White Dwarfs / Neutron Stars

Reading assignment

TUESDAY 12/8: 15.4 (Gamma ray bursts)

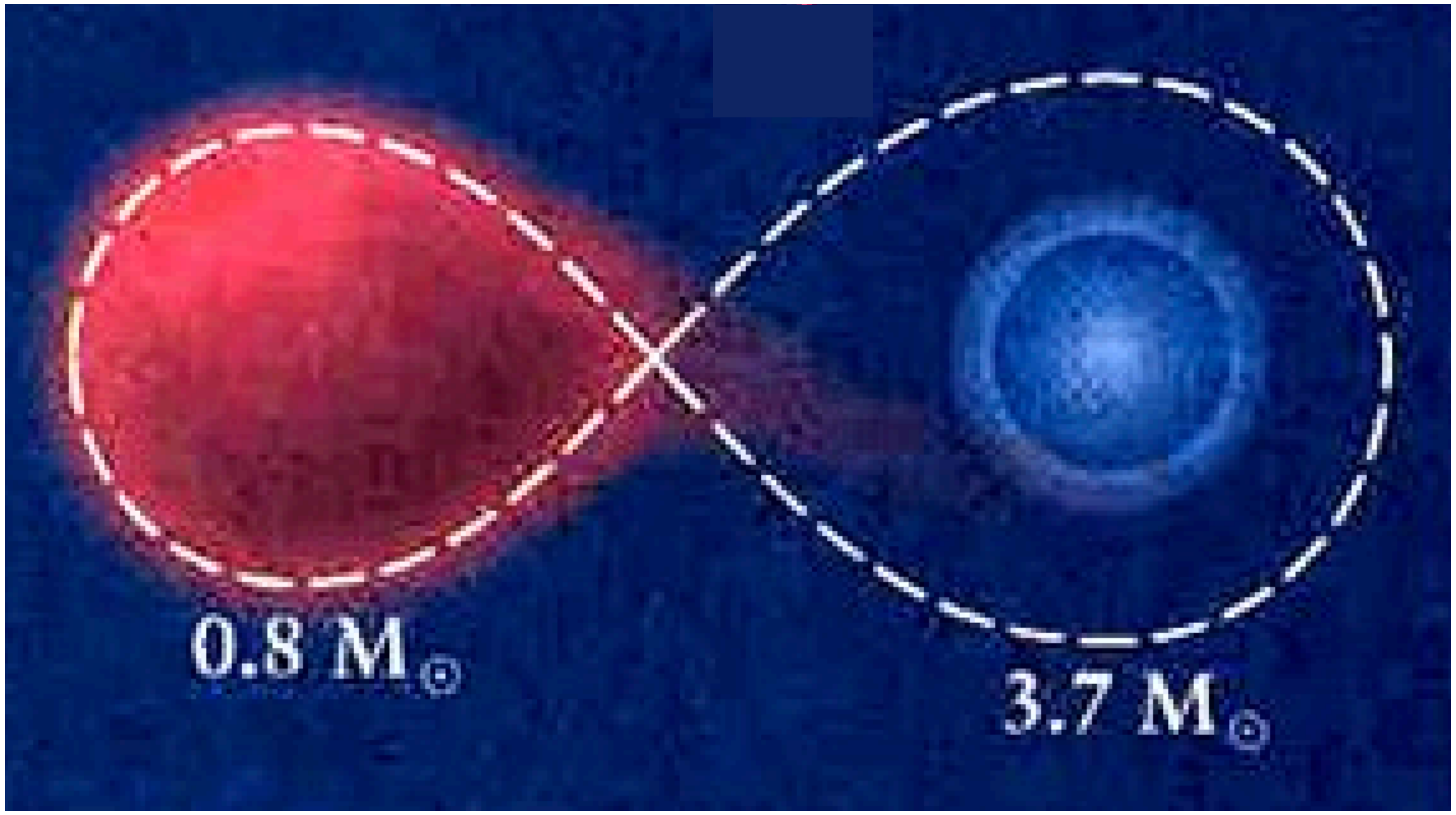
Homework Assignment #5 due by:

TUESDAY 12/15 before 9AM.

