The Continuous Spectrum of Light - Part II

Reading assignment for:

THURSDAY 9/24: Chapters 5.1, 5.2 (not in detail), 5.3, 5.4 (not in detail)

Homework assignment #1 due NOW

If the star is at a distance d from the instrument:

$$f_{p} = \frac{L_{p}}{4\pi d^{2}} = \frac{R^{2}}{4} F_{p}$$

$$L_{p} = 4\pi R^{2} F_{p}$$

 $\left(4 \Rightarrow f_{\lambda} = \frac{R^2}{J^2} F_{\lambda}\right)$

If the star is at a distance d from the instrument:

$$f_{ij} = \frac{L_{ij}}{4\pi d^2} = \frac{R^2}{4} F_{ij}$$

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$$\left(4 \Rightarrow f_{\lambda} = \frac{R^2}{d^2} F_{\lambda} \right)$$

Incident energy flux on the detector (a.k.a., apparent brightness)

a.k.a. the inverse square law of light

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Incident energy flux on the detector (a.k.a., apparent brightness)

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NOTE 1: this law is true only without interstellar absorption

NOTE 2: by measuring
$$d^2 f_{7} \longrightarrow \mathbb{R}^2 \overline{F_{7}}$$

ONLY
ONLY
OBSERVED
QUANTITIES
QUANTITIES

 $\left(4 = 1 \right) = \frac{R^2}{d^2} F_{\lambda}$

Assuming no interstellar absorption:

i.e., the apparent magnitude depends on both the intrinsic luminosity and the distance of the star Assuming no interstellar absorption:

$$M_{\nu} = -2.5 \text{ bg } f_{\nu} + C \text{ monochromatic apparent magnitude}$$

$$f_{\nu} = \frac{L\nu}{4\pi d^2} \longrightarrow M_{\nu} = -2.5 \text{ bg } \frac{L\nu}{4\pi d^2} + C$$
i.e., the apparent magnitude depends on both the intrinsic luminosity and the

distance of the star

ABSOLUTE MAGNITUDE M: For an absolute comparison of intrinsic brightness, it is common to discuss the magnitudes of stars would have IF they were all at the same distance of d=10 pc, i.e., M = m(@d=10pc):

$$M_{\nu} := -2.5 \log \frac{L_{\nu}}{4\pi (10 \, \text{pc})^2} + C$$

$$M_{p} - m_{p} = -2.5 \text{ bg} \frac{L_{p}}{4\pi (10 \text{ pc})^{2}} + C + 2.5 \text{ bg} \frac{L_{p}}{4\pi d^{2}} - C = 5 \text{ bg} (10 \text{ pc}) - 5 \text{ bg} d$$

$$M_{\gamma} - M_{\gamma} = 5 - 5 \log d[pc]$$

 $M_{bd} = -2.5 \log \left(\frac{1}{f_{p}} dp + c \right)$ $\frac{L}{4\pi d^2} = \frac{4\pi R^2}{11\pi d^2} = \frac{R^2}{12} \mp$ Mbo) := - 2.5 Bg L+C $M_{bol} - M_{bol} = 5 - 5 \log d[pc]$ $M_{bol} - M_{bol,0} = -2.5 \log \frac{L}{L_0}$ $M_{bol,0} = 4.74 \qquad M_{bol} = -2.5 \text{ by } \frac{L}{10} + 4.74$ $M_{\odot} = -26.83 \qquad M_{\odot} = 3.839 \times 10^{33} \frac{203}{10} = \frac{203}{10} + 4.74$

$$M_{bol} = -2.5 \ \mbox{egg} \int_{0}^{\infty} f_{p} dp + c$$

$$\frac{L}{4\pi d^{2}} = \frac{4\pi R^{2} \int_{0}^{\infty} f_{p} dv}{4\pi d^{2}} = \frac{R^{2}}{d^{2}} F$$

$$M_{bol} := -2.5 \ \mbox{egg} L + c$$

$$M_{bol} - m_{bol} = 5 - 5 \ \mbox{egg} dr_{pc}$$

$$M_{bol} - M_{bol} = -2.5 \ \mbox{egg} \frac{L}{L_{0}}$$

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$$M_{bol} = 4.74$$

$$M_{bol} = -26.83$$

$$M_{bol} = 3.839 \ \mbox{xl}^{33} \ \ \mbox{egg}}$$

NOTE: M_{bol} is not directly measurable. To obtain the total energy radiated from a star requires making a **bolometric correction**

$$F = \frac{L}{4\pi d^2} \qquad [F] = \frac{eQ}{s cm^2}$$

$$F_{\odot} = \frac{L_{\odot}}{4\pi d^2} = \frac{3.839 \times 10^{26} \text{ W}}{(1.496 \times 10^{11} \text{ m})^2} = 1365 \frac{W}{M^2}$$

In bolometric units:
a.k.a. the inverse square law of light
$$F = \frac{L}{4\pi d^2}$$
 $F = \frac{e^{20}}{G}$
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Solar irradiance

 $M = M_{\odot} - 2.5 \ \log \frac{L}{10}$

$$W/MO = 4.74$$

 $LO = 3.839 \times 10^{26} W$

$$M = M_{\odot} - 2.5 Gg \left(\frac{F}{F_{0,0}}\right)$$

W/ Fro, = RADIANT FLUX OF THE SUN @ d = 10 pc Given the Sun and another star:

