orbits (e>0):





The observed velocity curves are skewed; but in reality, many spectroscopic binaries have nearly circular orbits (due to tidal interactions between the two stars)

Assuming e<<<1: $v_1 = \frac{2\pi a_1}{P} = \text{const}$ $v_2 = \frac{2\pi a_2}{P} = \text{const}$

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$$\Rightarrow \boxed{\frac{m_1}{m_2}} = \frac{V_2}{V_1} = \frac{V_{2r}/\sin i}{V_{1r}/\sin i} = \boxed{\frac{V_{2r}}{V_{1r}}}$$

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Using it into Kepler's equation:

$$\Rightarrow \boxed{m_1 + m_2 = \frac{P}{2\pi G} \frac{(V_{1r} + V_{2r})^3}{\sin^3 i}}$$

i.e., the sums of the masses can be obtained only if v_{1r} and v_{2r} are measurable (i.e., double-line)

IF single-line spectroscopic binary, i.e., only v_{1r} is observable then $v_{2r} = \frac{m_1}{m_2} v_{1r}$

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$$\boxed{\frac{m_2^3}{(m_1+m_2)^2} \sin^3 i = \frac{P}{2\pi G} v_{1r}^3 < m_2}$$

Useful for statistical studies or if an estimate of the mass of at least one component is already known by some indirect mean.

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Mass function (all observable quantities)

If either m_1 or $\sin(i)$ is unknown, then the mass function sets a lower limit on m_2

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If either m_1 or sin(i) is unknown, then the mass function sets a lower limit on m_2

Without i, it is impossible to get exact values of m_1 and m_2. Since stars can be grouped according to their effective temperatures and luminosities (HR diagram), and assuming the existence of a relationship between these quantities and mass, then a statistical estimate of mass for each class may be found using an appropriate averaged value of $\sin^3(i)$.

IF single-line spectroscopic binary, i.e., only v_{1r} is observable then $v_{2r} = \frac{m_1}{m_2} v_{1r}$

$$\Rightarrow m_1 + m_2 = \frac{P}{2\pi G} \frac{v_{1r}^3}{\sin^3 i} \left(1 + \frac{m_1}{m_2}\right)^3$$

