The Continuous Spectrum of Light

Reading assignment for next week:

THURSDAY: NO LECTURE
TUESDAY 9/22: Chapters 3.4, 3.5, 3.6

Homework assignment:
Posted on the AST-31 website
Due date: Tuesday, September 22 (before class)

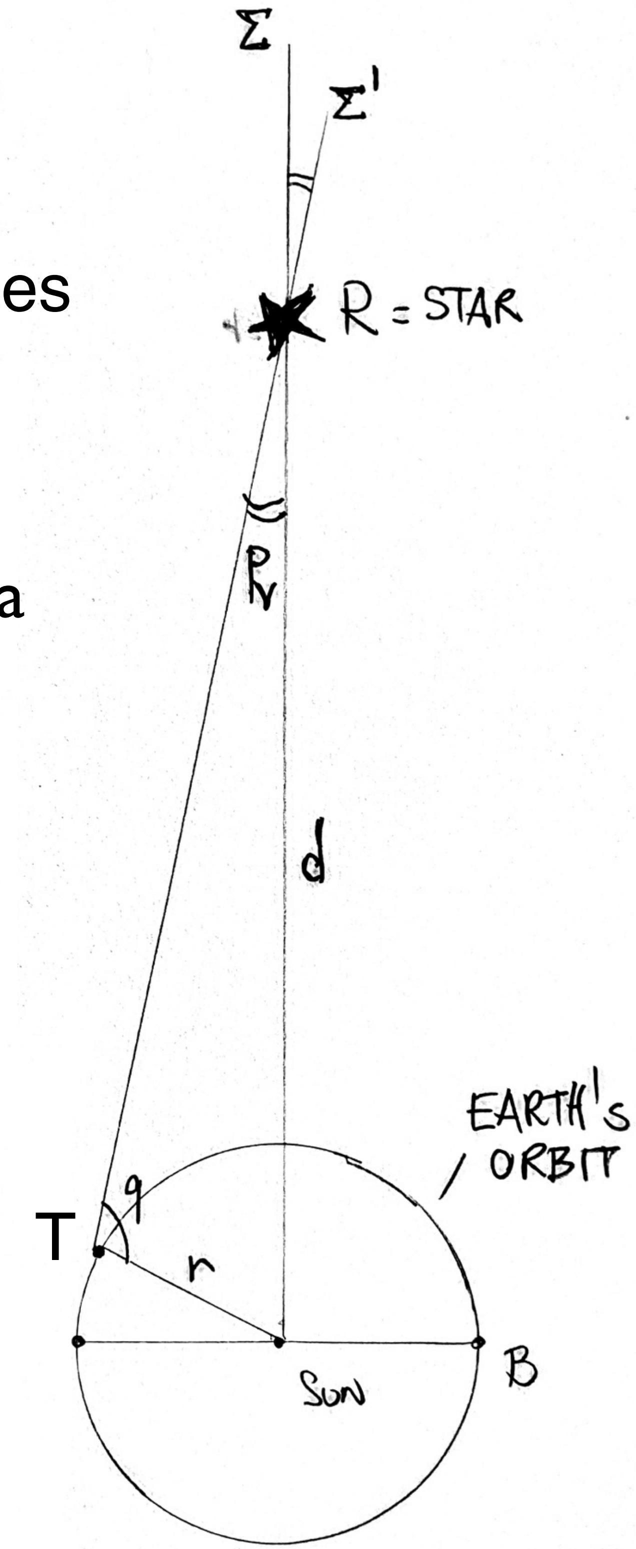
Stellar Parallax

To measure the intrinsic brightness of stars, we need:

I. Observed flux/brightness

II. distance

➡ **Stellar parallax**: the only direct method to measure distances

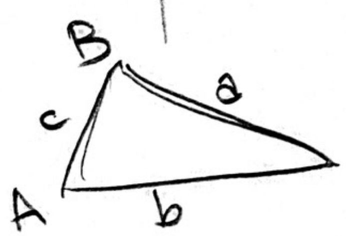


$\hat{\Sigma\Sigma} = T\hat{R}S \equiv P_v$ ANNUAL PARALLAX OF THE STAR (variable quantity)

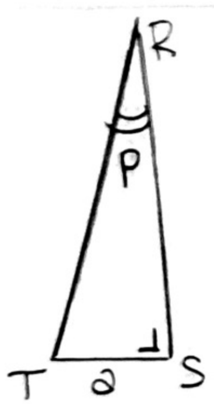
$r = \overline{TS}$ $d = \overline{RS}$ $q = S\hat{T}R$ $\overline{AS}$ = SEMI-MAJOR AXIS OF EARTH'S ORBIT = a