 $M = M_{\odot} - 2.5$ Bug

$$W/MO = 4.74$$

 $LO = 3.839 \times 10^{26} W$

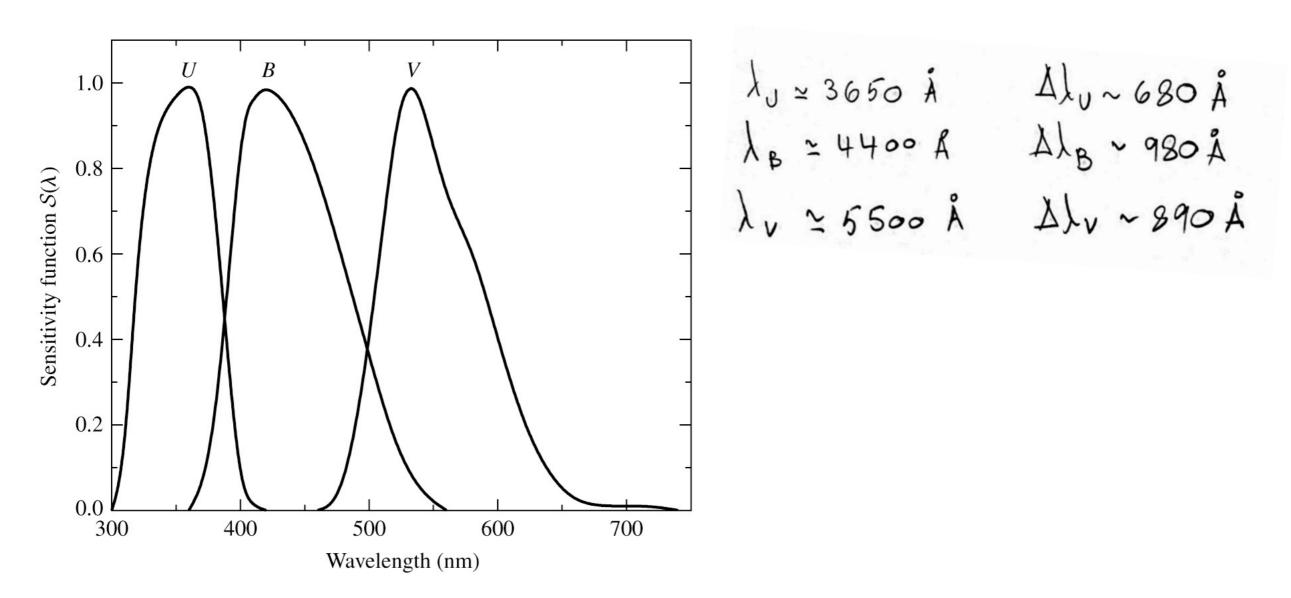
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1. Photographic system: m_{pg}=m_{visual} for A0 stars (Vega)

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- 2. Photometric system: UBV

 $U=m_U$, $B=m_B$ (similar to photographic plate magnitude), $V=m_V$ zero-point determined using the sequence of standard stars (A0 spectral type stars)

For Vega (A0 star, T_{surface}=10,000K), U-B=B-V=0

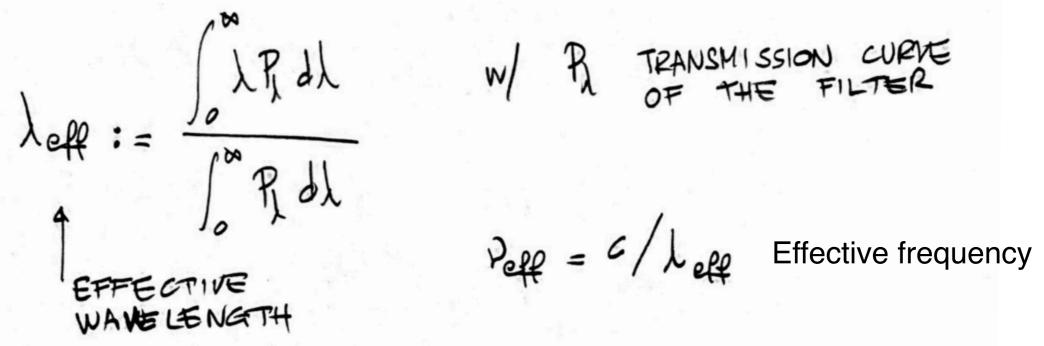


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3. Spectro-Photometric system



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For Vega (A0 star, T_{surface}=10,000K), U-B=B-V=0

- 3. Spectro-Photometric system
- 4. AB system (this has become the standard magnitude system):

$$M_{p}(AB) := -2.5 \log f_{p} - 48.6$$

IF $[f_{v}] = \frac{erg}{s cm^{2} Hz}$ Apparent brightness

$$U-B := M_U - M_B$$

 $B-V := M_B - M_V$

The smaller (or more negative) the color index, the bluer a star is. The color index is independent on distance of the star.

BOLOMETRIC CORRECTION BC := MBOL - MV

Astronomers measure the apparent magnitude (m) and the distance (d) of a star. The absolute magnitude (M) is computed by mentally moving the star at a distance d=10pc. The absolute magnitude is then converted to an absolute bolometric magnitude (M_{bol}) using the bolometric correction (BC).

$$M_{BOL} = M_V + BC =$$

$$= -2.5 \text{ BgL} + C = -2.5 \text{ BgL} \left[\frac{99}{5} \right] + 88.70$$

$$= -2.5 \text{ Bg} \frac{L}{L} + 4.72$$

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BC:= -2.5 Bg incident energy flux
from all frequencies
recorded every flux
= -2.5 Bg
$$\frac{\int_{0}^{\infty} f_{y} dv}{\int_{0}^{\infty} f_{p} P_{y} dv}$$

L filter response
curve

The BC depends on the spectral type of the star; known the spectral type, I can derive its absolute bolometric magnitude M_{bol} given BC (tabulated as a function of different spectral types) and M_V .

