The Interaction of Light & Matter

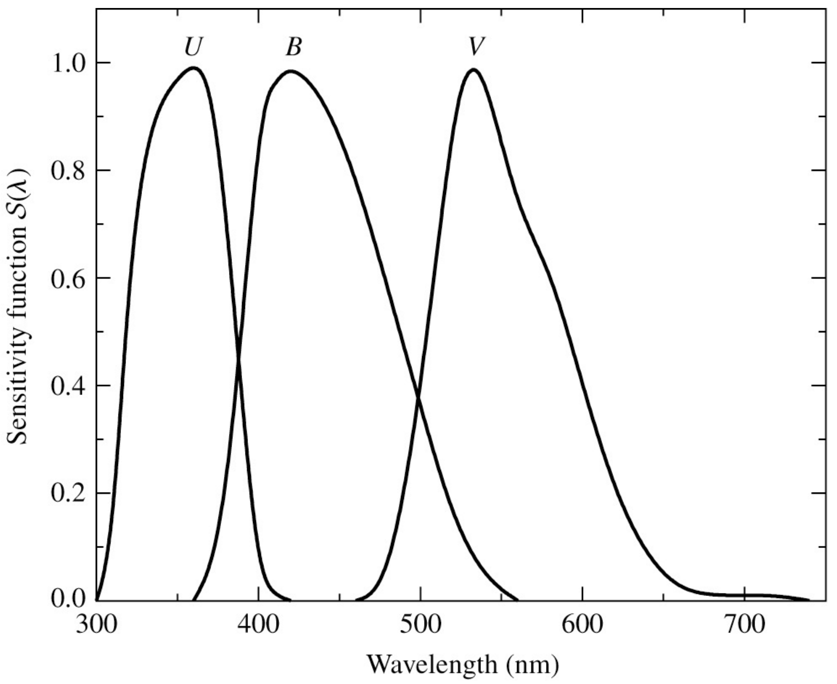
Reading assignment:

TUESDAY 9/29: Chapters 7.1, 7.2, 7.3

Homework Assignment #2

Due by TUESDAY 1/6

COLOR INDICES

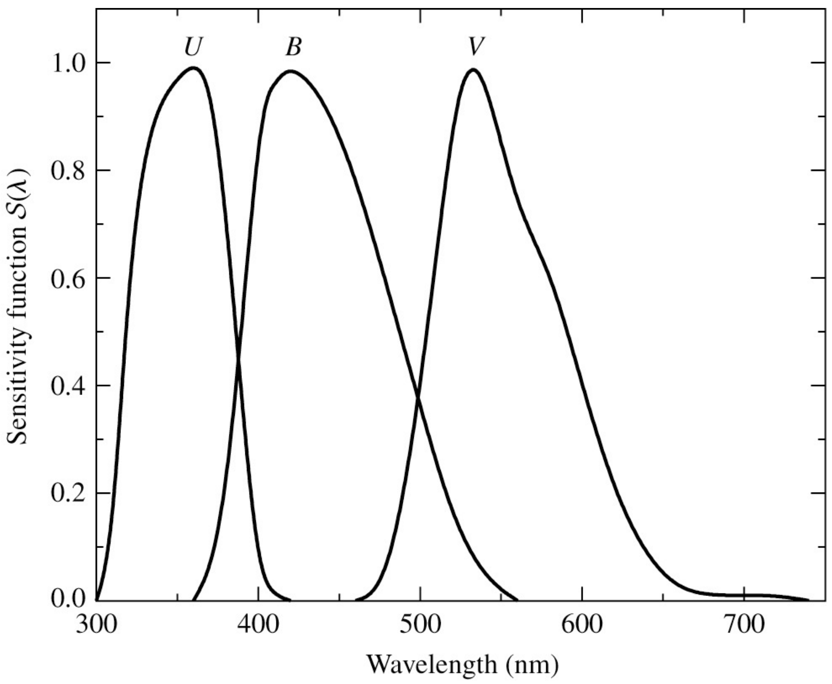


COLOR INDICES

$m_U =$ $$U = -2.5 \log \int_0^\infty S_U f_\lambda \, d\lambda + C_U$$

transmission function

$S(\lambda) \equiv P_\lambda$



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transmission function

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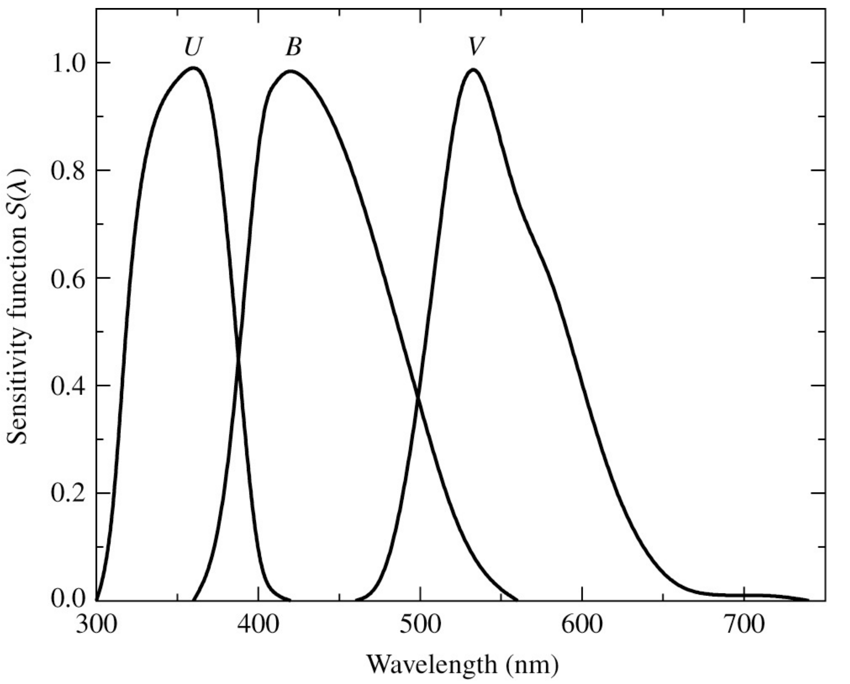
$m_B = B$
$m_V = V$



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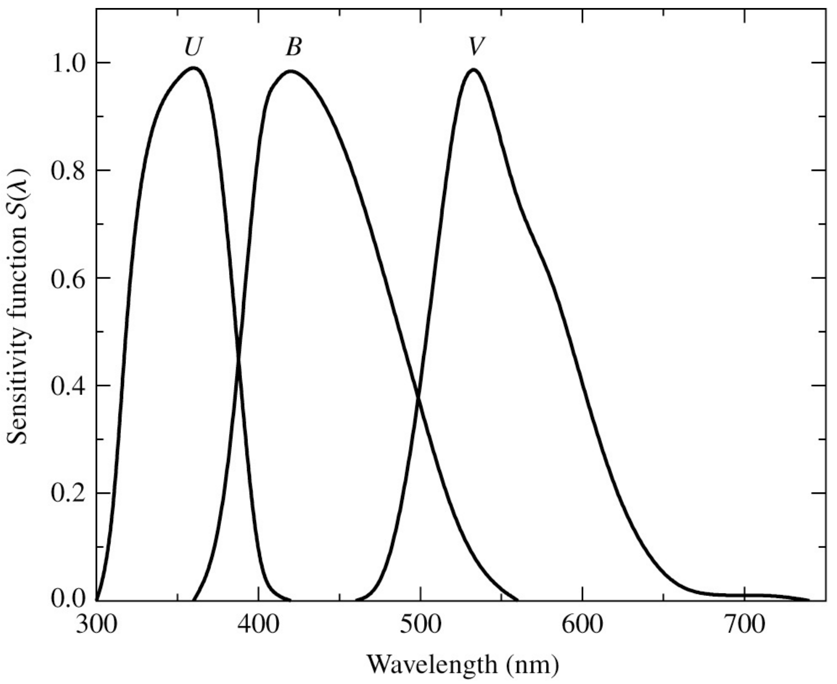

m_U-m_B = U-B

$$U-B = -2.5 \log \frac{\int_0^\infty S_U f_\lambda \, d\lambda}{\int_0^\infty S_B f_\lambda \, d\lambda} + \underbrace{C_{U-B}}_{\equiv C_U - C_B}$$