T=Earth

$\Rightarrow \sin P_v = \frac{r}{d} \sin q$



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



I DEFINE THE ANGLE P SUCH THAT $\tan P = \frac{a}{d}$

IE: P IS THE ANGLE CORRESPONDING TO THE SEMI-MAJOR AXIS OF EARTH'S ORBIT IF OBSERVED FROM THE STAR R

$\Rightarrow$ P (PARALLAX ANGLE) $\equiv \max(P_v)$

SINCE P IS SMALL FOR ALL STARS $\Rightarrow \tan P \simeq P$

⬇

$P = \frac{a}{d}$

$$P = \frac{a}{d} \Rightarrow P[''] = \frac{a}{d} R'', \quad \text{with R}'' = 206265 \text{ number of arcsec in a radian}$$

a = 1 AU = 1.496 x 10^8 km $\qquad$ a 206265 = 3.086 x 10^{13} km = 1 parsec

$$d[pc] = \frac{1}{p['']}$$

NOTE: The closest star, alpha-Cen, has p=0.76”

NOTE: Uncertainties and distance limit for applying the parallax method

$$\frac{\Delta p}{p} = \frac{\Delta d}{d}$$

From the ground, error in p is ~0.007”; to have relative error <35% (0.35), then **d<50 pc**

ESA Hipparcos, error in p is ~0.001”, distances of ~120K stars at **d<1 kpc**

ESA Gaia, error in p is ~0.00001”, distances out to **d<100 kpc**

NASA SIM PlanetQuest, error in p is ~0.000004”, distances out to **d<250 kpc**

The Electromagnetic Spectrum

Wavelength: $\lambda = \frac{c}{\nu}$ Energy: $E = h\nu$

Gamma-ray	$\lambda < 0.01\ nm$
X-ray	$0.01\ nm < \lambda < 10\ nm$
Ultraviolet	$10\ nm < \lambda < 400\ nm$
Visible (or optical)	$400\ nm < \lambda < 700\ nm$
Infrared	$700\ nm < \lambda < 1\ mm$
Microwave	$1\ mm < \lambda < 10\ cm$
Radio	$10\ cm < \lambda$

These ranges are only indicative, not strict...

Magnitude scale

Let's start considering bolometric magnitudes, i.e., over all wavelengths of light (i.e., integrated over all wavelengths or frequencies)

Pogson's formula: the human eye has a nearly logarithmic subjective response to radiant energy flux, i.e.:

$$m_2 - m_1 \equiv -2.5 \log{(l)} + c$$

where "l" is the apparent brightness; "c" is the zero point constant, which depends on the specific of the adopted magnitude system and the units of the brightness.

NOTE: 1(5) mag difference —> 2.5(100)x difference in brightness
NOTE: Typical absolute uncertainty in apparent magnitude is ~0.01 mag; typical relative uncertainty in apparent magnitude is ~0.002 mag.

Total power of output of a star

$$L_{\text{tot}} = L_{\gamma} + L_{\nu} + L_{\dot{m}}$$

Photon luminosity; [L] = erg/s luminosity is power, i.e., energy emitted per second

Neutrino luminosity: mean free path of neutrinos in water is 10^9 R_{Sun}, i.e., neutrino escape without interacting

Mass-loss luminosity (catastrophic, w/ rapid and violent change in structure of star, e.g., SNe), planetary nebula, red giants, T-Tauri. For the corona of Sun: $L_{\dot{m}} \approx 2 \times 10^{-6} L_{\odot}$