 $M_{\rm V,\odot} = 4.83$

Blackbody radiation:

Any object with a temperature above absolute zero emits light at all wavelengths with varying degrees of efficiency. An ideal blackbody is an object that absorbs all of the light energy incident upon it, and reradiates this energy with the characteristic spectrum. The radiation emitted by a blackbody is called blackbody radiation. Stars, planets, and humans are blackbodies, to a rough first approximation.

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Considering the power (energy per unit of time) radiate per unit of area per unit of wavelength by a surface in thermodynamic equilibrium (i.e., having already integrated the specific intensity over the solid angle):

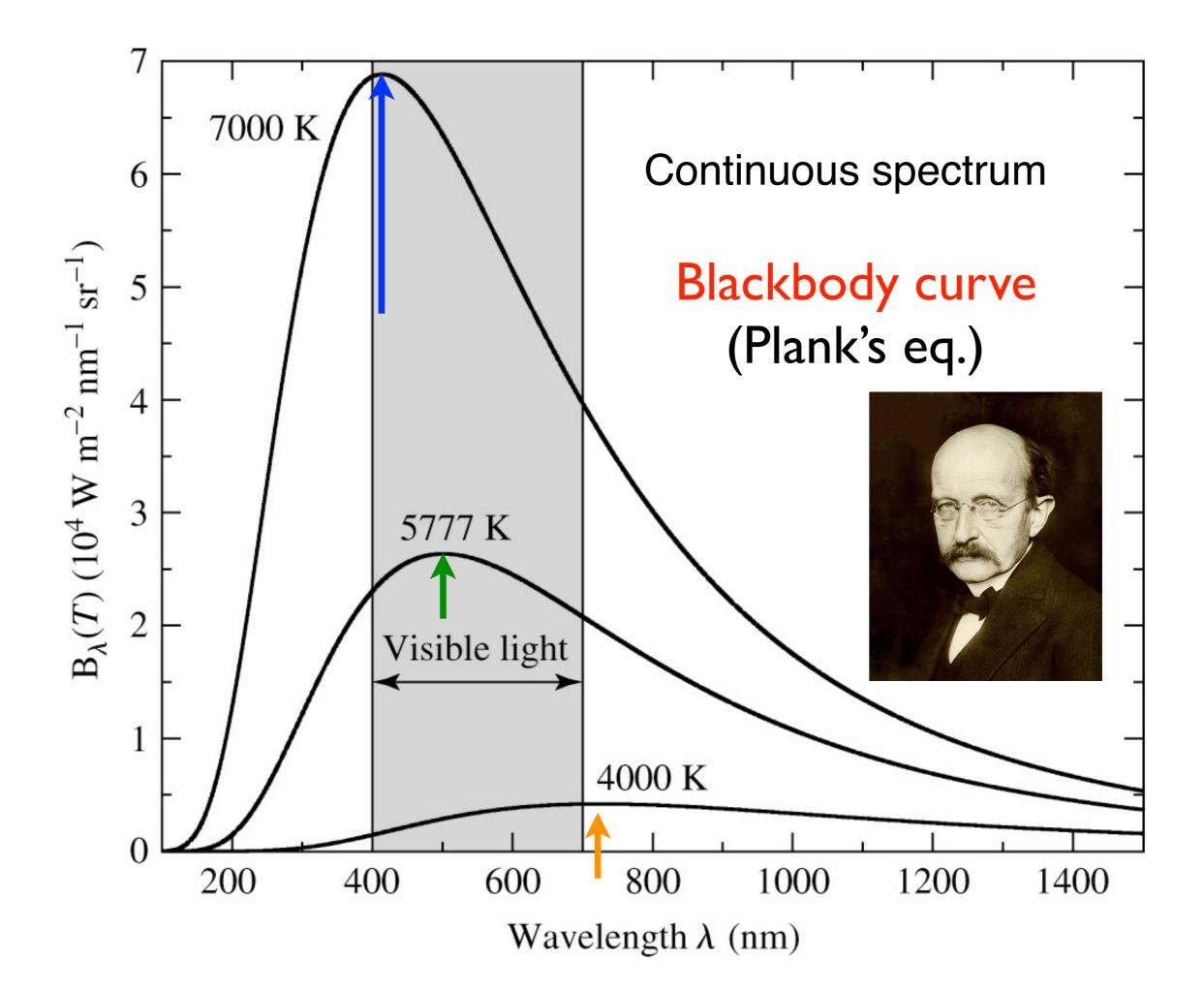
Planck's function 2TT hp 1 hp/k $=\frac{2\pi c^2 h}{15} \frac{hc/lkT}{1}$

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Planck's function $F_{L} = \frac{2\pi c^{2}h}{\lambda^{5}} \frac{hc/\lambda kT}{hc/\lambda kT} \qquad F_{p} = \frac{2\pi hp^{3}}{c^{2}} \frac{hp/kT}{p}$ $[F_{\lambda}] = \frac{\text{erg}}{\text{s cm}^2 \text{ Å}}$ $[F_{\nu}] = \frac{\mathrm{erg}}{\mathrm{s} \mathrm{cm}^2 \mathrm{Hz}}$