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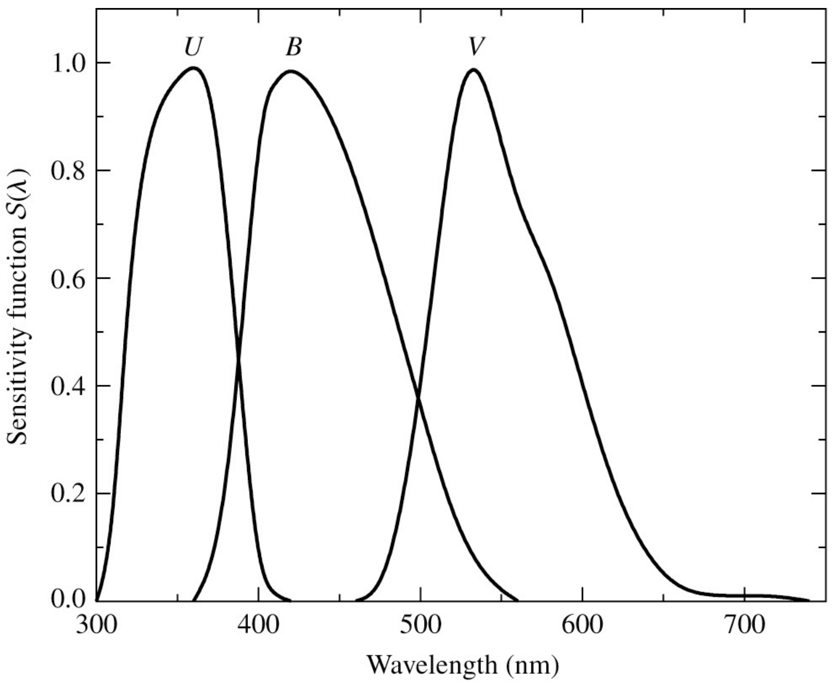

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m_B-m_V = B-V

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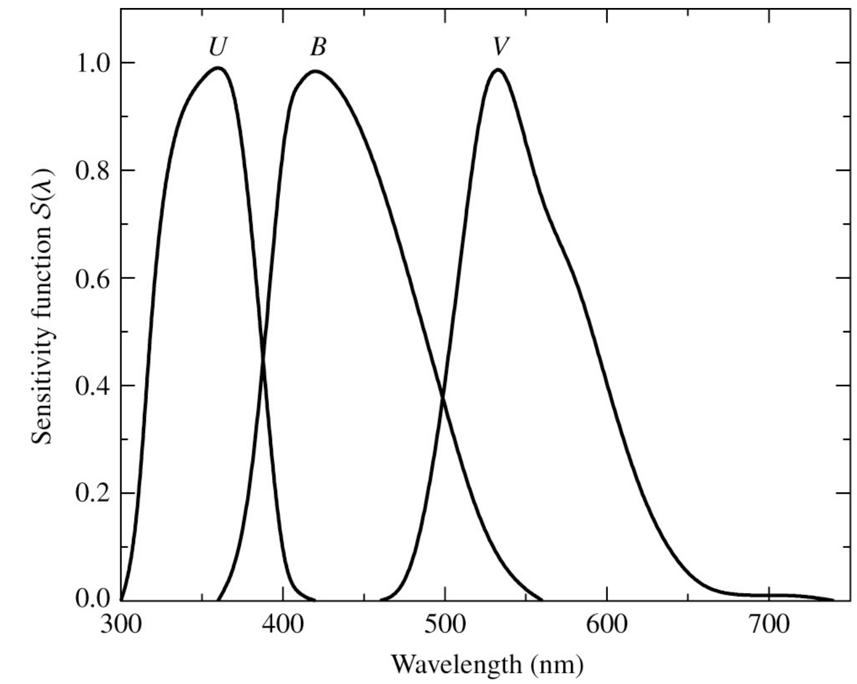

m_U-m_B = U-B

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m_B-m_V = B-V

NOTE: the color does not depend on $(R/d)^2$ because this term cancels out in the above equation
The color is solely dependent on the temperature of a model blackbody star

$$m_1 - m_2 = -2.5 \log\left(\frac{f_{\lambda_1}}{f_{\lambda_2}}\right) = -2.5 \log\left(\frac{F_{\lambda_1}}{F_{\lambda_2}}\right) + C$$

$$= A + \frac{1.56}{T_{color}}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) + f(T_{cd})$$

w/ $$f(T_c) = 2.5 \log\left(\frac{1 - e^{-hc/\lambda_2 k T_{col}}}{1 - e^{-hc/\lambda_1 k T_{col}}}\right)$$

$$\Rightarrow \quad B-V \simeq -0.586 + \frac{6850}{T_{col}[K]} + f(T_{cd})$$

$$B-V\big|_{\odot} = 0.64$$

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The hotter a star, the smaller (or more negative) the B-V color, hence the bluer the star

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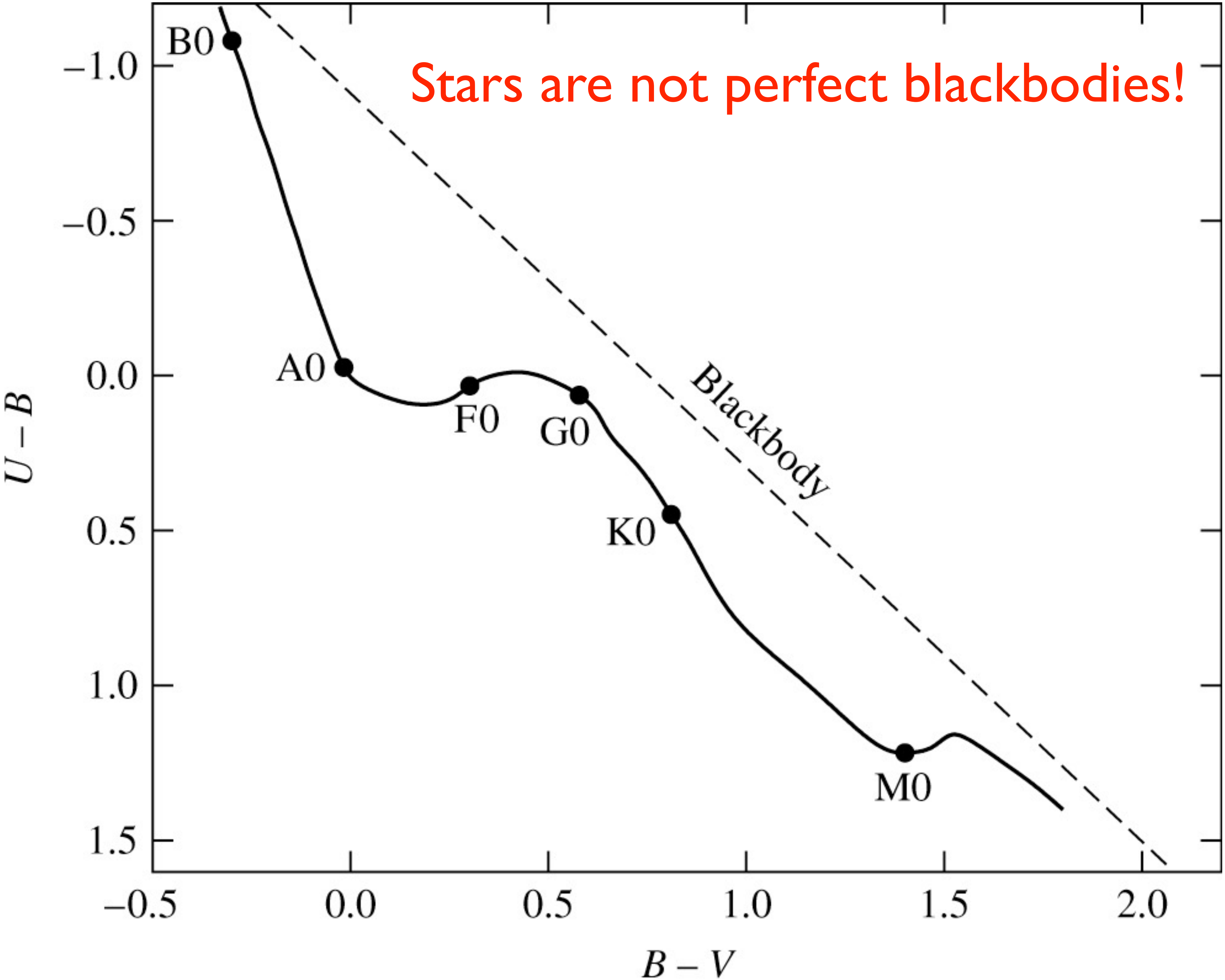
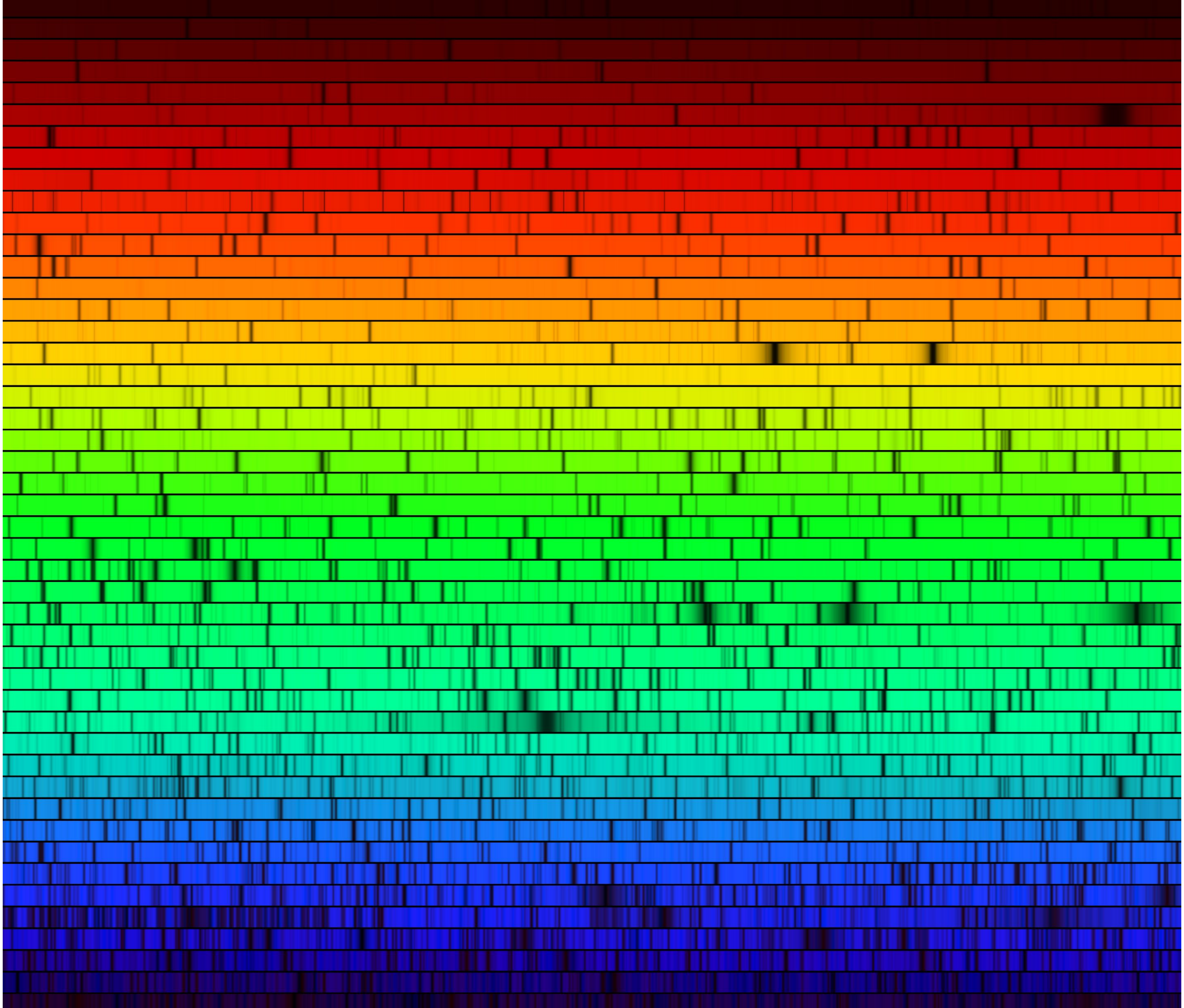
$$B - V\big|_{\odot} = 0.64$$