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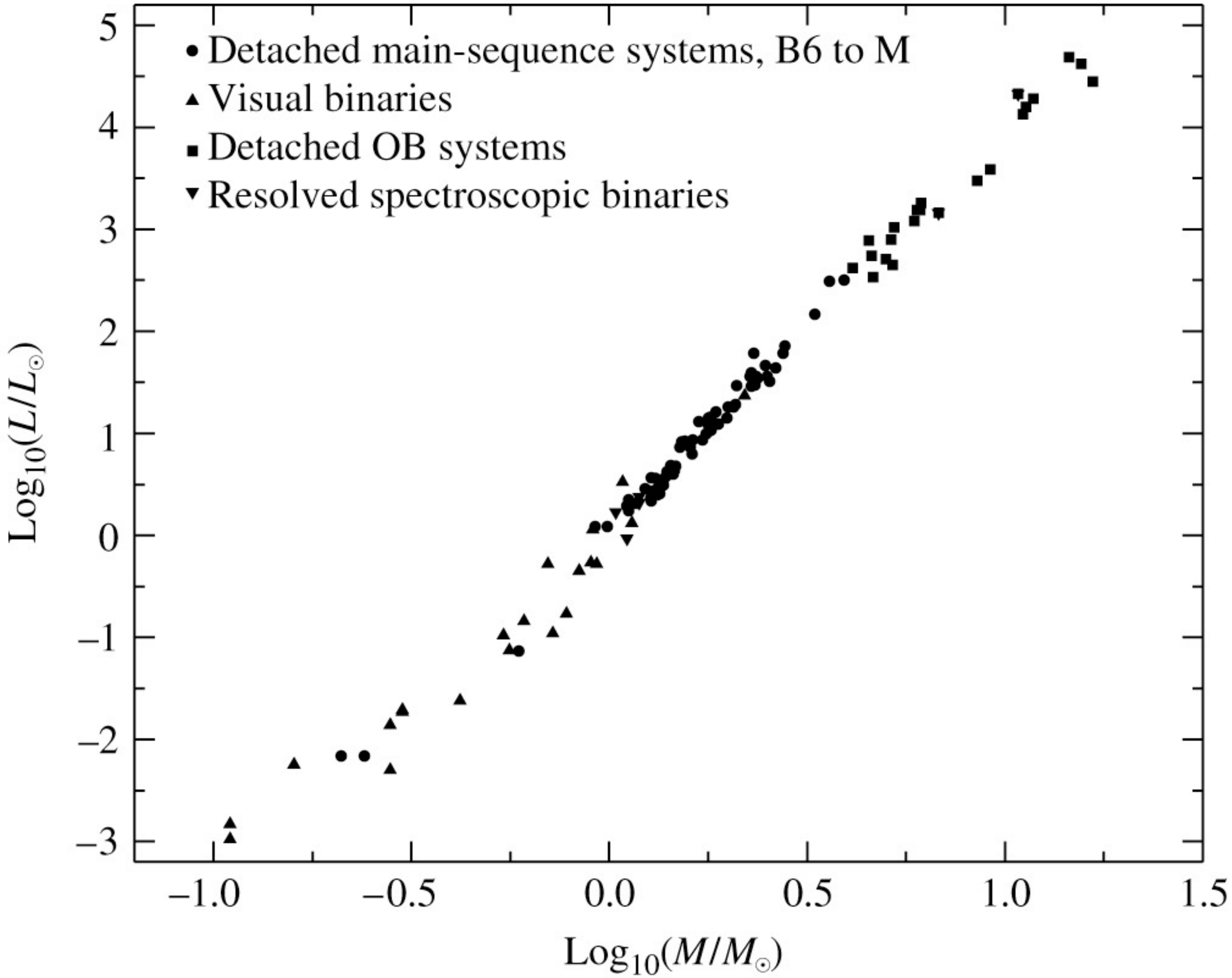
But accounting for systematic/selection effects as i>0 for a spectroscopic binary: $< \sin^3 i > \approx 2/3$