Total power (energy per unit of time) radiated per unit of area, i.e., the surface flux:

$$F = \int_{0}^{\infty} F_{\lambda} d\lambda = \frac{2\pi^{5}}{15} \frac{1}{c^{2}} \frac{K^{4}}{h^{3}} T^{4} = G T^{4}$$

$$x = \frac{hc}{\lambda kT} \qquad W/ = 5.6704 \times 10^{-8} \frac{W}{m^{2} s' K^{4}}$$

$$\int_{0}^{\infty} \frac{x^{3}}{e^{x} - 1} = \frac{\pi^{4}}{15} \qquad = 5.67 \times 10^{-5} \frac{erg}{cm^{2} s' k^{4}}$$

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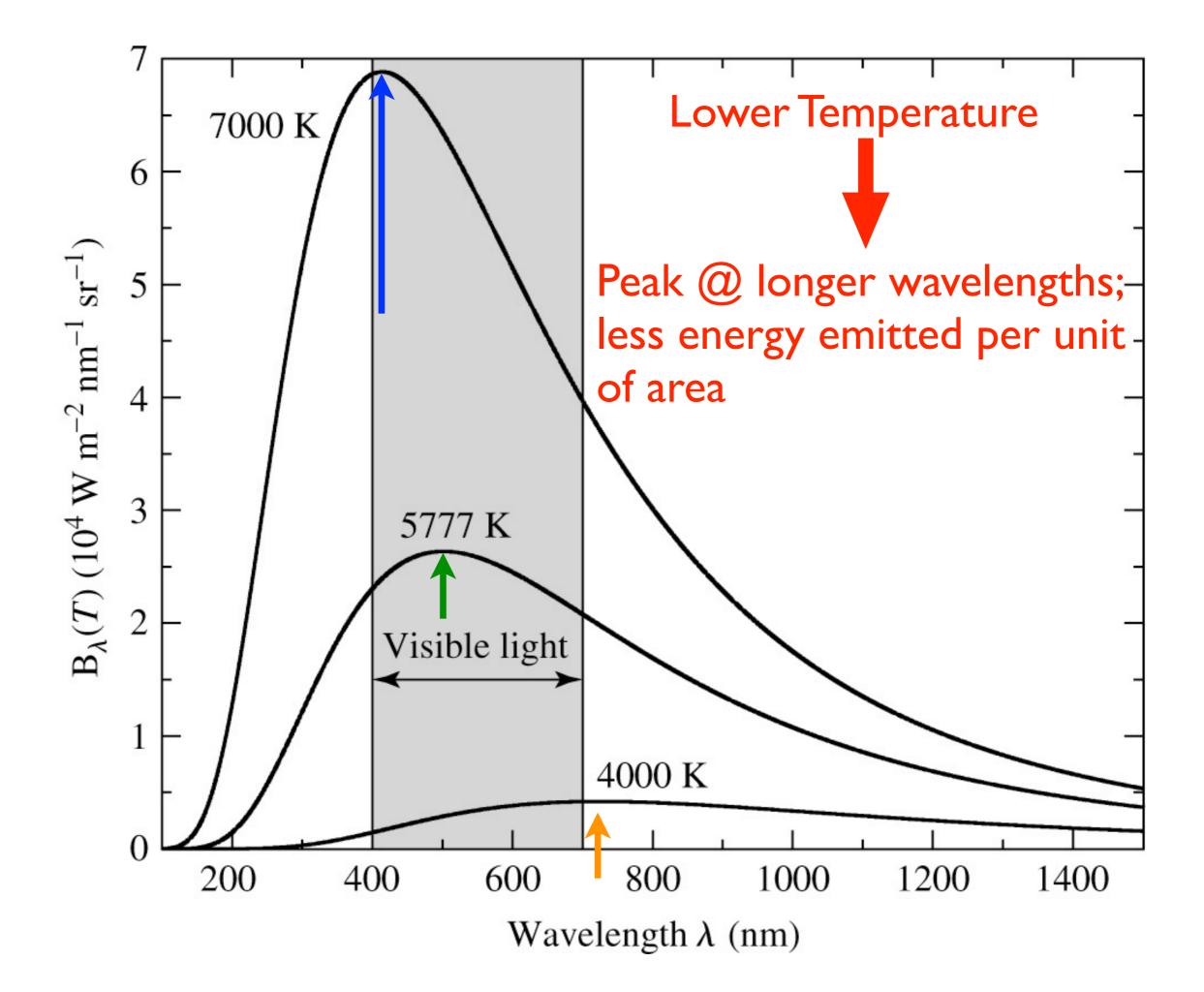
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$$[F] = \frac{\text{erg}}{\text{s cm}^{2}} \int_{\sigma}^{\infty} \frac{x^{3}}{e^{x}-1} = \frac{\pi^{4}}{15} \qquad = 5.672 \times 10^{-5} \frac{\text{erg}}{cm^{2}s' k^{4}}$$

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 $L_{L}dl = 4\pi R^2 F_{L}dl$ AREA OF SURFACE OF STAR

 $L = 4\pi R^2 F = 4\pi R^2 \sigma T_{eff}^4$

Luminosity:

ity: $L_{\lambda} d\lambda = 4\pi R^2 F_{\lambda} d\lambda$ AREA OF SURFACE OF STAR $= 4\pi R^2 F = 4\pi R^2 \sigma T_{opp}^4$

Luminosity:

This relation is used to estimate stellar radii by measuring the luminosity L and knowing the surface temperature T_{eff} .