The hotter a star, the smaller (or more negative) the B-V color, hence the bluer the star

The cooler a star, the larger the B-V color, hence the redder the star


Stars are not perfect blackbodies!
B0
A0
F0
G0
K0
M0
Blackbody
U − B
B − V
−1.0
−0.5
0.0
0.5
1.0
1.5
−0.5
0.0
0.5
1.0
1.5
2.0

The Interaction of Light & Matter

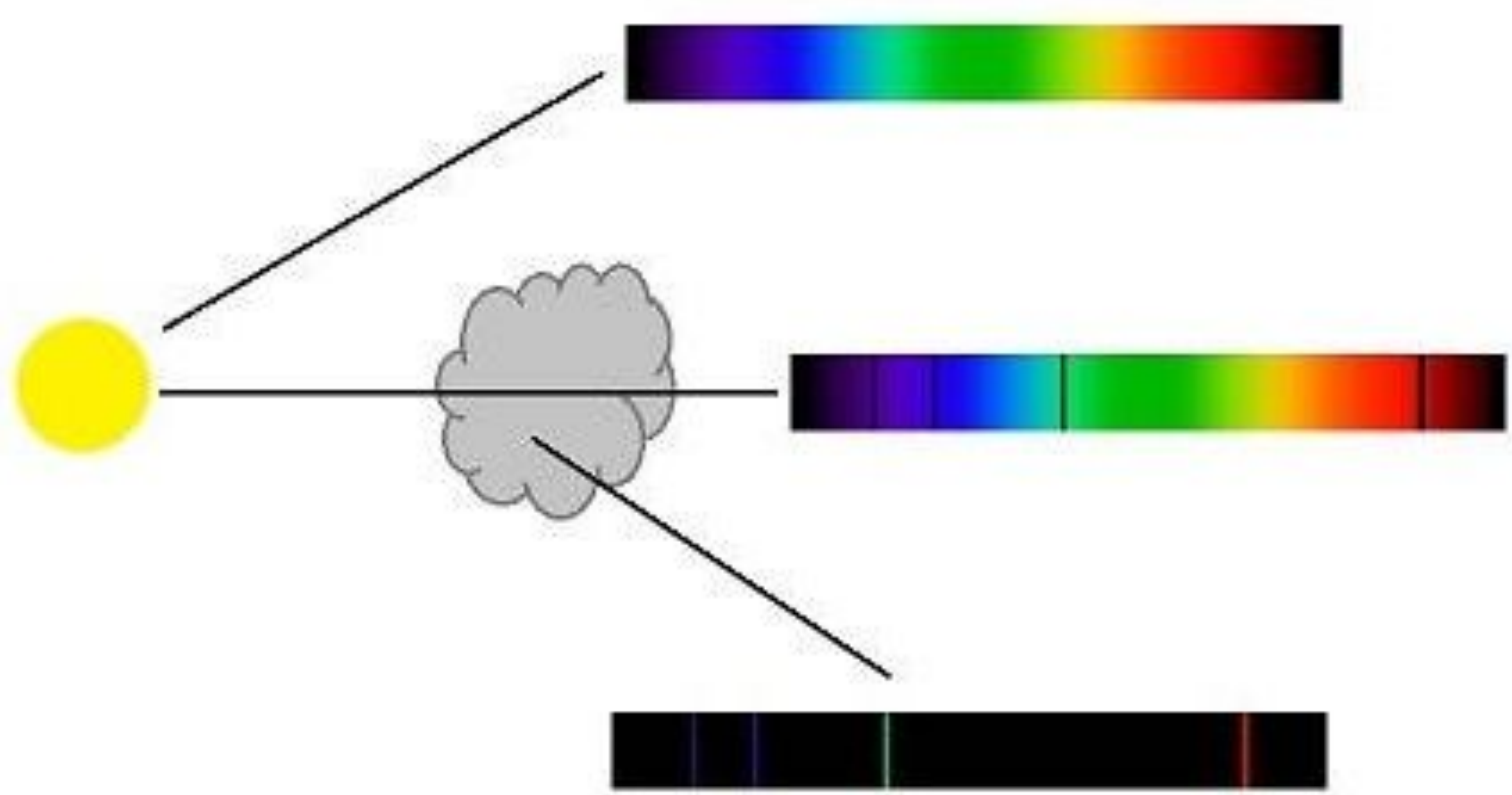


Kirchhoff's Laws:

1. A hot, dense gas or hot solid object produces a continuous spectrum with no dark spectral lines (e.g., the blackbody radiation): continuous spectrum

2. A hot, diffuse gas produces bright spectral lines (emission lines): emission line spectrum

3. A cool, diffuse gas in front of a source of a continuous spectrum produces dark spectral lines (absorption lines) in the continuous spectrum: absorption line spectrum