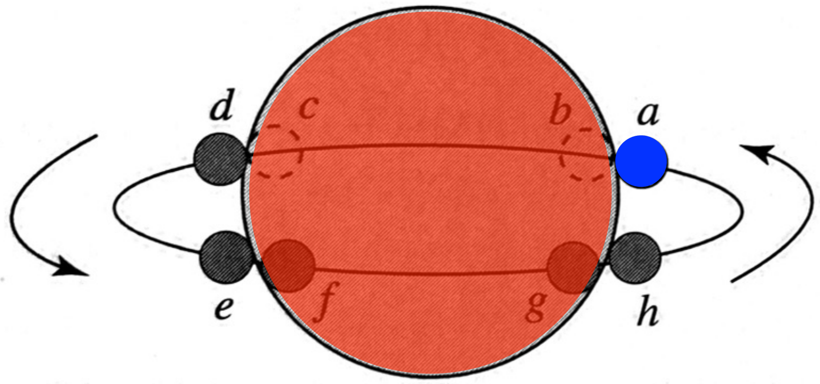
Mass-Luminosity relation

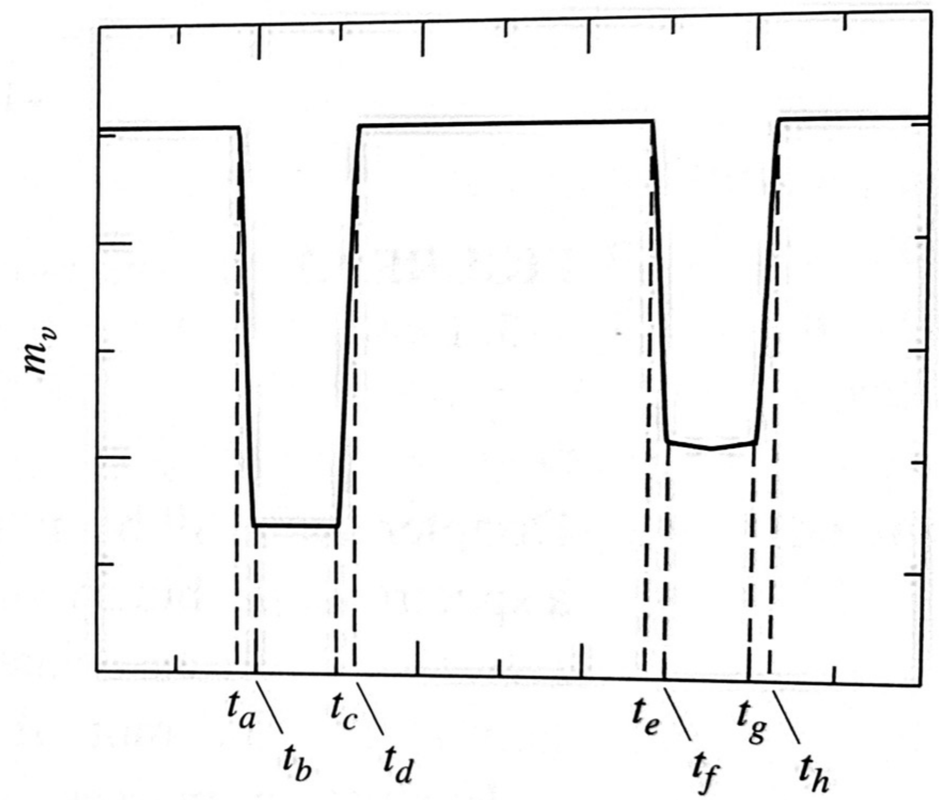


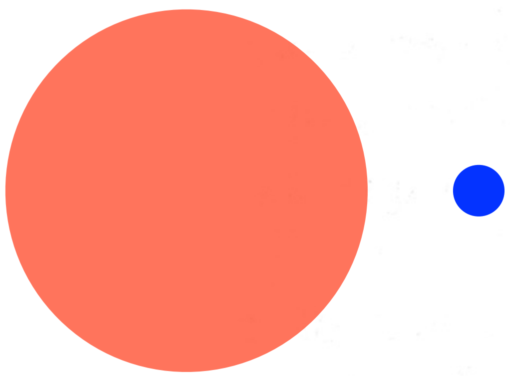
$\mathrm{L} = \mathrm{constant}\ \mathrm{M}^{\beta}$ $\quad\beta \in (3.5, 4)$ for main sequence stars

(this law is violated by white dwarfs, giants, etc…)

ECLIPSING BINARIES to determine radii and ratios of temperatures: if the smaller star is completely eclipsed by the larger one, a nearly constant minimum will occur







$$R_{small} = \frac{v}{2}(t_b - t_a) \qquad i \simeq 90^\circ$$

$V = v_{small} + v_{large}$

RELATIVE VELOCITY OF TWO STARS

$$R_{large} = \frac{v}{2}(t_c - t_a) = R_{small} + \frac{v}{2}(t_c - t_b)$$

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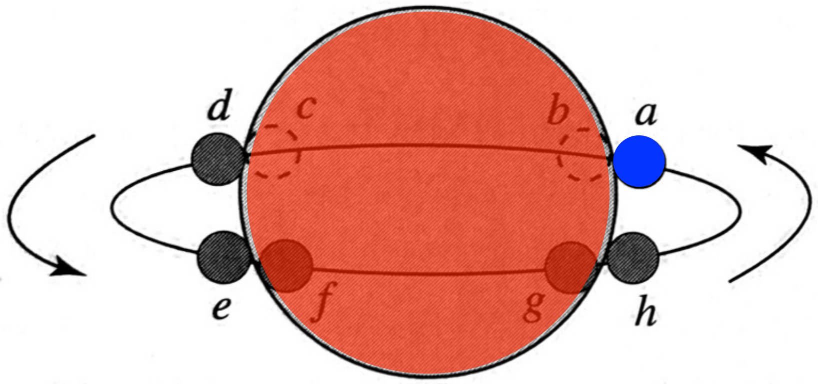