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Effective temperature T_{eff} is defined as the temperature of a blackbody with the same radiated power per unit of area F (1st definition of temperature)

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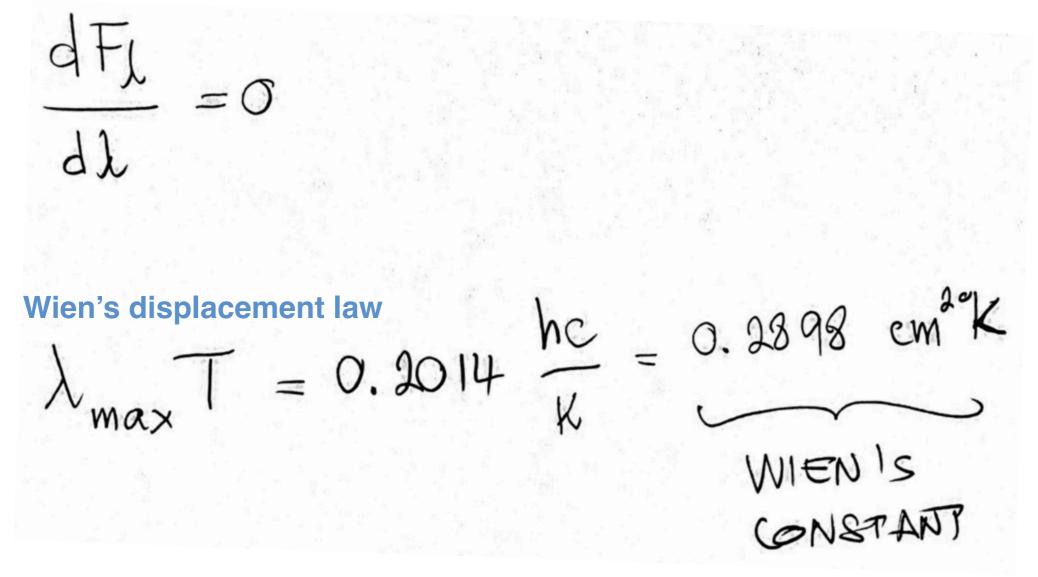
Monochromatic flux for an observer at a distance d:

$$f_{1} dl = \frac{L_{\lambda} dl}{4\pi d^{2}} = \left(\frac{R}{d}\right)^{2} \frac{4\pi c^{2}h}{\lambda^{5}} \frac{dl}{e^{ne/lkT}}$$

$$f = \left(\frac{R}{d}\right)^{2} \sigma T^{4}$$

Energy of starlight with wavelength between λ and $\lambda + d\lambda$ arriving per second on the unit of surface of detected aimed at the star. $[f_{\lambda}d\lambda] = \frac{\text{erg}}{\text{s} \text{ cm}^2}$ Position in wavelength of the maximum (peak) of the blackbody radiation:

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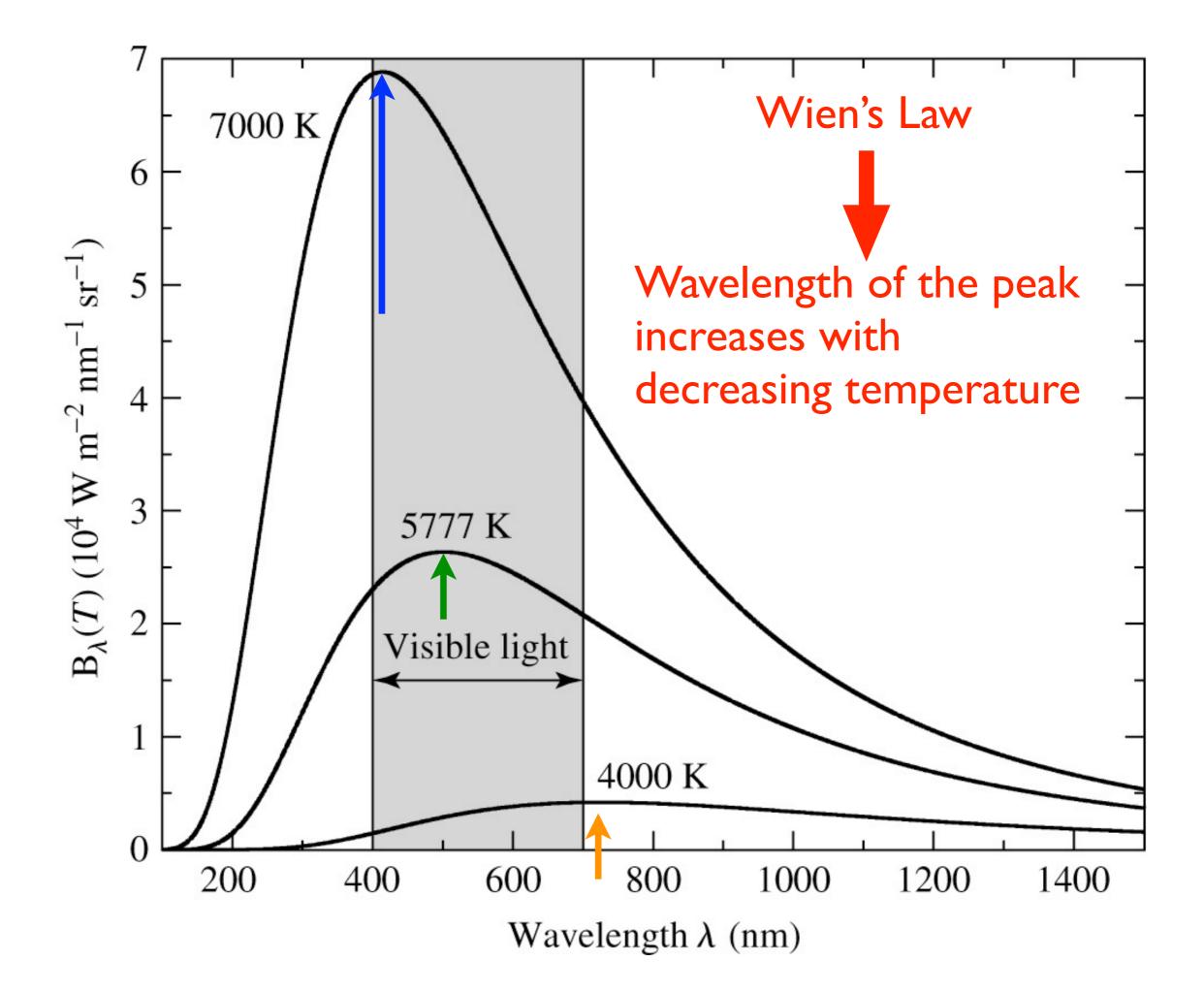
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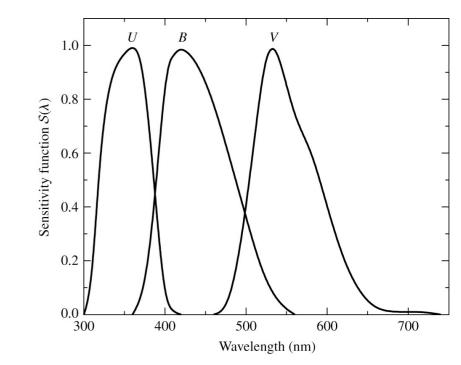
$$\frac{dF_{L}}{d\lambda} = 0$$
Wien's displacement law
$$\lambda_{max}T = 0.2014 \quad \frac{hc}{k} = 0.2898 \quad cm^{20}K$$
WIEN's
WIEN'S
CONSTANT

.

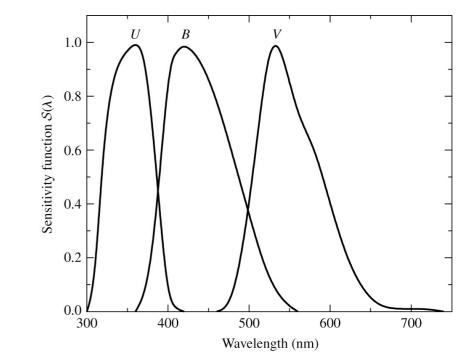
NOTE:

For
$$\lambda$$
 LARGE $\frac{1}{hc/\lambda kT} \rightarrow \frac{1}{hc/\lambda kT}$
 $e^{-1} \rightarrow hc/\lambda kT$
 $e^{-1} \rightarrow \frac{2\pi c kT}{\lambda^4}$
Rayleigh-
wavelength
radiation of





$$m_{U} = U = -2.5 \text{ Gg} \int_{0}^{10} S_{0} f_{L} dL + C_{0} f_{0} f_{0}$$



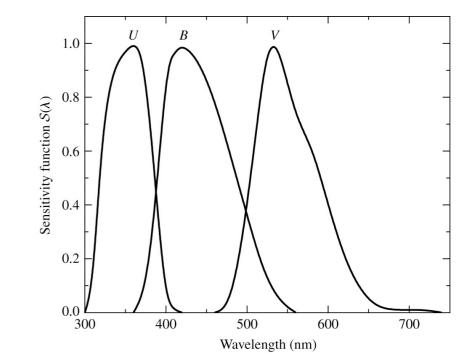
$$m_{U} = U = -2.5 \text{ bg} \int_{0}^{10} S_{0} f_{L} dL + C_{0}$$

$$m_{B} = B$$

$$m_{V} = V$$

$$Transmission function$$

$$S(L) = PL$$



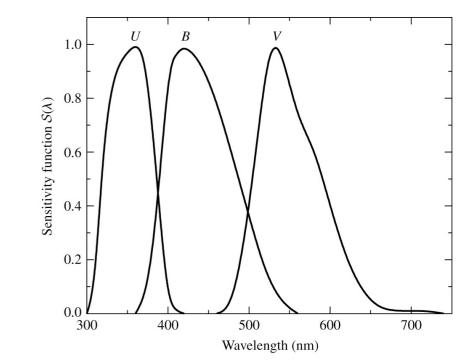
$$m_{U} = U = -2.5 \text{ bg} \int_{0}^{W} S_{0} f_{L} dL + C_{0}$$

$$m_{B} = B$$

$$m_{V} = V$$

$$Transmission function$$

$$S(L) = PL$$



$$m_{U}-m_{B} = U-B$$

$$U-B = -2.5 \text{ fog} \frac{\int_{0}^{\infty} S_{J} f_{J} dJ}{\int_{0}^{\infty} S_{B} f_{J} dJ} + C_{V-B}$$