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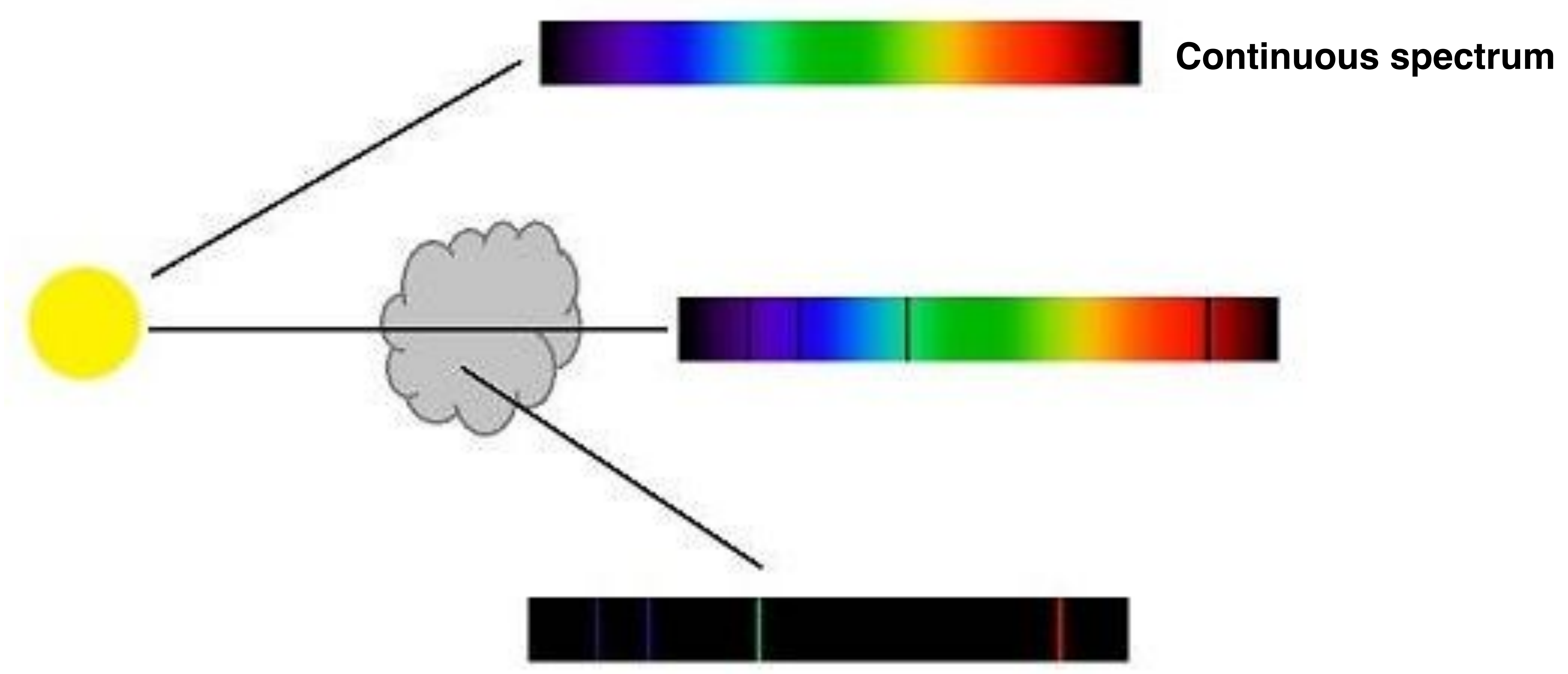

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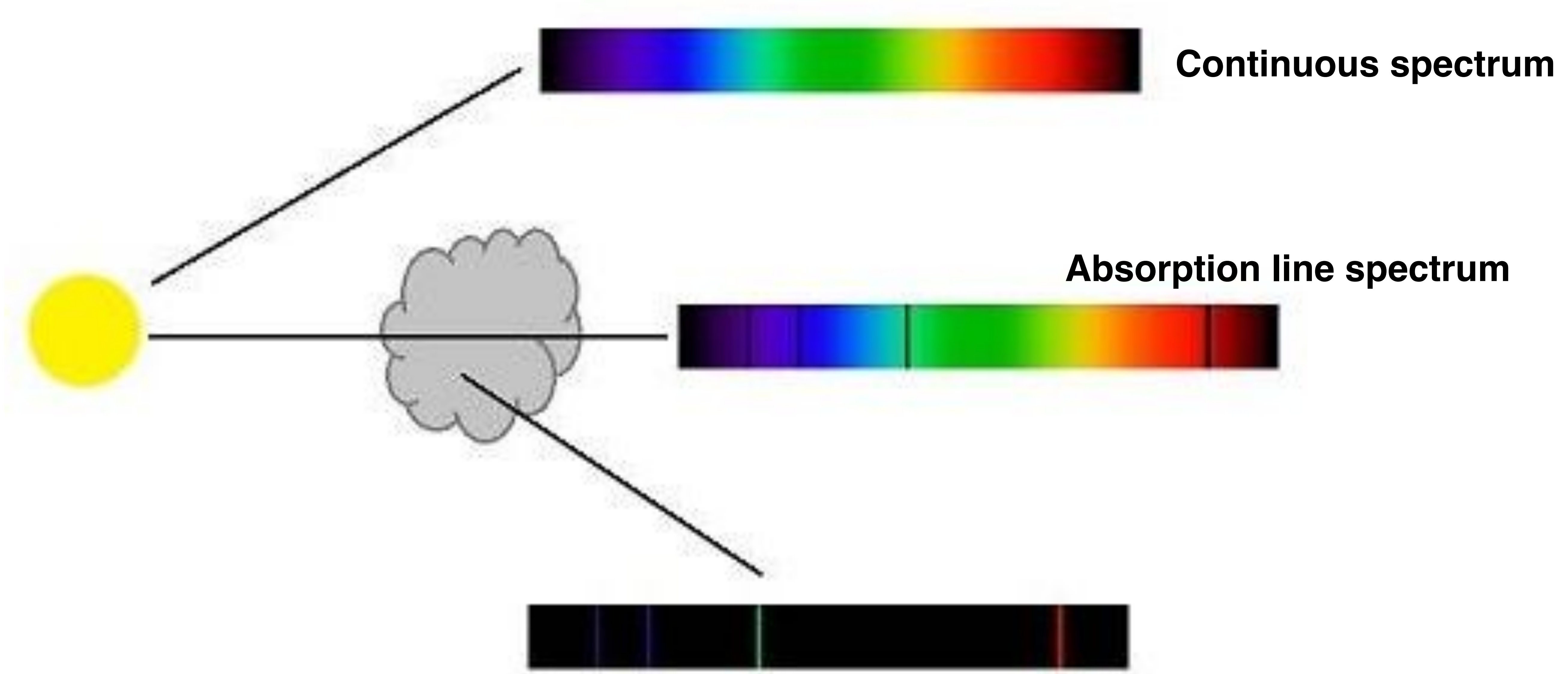