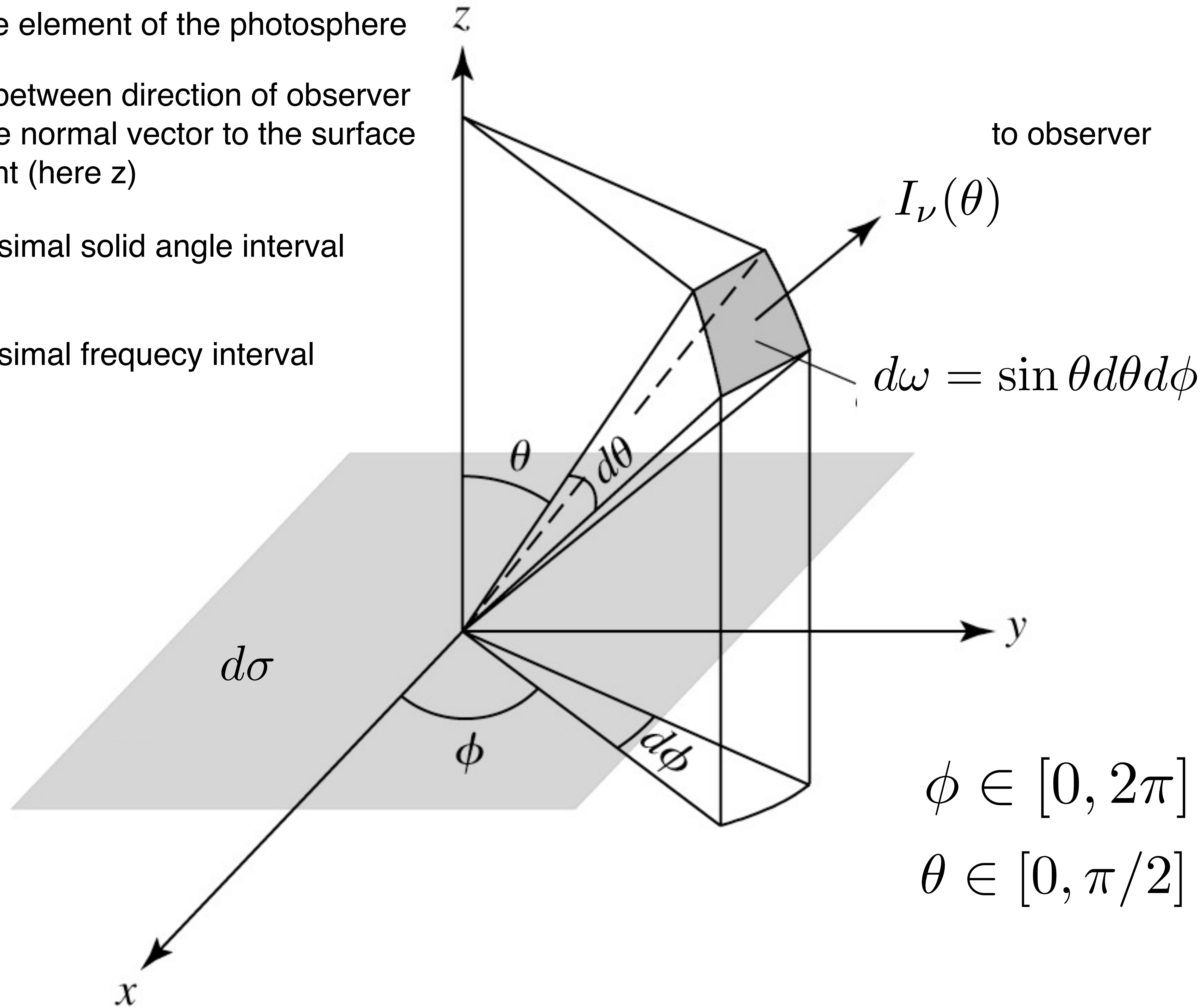
Let's focus on the photon luminosity (L), which originates from the photosphere of stars (i.e., the surface); from the interior of a star, we receive no photons, due to the total opacity being very large, except neutrinos.

$d\sigma$ surface element of the photosphere

θ angle between direction of observer and the normal vector to the surface element (here z)

$d\omega$ infinitesimal solid angle interval

$d\nu$ infinitesimal frequecy interval



If we define the **SPECIFIC INTENSITY OF THE RADIATION** AS the energy emitted per unit of time, per unit of surface, per unit of solid angle, and per unit of frequency

$$[I_\nu(\theta)] = \frac{erg}{s\ cm^2\ str\ Hz}$$

⬇

Energy emitted by the surface element in the unit of time, in the infinitesimal solid angle around the direction to the observer in the infinitesimal frequency interval:

$$dE_\nu = I_\nu(\theta) \cos\theta d\sigma d\nu d\omega$$

since we want the component along the direction to the observer, i.e., we are interested only in the photons arriving perpendicularly to our detector

ENERGY FLUX: $$F_\nu = \int I_\nu(\theta)\cos\theta d\omega$$

Assuming spherical symmetric (only the semi-sphere): $d\omega = 2\pi\sin\theta d\theta$

$$F_\nu = 2\pi\int_0^{\pi/2} I_\nu(\theta)\sin\theta\cos\theta d\theta$$

i.e., integrating for the outward hemisphere

Energy flux: total energy emitted in the unit of time, unit of surface, and unit of frequency

$$[F_\nu] = \frac{erg}{s\ cm^2\ Hz}$$

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Energy flux: total energy emitted in the unit of time, unit of surface, and unit of frequency

$$[F_\nu] = \frac{erg}{s\ cm^2\ Hz}$$

NOTE: $\nu = c/\lambda$ ➡ $d\nu/d\lambda = -c/\lambda^2$

$$F_\lambda d\lambda = F_\nu d\nu$$

➡ $$F_\lambda = \frac{c}{\lambda^2} F_\nu$$

To go from energy flux in frequency to energy flux in wavelength

Since I am interested in the energy emitted by the photosphere in the unit of time (i.e., L), I need to integrate over the surface and the frequency

Monochromatic photon luminosity

$$L_\nu = 4\pi R^2 F_\nu \qquad [L_\nu] = \frac{\text{erg}}{\text{s Hz}}$$

surface of sphere

NOTE: A star is alway seen as a point-like object (except for the Sun and a handful of other cases). Therefore, observations always return the monochromatic photon luminosity (or the monochromatic energy flux if the radius of the star is known). Only for the Sun I am able to determine the specific intensity.