$$= C_{U}-C_{B}$$

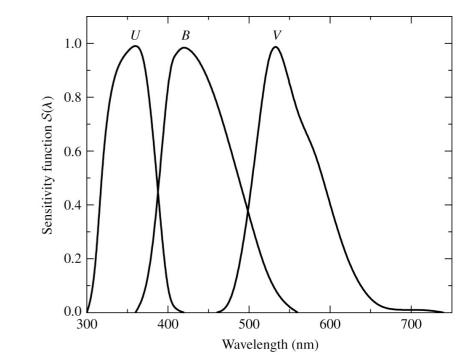
$$m_{U} = U = -2.5 \text{ bg} \int_{0}^{10} S_{0} f_{L} dL + C_{0}$$

$$m_{B} = B$$

$$m_{V} = V$$

$$Transmission function$$

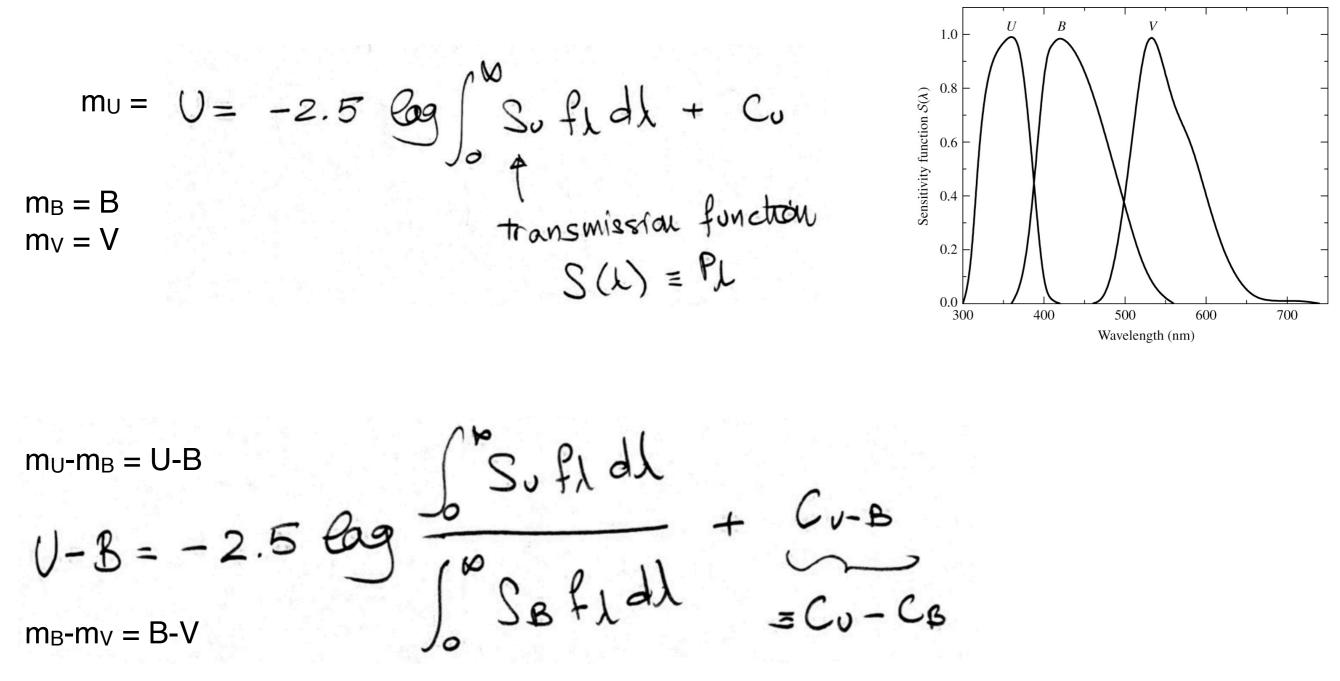
$$S(L) = PL$$



$$m_{U}-m_{B} = U-B$$

$$U-B = -2.5 \text{ Gag} \frac{\int_{0}^{\infty} S_{J} f_{J} dJ}{\int_{0}^{\infty} S_{B} f_{J} dJ} + C_{V-B}$$

$$m_{B}-m_{V} = B-V \qquad \qquad \int_{0}^{\infty} S_{B} f_{J} dJ = C_{U}-C_{B}$$