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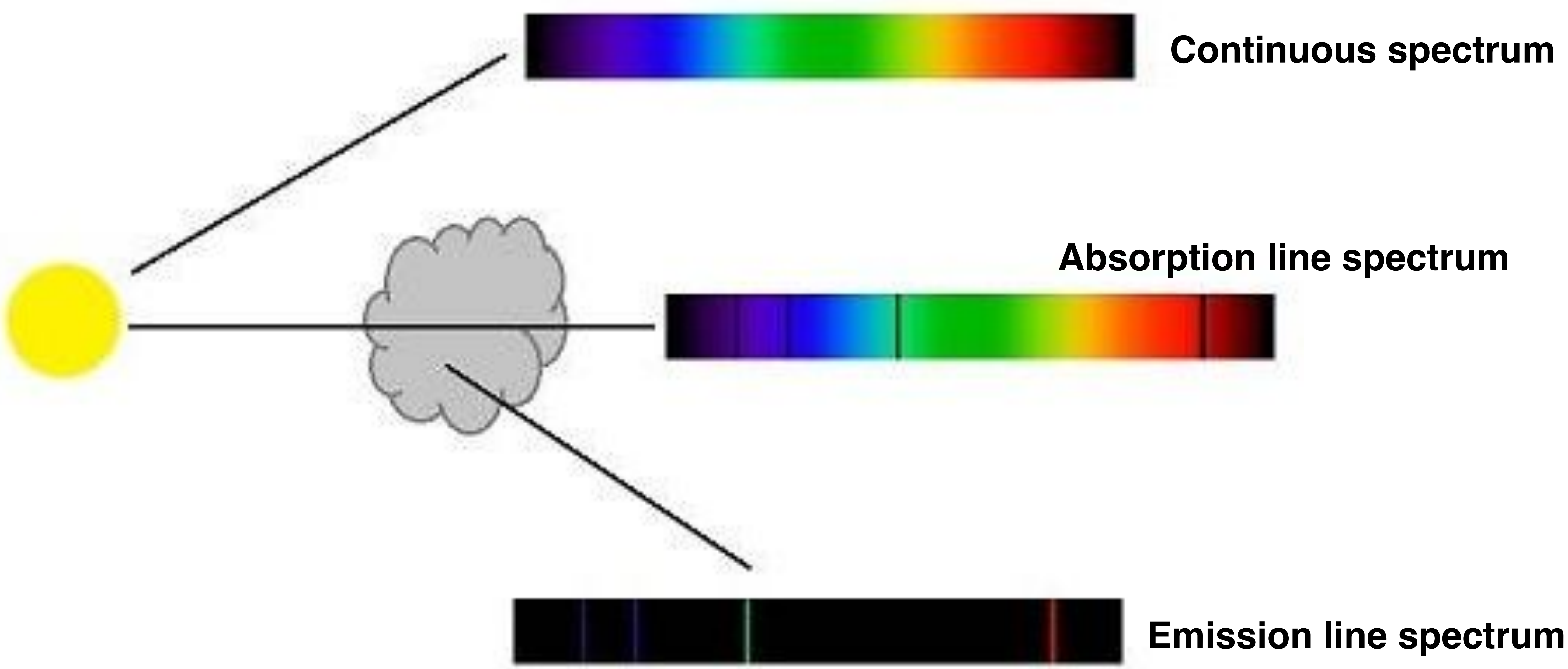
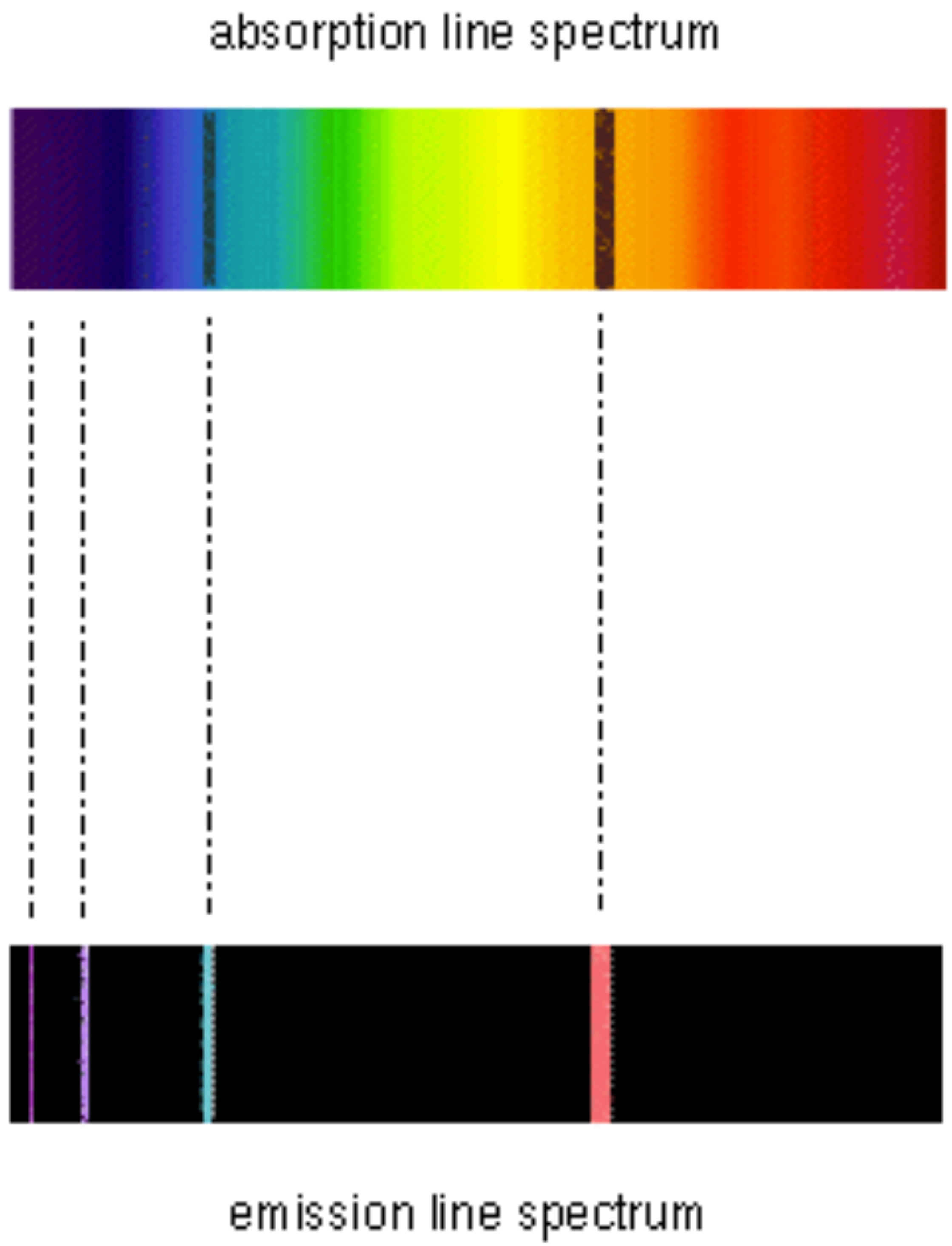
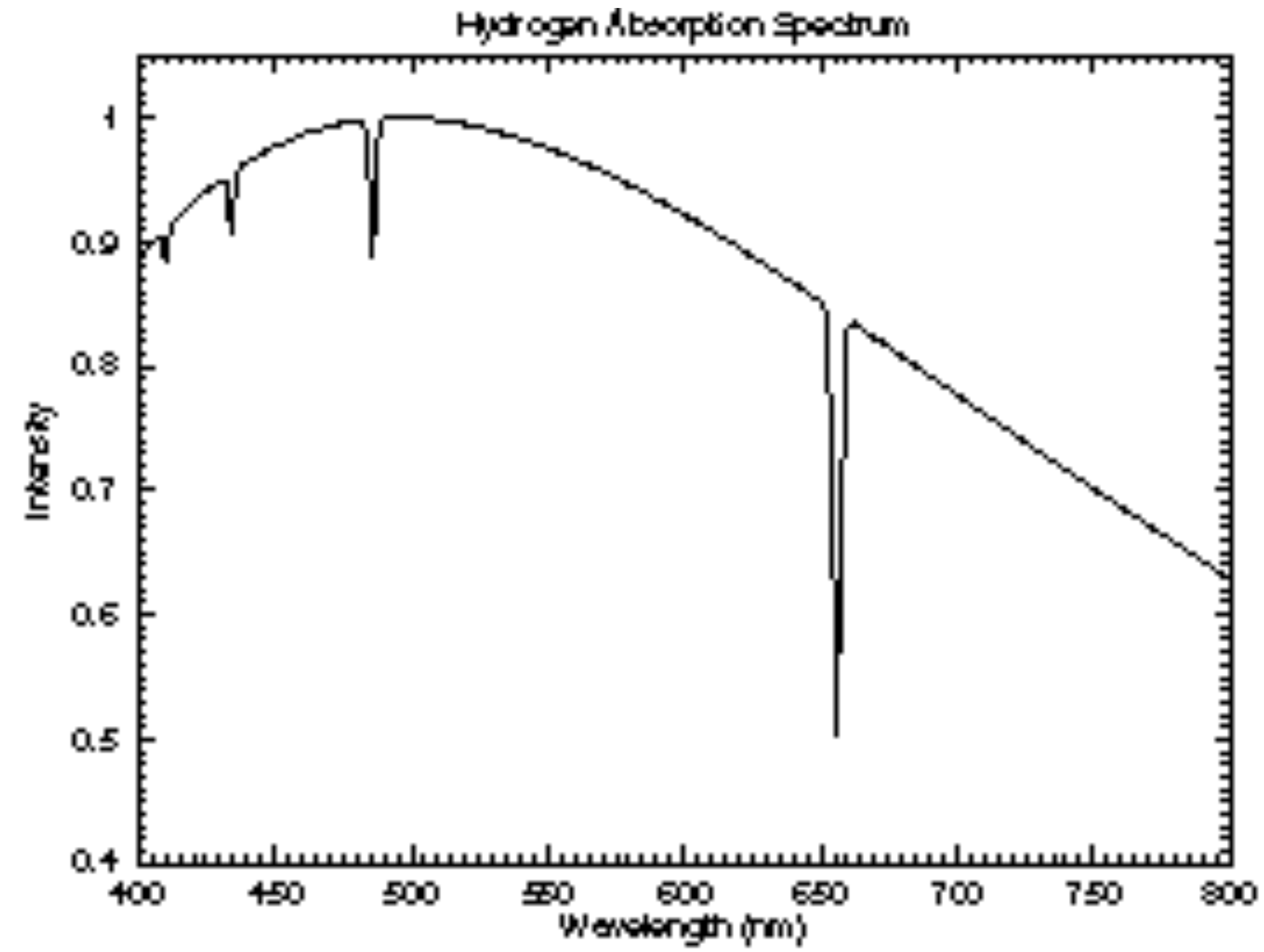
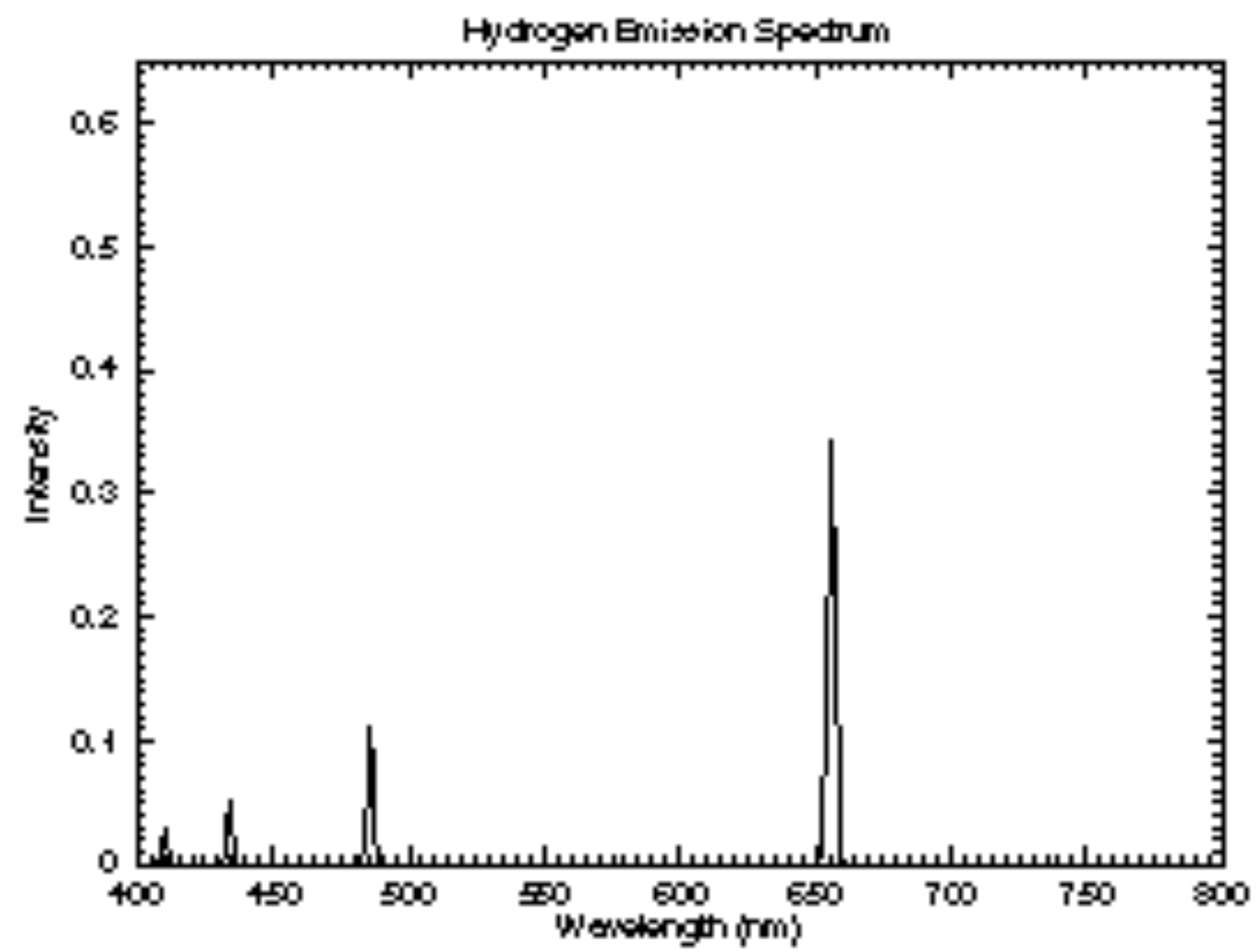