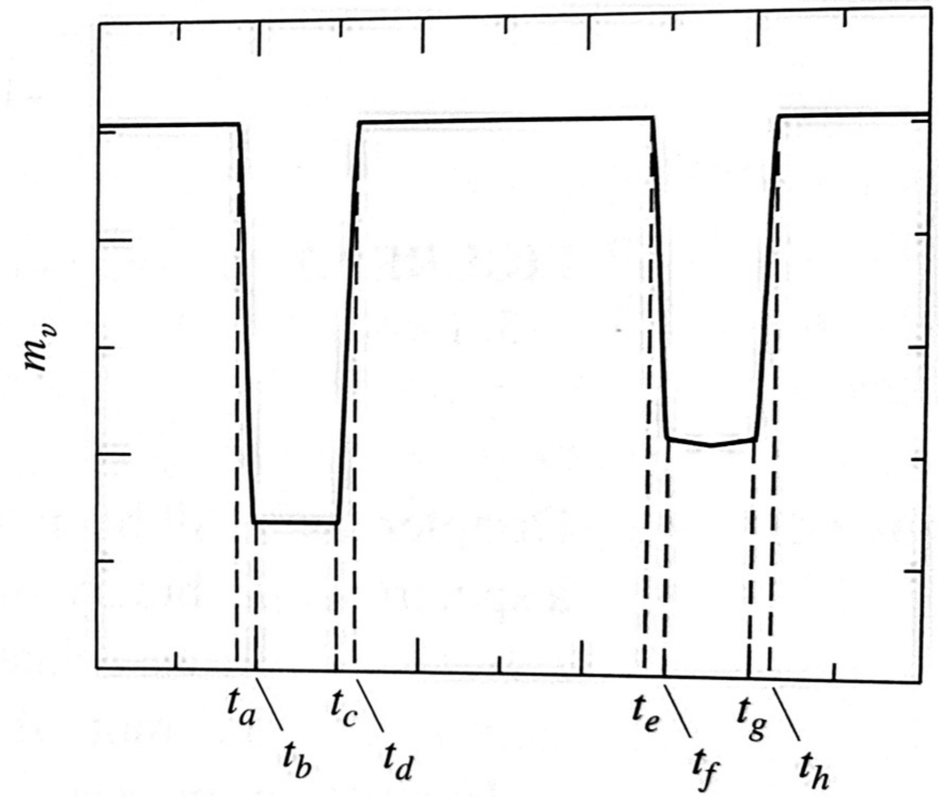

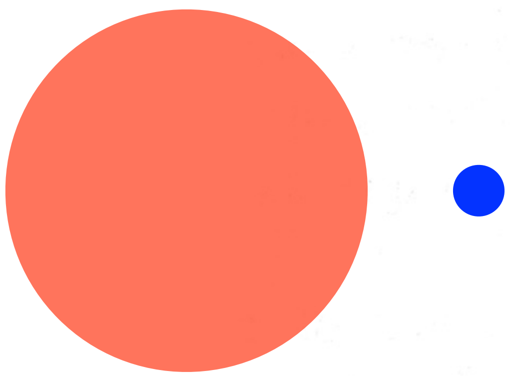

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RELATIVE VELOCITY OF TWO STARS