Integrating over all frequencies, I get the **total photon luminosity**, i.e., the total photon energy emitted by the star in the unit of time:

$$L = 4\pi R^2 \int_0^\infty F_\nu \, d\nu = 4\pi R^2 \int_0^\infty F_\lambda \, d\lambda$$

$$[L] = \frac{\text{erg}}{\text{s}}$$

If the star is at a distance d from the instrument:

$$f_\nu = \frac{L_\nu}{4\pi d^2} = \frac{R^2}{d^2} F_\nu \qquad \left(\Leftrightarrow f_\lambda = \frac{R^2}{d^2} F_\lambda \right)$$

$$L_\nu = 4\pi R^2 F_\nu$$

Incident energy flux on the detector (a.k.a., apparent brightness)

a.k.a. the inverse square law of light

NOTE 1: this law is true only without interstellar absorption

NOTE 2: by measuring $d^2 f_\nu \longrightarrow R^2 F_\nu$

$d^2 f_\nu$: ONLY OBSERVED QUANTITIES

$R^2 F_\nu$: ONLY PHYSICAL (INTRINSIC) QUANTITIES

Assuming no interstellar absorption:

$$m_\nu = -2.5 \log f_\nu + c$$ monochromatic apparent magnitude

$$f_\nu = \frac{L_\nu}{4\pi d^2} \longrightarrow m_\nu = -2.5 \log \frac{L_\nu}{4\pi d^2} + c$$

i.e., the apparent magnitude depends on both the intrinsic luminosity and the distance of the star

ABSOLUTE MAGNITUDE M: For an absolute comparison of intrinsic brightness, it is common to discuss the magnitudes of stars would have IF they were all at the same distance of d=10 pc, i.e., **M = m(@d=10pc):**

$$M_\nu := -2.5 \log \frac{L_\nu}{4\pi (10\,pc)^2} + c$$

$$M_\nu - m_\nu = -2.5 \log \frac{L_\nu}{4\pi (10\,pc)^2} + C + 2.5 \log \frac{L_\nu}{4\pi d^2} - C =$$

$$= \dots = 5 \log (10\,pc) - 5 \log d$$

$$\Rightarrow \boxed{M_\nu - m_\nu = 5 - 5 \log d[pc]}$$

$$m_\nu - M_\nu := \text{DISTANCE MODULUS} = 5 \log d[pc] - 5$$

$$m_{bol} = -2.5 \log \int_0^\infty f_\nu \, d\nu + c$$

$$\frac{L}{4\pi d^2} = \frac{4\pi R^2 \int_0^\infty F_\nu \, d\nu}{4\pi d^2} = \frac{R^2}{d^2} F$$

$$M_{bol} := -2.5 \log L + c$$

$$\Rightarrow \quad M_{bol} - m_{bol} = 5 - 5 \log d[pc]$$

$$M_{bol} - M_{bol,\odot} = -2.5 \log \frac{L}{L_\odot}$$

$$M_{bol,\odot} = 4.74$$

$$m_\odot = -26.83$$

$$\Rightarrow \quad M_{bol} = -2.5 \log \frac{L}{L_\odot} + 4.74$$

$$L_\odot = 3.839 \times 10^{33} \ \frac{erg}{s}$$

NOTE: M_{bol} is not directly measurable. To obtain the total energy radiated from a star requires making a **bolometric correction**

In bolometric units:

a.k.a. the inverse square law of light

$$F = \frac{L}{4\pi d^2}$$

$$[F] = \frac{\text{erg}}{\text{s cm}^2}$$

$$[L] = \frac{\text{erg}}{\text{s}}$$

$$F_\odot = \frac{L_\odot}{4\pi d^2} = \frac{3.839 \times 10^{26}\ \text{W}}{(1.496 \times 10^{11}\ \text{m})^2} = 1365\ \frac{\text{W}}{\text{m}^2}$$

Solar irradiance

Given the Sun and another star:

$$M = M_\odot - 2.5 \log \frac{L}{L_\odot}$$

w/ $M_\odot = 4.74$

$L_\odot = 3.839 \times 10^{26}$ W

$$m = M_\odot - 2.5 \log \left(\frac{F}{F_{10,\odot}} \right)$$

w/ $F_{10,\odot}$ = RADIANT FLUX OF THE SUN @ d = 10 pc

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