absorption line spectrum
emission line spectrum


Hydrogen Absorption Spectrum
Intensity
Wavelength (nm)
1
0.9
0.8
0.7
0.6
0.5
0.4
400
450
500
550
600
650
700
750
800


Hydrogen Emission Spectrum
Intensity
Wavelength (nm)
0.6
0.5
0.4
0.3
0.2
0.1
0
400
450
500
550
600
650
700
750
800

Doppler shift:

$$\frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{rest}}}{\lambda_{\mathrm{rest}}} = \frac{\Delta\lambda}{\lambda_{\mathrm{rest}}} = \frac{v_{\mathrm{R}}}{c}$$

Radial velocity v_R

$v_{\mathrm{R}} << c$

Doppler shift:

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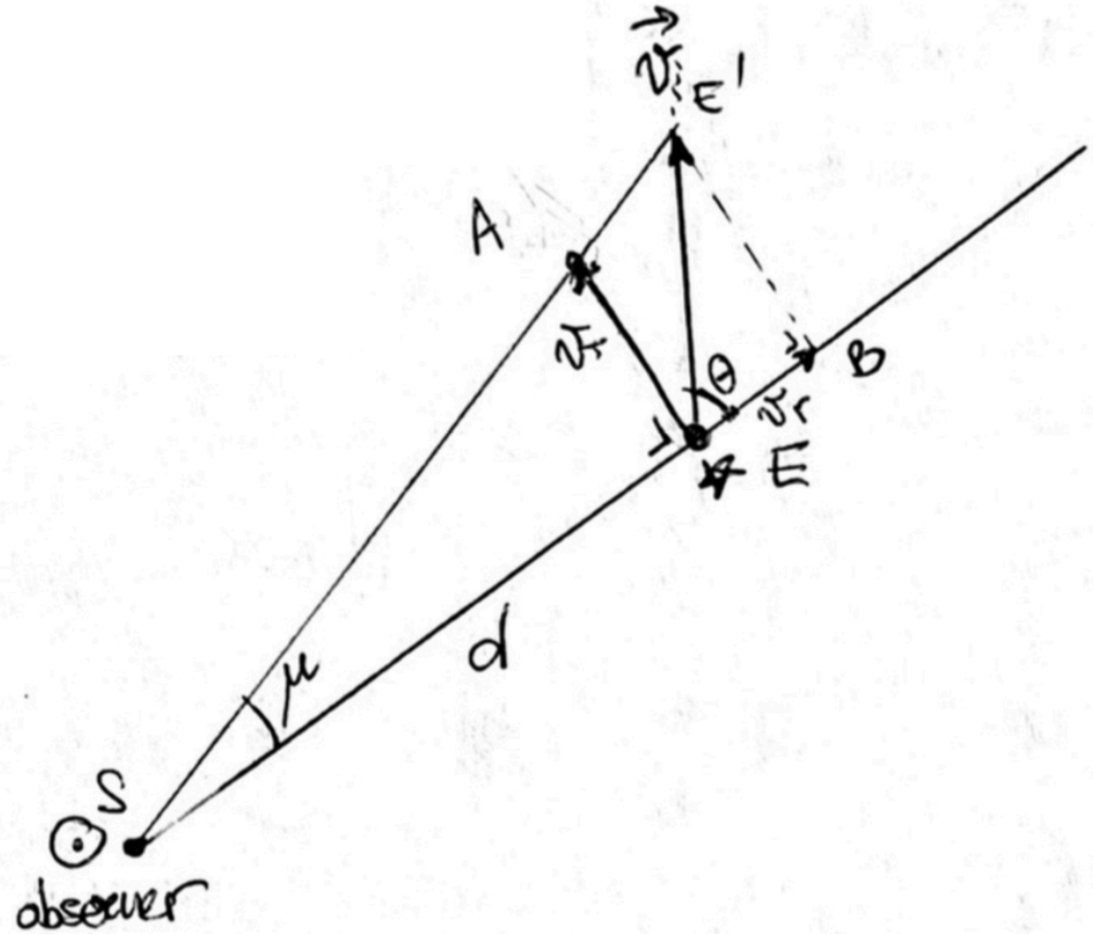
Radial velocity v_R

$v_{\rm R} << c$

$$\boxed{v_{\rm R} = c\frac{\Delta\lambda}{\lambda_{\rm rest}}}$$

Proper motion: motion of a star with respect to the Sun



E: star

S: Sun (observer)

v : velocity of the star in the direction EE'

EE': distance traveled in a year

SE : line of sight

μ: apparent angle (proper motion of the star in a year)