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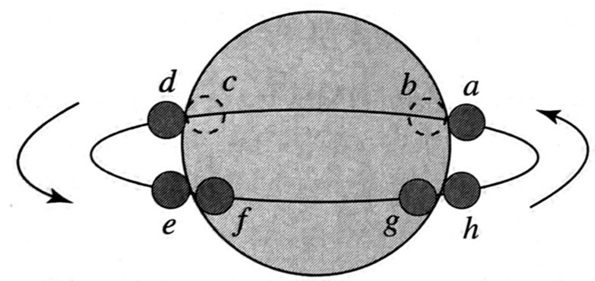
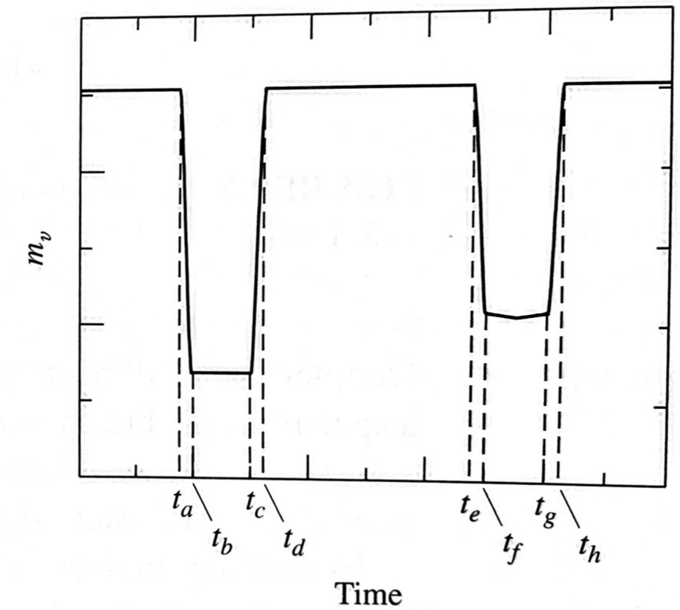
The dip in the light curve is deeper when the hotter star is behind its companion, from the Stefan-Boltzmann law and from the fact that the same total cross-sectional area is eclipsed.

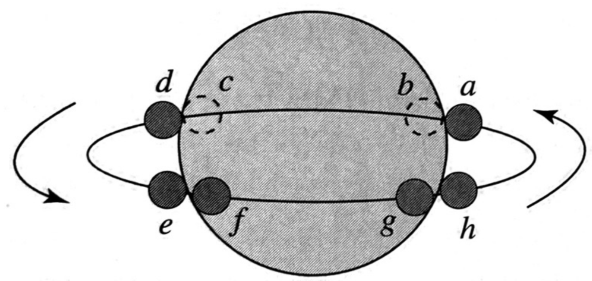
$$F = \sigma T_{eff}^4 \qquad [\text{erg s}^{-1}\text{ cm}^{-2}]$$

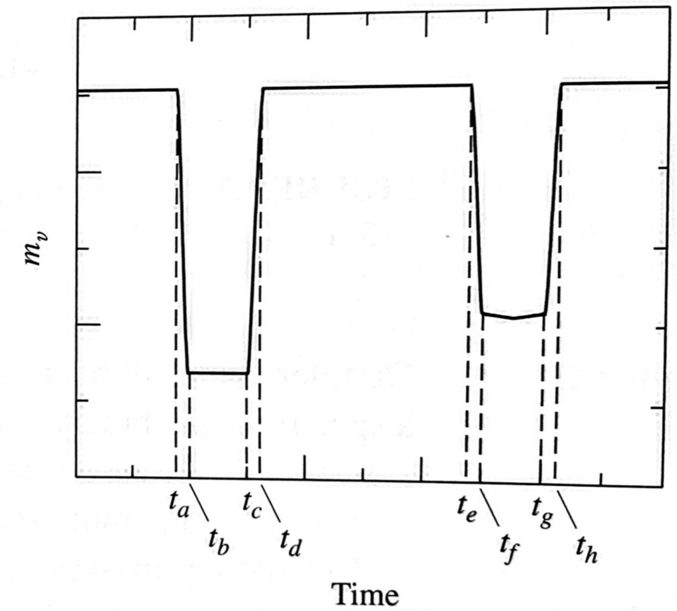




When both stars are visible:

$$f_{tot} = K \left(\pi R_{small}^2 F_{small} + \pi R_{large}^2 F_{large} \right)$$