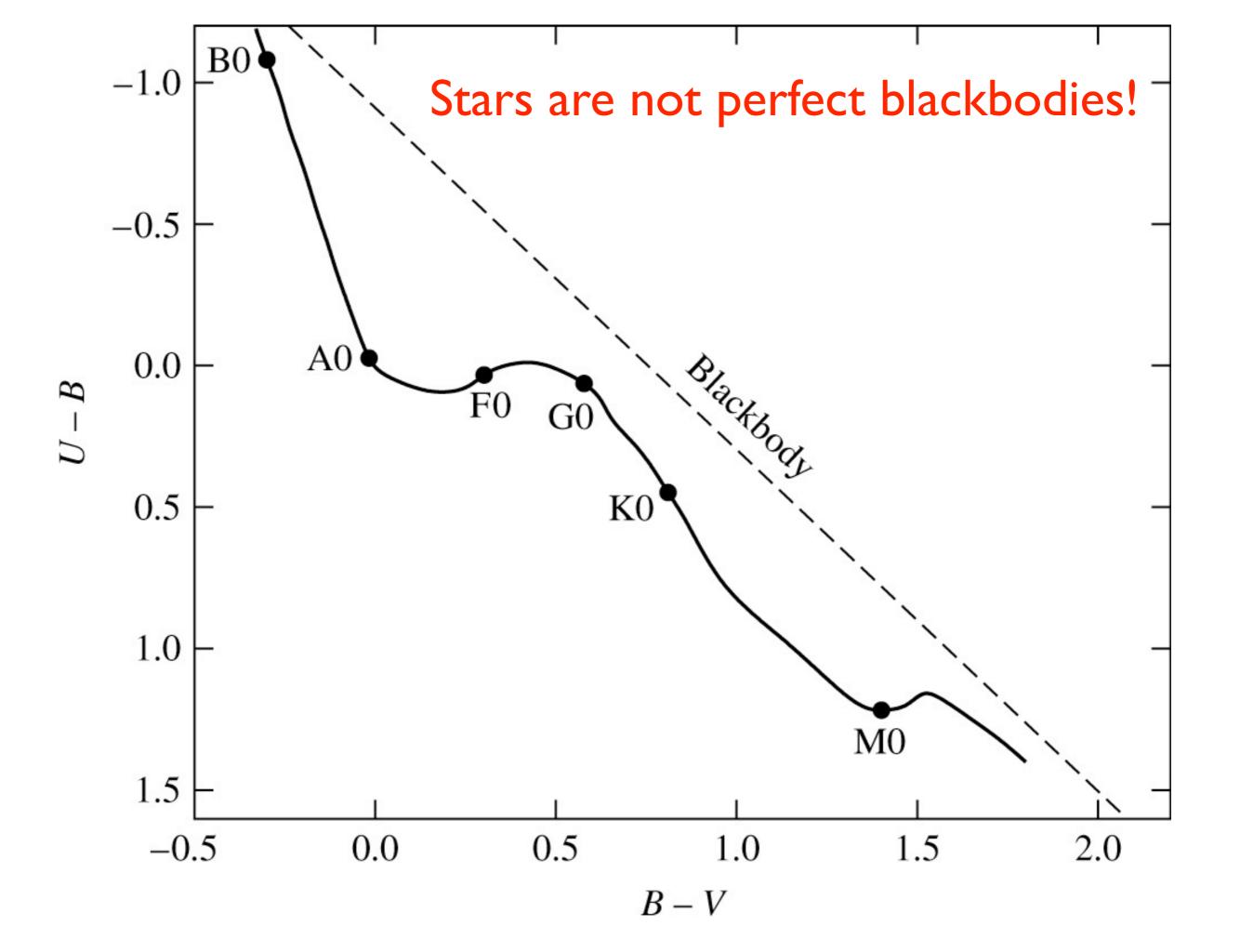
NOTE: the color does not depend on (R/d)² because this term cancels out in the above equation **The color is solely dependent on the temperature of a model blackbody star**

$$\begin{split} m_{1} - m_{2} &= -2.5 \ \text{egg}\left(\frac{f_{\lambda_{1}}}{f_{\lambda_{2}}}\right) = -2.5 \ \text{egg}\left(\frac{F_{\lambda_{1}}}{F_{\lambda_{2}}}\right) + c \\ &= A + \frac{1.56}{T_{color}} \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right) + f(T_{cd}) \\ w/ \quad f(T_{c}) &= 2.5 \ \text{egg}\left(\frac{1 - e^{-hc/\lambda_{2}KT_{col}}}{1 - e^{-hc/\lambda_{1}KT_{col}}}\right) \\ &= B - V \simeq -0.586 + \frac{6350}{T_{col}F_{cl}} + f(T_{cd}) \\ B - V &= 0.64 \end{split}$$

$$\begin{split} m_{1} - m_{2} &= -2.5 \log\left(\frac{f\lambda_{1}}{f\lambda_{2}}\right) = -2.5 \log\left(\frac{F\lambda_{1}}{F\lambda_{2}}\right) + c \\ &= A + \frac{1.56}{T_{color}} \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right) + f(T_{cd}) \\ w/ f(T_{c}) &= 2.5 \log\left(\frac{1 - e}{1 - e^{-hc/\lambda_{2}KT_{col}}}\right) \\ &= B - V \simeq -0.586 + \frac{6350}{T_{col}FK} + f(T_{cd}) \\ B - V &\equiv -0.64 \end{split}$$

$$\begin{split} m_{1} - m_{2} &= -2.5 \ \text{log}\left(\frac{f_{\lambda_{1}}}{f_{\lambda_{2}}}\right) = -2.5 \ \text{log}\left(\frac{F_{\lambda_{1}}}{F_{\lambda_{2}}}\right) + C \\ &= A + \frac{1.56}{T_{\text{color}}} \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right) + f(T_{\text{cd}}) \\ & \text{w/} \quad f(T_{\text{c}}) = 2.5 \ \text{log}\left(\frac{1 - e^{-hc/\lambda_{2}KT_{\text{col}}}}{1 - e^{-hc/\lambda_{1}KT_{\text{col}}}}\right) \\ & \text{B-V} \simeq -0.586 + \frac{6850}{T_{\text{col}}\Gamma_{\text{K}}} + f(T_{\text{cd}}) \\ & \text{B-V} \mid_{\odot} = 0.64 \end{split}$$

The hotter are star, the smaller (or more negative) the B-V color, hence the bluer the star



The Interaction of Light & Matter

Reading assignment for:

THURSDAY 9/24: Chapters 5.1, 5.2 (not in detail), 5.3, 5.4 (not in detail)