EA : tangential direction/component

EB : radial direction/component

Since $\mu \lll 1 \rightarrow \hat{SAE} \simeq 90°$

$AE' \approx EB$

$[v_R]=[v_T]=$km/s; n=number of seconds in a year

$EE' = nv$

$EA = nv_T$

$EB = E'A = nv_R$

Since $E\hat{A}E' \simeq 90^\circ \Rightarrow EA = EE' \sin\theta$

$AE' = EE' \cos\theta$

Substituting $EA = n\, v_T$

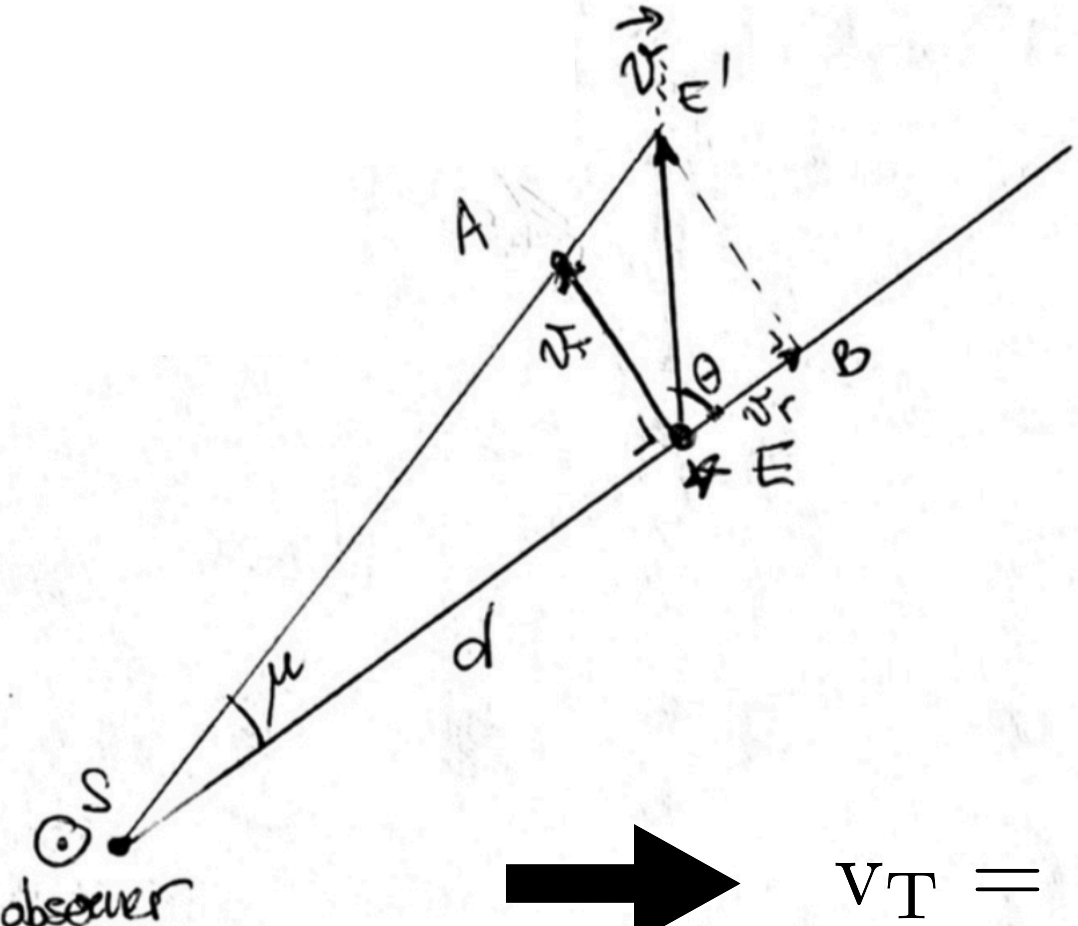
$E'A = n\, v_R$ $\Rightarrow$ $\boxed{\begin{array}{l} v_T = v \sin\theta \\ v_R = v\cos\theta \end{array}}$

i.e., knowing v_R and v_T, I can infer v and theta

v_R can be measured with the Doppler shifts

v_T is more complicated as it requires knowing the proper motion of the star and the distance d of the star from the Sun.

Proper motion: motion of a star with respect to the Sun



Consider angle μ and [d]=km

$$\mathrm{EA} = \mathrm{n v_T} \underset{\mu << 1}{=} \mathrm{d}\mu = \frac{\mathrm{d}}{\mathrm{R}''}\mu''$$

With R" number of arcsec in a radian, and the proper motion in a year expressed in arcsec/yr

$$\Rightarrow \quad \mathrm{v_T} = \frac{\mathrm{d}}{\mathrm{n}}\frac{\mu''}{\mathrm{R}''}$$

From parallax: $d = \dfrac{aR''}{P''}$ $\Rightarrow$ $\boxed{\mathrm{v_T} = \dfrac{\mathrm{a}}{\mathrm{n}}\dfrac{\mu''}{\mathrm{P}''} = \dfrac{\mathrm{a}}{\mathrm{n}}\dfrac{\mu}{\mathrm{P}}}$

w/ angles expressed in arcsec

Since a=1.496x10^8 km, n=3.156x10^7 s

$$\Rightarrow \quad \boxed{\mathrm{v_T} = \mathrm{v_R}\tan\theta = \mathrm{v}\sin\theta = 4.74\frac{\mu}{\mathrm{P}}}$$

$$\Leftrightarrow \quad \boxed{\frac{1}{\mathrm{P}} = \mathrm{d} = \frac{\mathrm{v_R}\tan\theta}{4.74\mu}}$$

With proper motion measured in "/yr, v_R in km/s, and d in parsec

Doppler shift:

$$\frac{\lambda_{\rm obs} - \lambda_{\rm rest}}{\lambda_{\rm rest}} = \frac{\Delta\lambda}{\lambda_{\rm rest}} = \frac{v_{\rm R}}{c}$$

Radial velocity v_R

$v_{\rm R} << c$

$$\boxed{v_{\rm R} = c\frac{\Delta\lambda}{\lambda_{\rm rest}}}$$

$$\mathrm{v_T} = 4.74\frac{\mu[''/\mathrm{yr}]}{\mathrm{p}['']}$$

$$\mathrm{v_T} = \mu\mathrm{d}$$

$$\boxed{\mathrm{v_T[km/s]} = 4.74\mu[''/\mathrm{yr}]\mathrm{d[pc]}}$$

$$v = \sqrt{v_{\rm T}^2 + v_{\rm R}^2}$$

Velocity of the star through space relative to the Sun

Photons:

$$E_{\mathrm{photon}} = h\nu = hc/\lambda = pc$$

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Bohr's model of Hydrogen atom:

$$\frac{1}{\lambda} = R_{\rm H}\Big(\frac{1}{m^2} - \frac{1}{n^2}\Big) = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\Big(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\Big)$$

Wavelength of emission/
absorption lines

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Permeability of free space

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Permeability of free space

Electric charge

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Permittivity of free space

Wavelength of emission/
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m<n, integers

Permittivity of free space

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Permittivity of free space

Wavelength of emission/
absorption lines

R_H=1.096775883x10^7 m^{-1}
Rydberg constant for H

Photons:

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Permeability of free space

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Permittivity of free space

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Rydberg constant for H

m=1, n=2, 3, 4, … Lyman series (UV)
m=2, n=3, 4, 5,… Balmer series (visible, near-UV)
m=3, n=4, 5, 6, … Paschen series (NIR)
m=4, n=5, 6, 7, … Brackett series (NIR, K-band)

Photons:

$$E_{\text{photon}} = h\nu = hc/\lambda = pc$$

Bohr's model of Hydrogen atom:

Permeability of free space

Electric charge

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

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Permittivity of free space

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$R_H = 1.096775883 \times 10^7$ m^{-1}
Rydberg constant for H

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m=4, n=5, 6, 7, … Brackett series (NIR, K-band)