When both stars are visible:

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If the smaller star is also the hotter, when the hotter star is entirely eclipsed:

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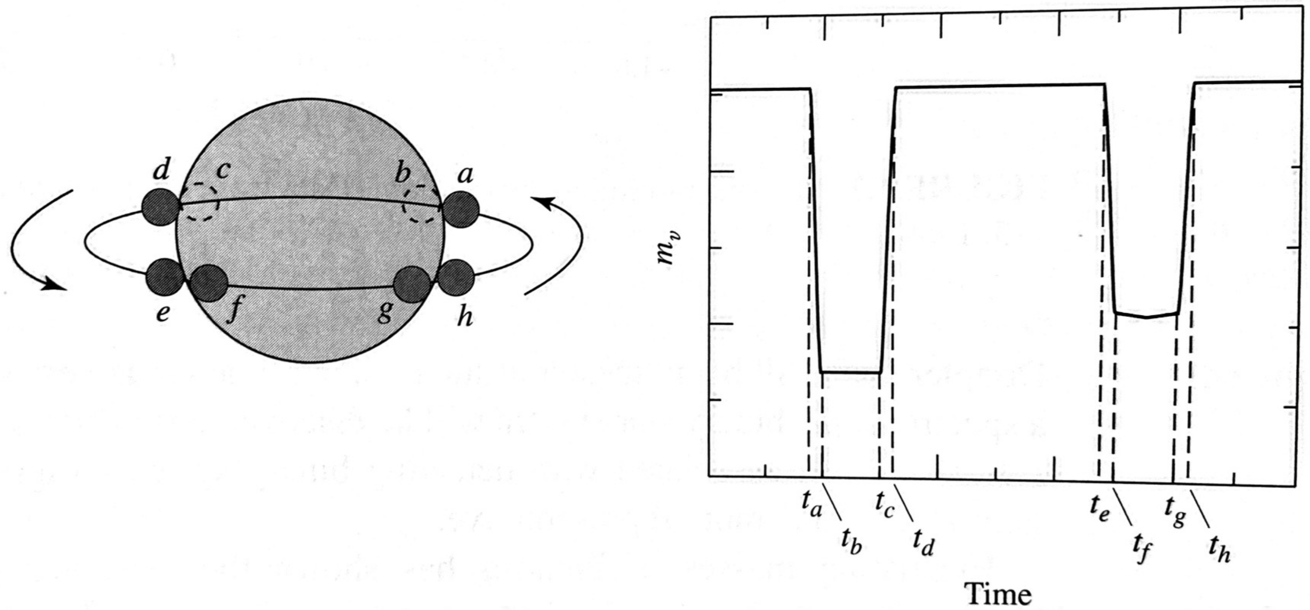

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When the cooler star is behind:

$$f_{\substack{secondary\\dip}} = K\left(\boxed{\pi R_{large}^2 - \pi R_{small}^2}\right) F_{large} + K\pi R_{small}^2 F_{small}$$

Area of larger star that is eclipsed by the smaller star

$$\frac{f_{tot} - f_{\substack{primary\\deep}}}{f_{tot} - f_{\substack{secondary\\dip}}} = \frac{F_{small}}{F_{large}} = \left(\frac{T_{small}}{T_{large}}\right)^4$$

i.e., from the measurements of the light curve of the system, one can determine the ratio of temperatures of the two stars

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MEASURABLES: L, $T_{surface}$, R, M

spectral features —> chemical composition at the stellar surface

==> theory of stellar structure and stellar evolution

Binary Systems and Stellar Parameters

Reading assignment:
THURSDAY 10/1: Chapters 8.1(quite long)

Homework Assignment #2
Due by TUESDAY 10/6