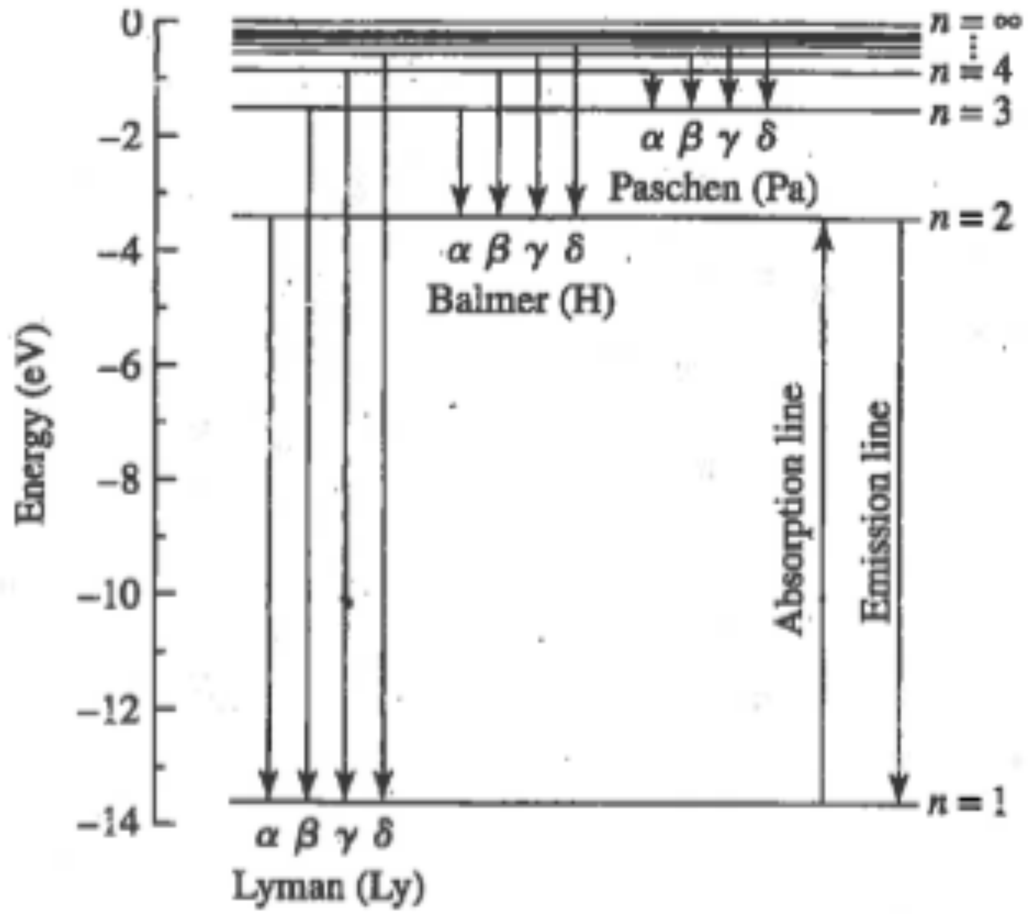
Allowed energy levels:

$$E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -13.6eV\frac{1}{n^2}$$

with n principal quantum number;
n=1 —> ground state —> 13.6 eV needed to ionized the H atom.

$$E_{\text{photon}} = h\nu = hc/\lambda = pc$$

$$E_{\text{photon}} = \Delta E = E_{\text{high}} - E_{\text{low}} = -13.6eV\left(\frac{1}{n_{\text{high}}^2} - \frac{1}{n_{\text{low}}^2}\right)$$



Emission

Absorption

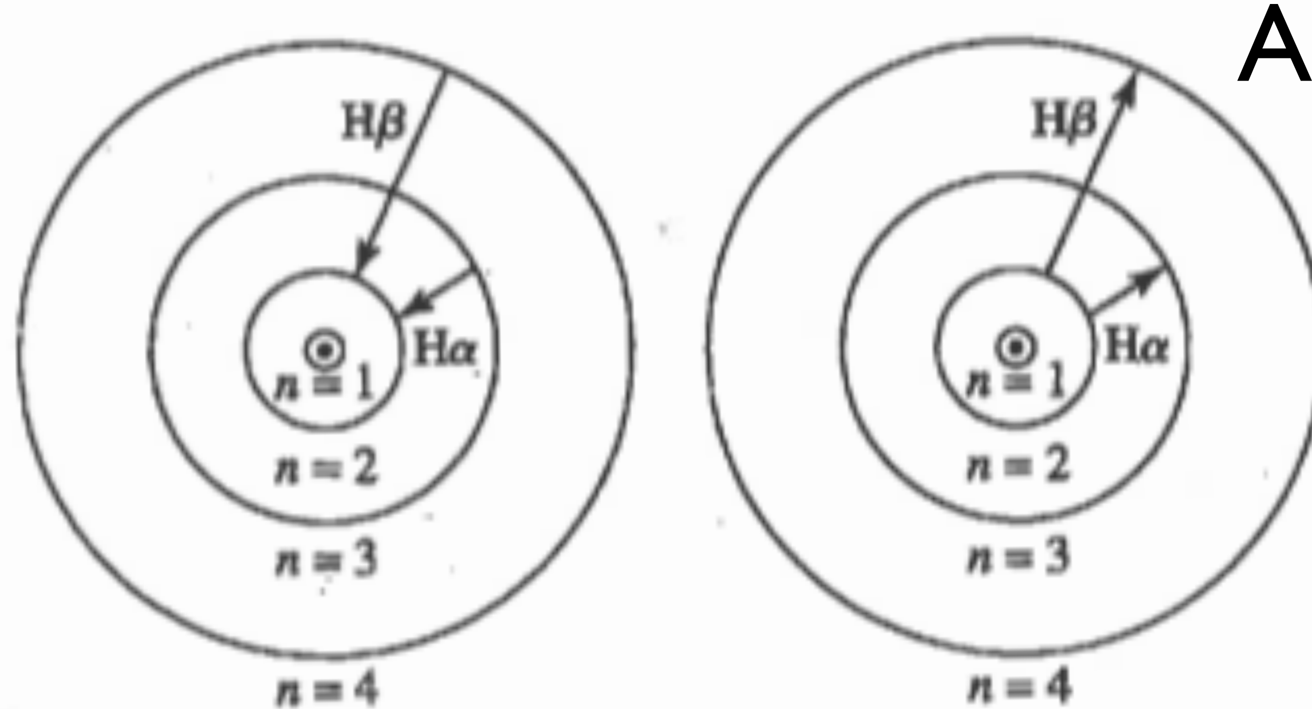


TABLE 5.2 The wavelengths of selected hydrogen spectral lines in air. (Based on Cox, (ed.), *Allen's Astrophysical Quantities*, Fourth Edition, Springer, New York, 2000.)

Series Name	Symbol	Transition	Wavelength (nm)	Medium
Lyman	Lyα	$2 \leftrightarrow 1$	121.567	vacuum
	Lyβ	$3 \leftrightarrow 1$	102.572	vacuum
	Lyγ	$4 \leftrightarrow 1$	97.254	vacuum
	$\mathrm{Ly}_{\mathrm{limit}}$	$\infty \leftrightarrow 1$	91.18	vacuum
Balmer	Hα	$3 \leftrightarrow 2$	656.281	air
	Hβ	$4 \leftrightarrow 2$	486.134	air
	Hγ	$5 \leftrightarrow 2$	434.048	air
	Hδ	$6 \leftrightarrow 2$	410.175	air
	Hϵ	$7 \leftrightarrow 2$	397.007	air
	H_8	$8 \leftrightarrow 2$	388.905	air
	$\mathrm{H}_{\mathrm{limit}}$	$\infty \leftrightarrow 2$	364.6	air
Paschen	Paα	$4 \leftrightarrow 3$	1875.10	air
	Paβ	$5 \leftrightarrow 3$	1281.81	air
	Paγ	$6 \leftrightarrow 3$	1093.81	air
	$\mathrm{Pa}_{\mathrm{limit}}$	$\infty \leftrightarrow 3$	820.4	air

TABLE 5.1 Wavelengths of some of the stronger Fraunhofer lines measured in air near sea level. The atomic notation is explained in Section 8.1, and the equivalent width of a spectral line is defined in Section 9.5. The difference in wavelengths of spectral lines when measured in air versus in vacuum are discussed in Example 5.3.1. (Data from Lang, *Astrophysical Formulae*, Third Edition, Springer, New York, 1999.)

Wavelength (nm)	Name	Atom	Equivalent Width (nm)
385.992		Fe I	0.155
388.905		H_8	0.235
393.368	K	Ca II	2.025
396.849	H	Ca II	1.547
404.582		Fe I	0.117
410.175	h, $H\delta$	H I	0.313
422.674	g	Ca I	0.148
434.048	G′, $H\gamma$	H I	0.286
438.356	d	Fe I	0.101
486.134	F, $H\beta$	H I	0.368
516.733	b_4	Mg I	0.065
517.270	b_2	Mg I	0.126
518.362	b_1	Mg I	0.158
588.997	D_2	Na I	0.075
589.594	D_1	Na I	0.056
656.281	C, $H\alpha$	H I	0.402

Binary Systems and Stellar Parameters

Reading assignment:
TUESDAY 9/29: Chapters 7.1, 7.2, 7.3

Homework Assignment #2
Due by TUESDAY 1/6