ALGOL binary system:

A (more evolved) sub-giant star of 0.8 M_{Sun} with a main-sequence companion of 3.7 M_{Sun}.

What's wrong with this system?



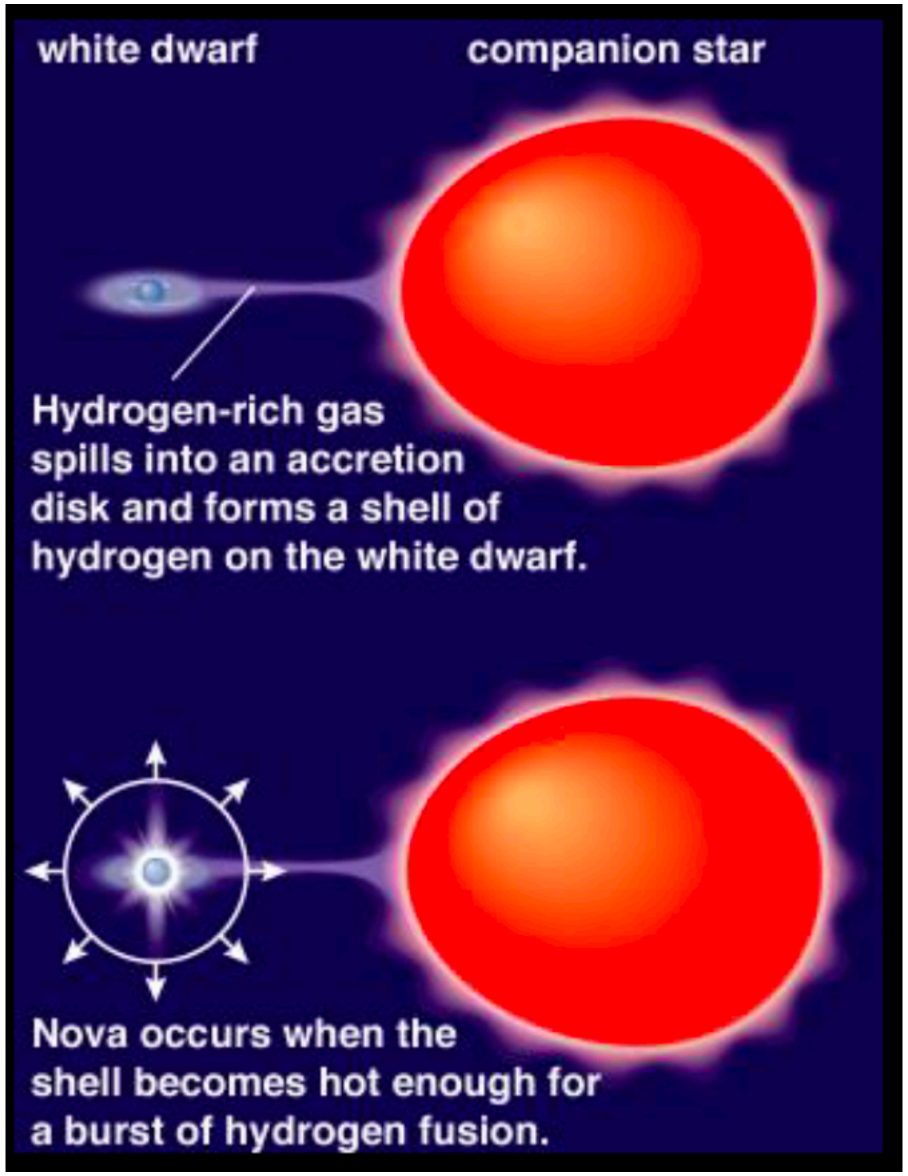
When a WD is the primary component of a semi-detached binary system:

- Dwarf nova
- Classical nova
- Supernova (type Ia) [single-degenerate]

When a WD is the primary component of a semi-detached binary system:

- Dwarf nova
- Classical nova
- Supernova (type Ia) [single-degenerate]

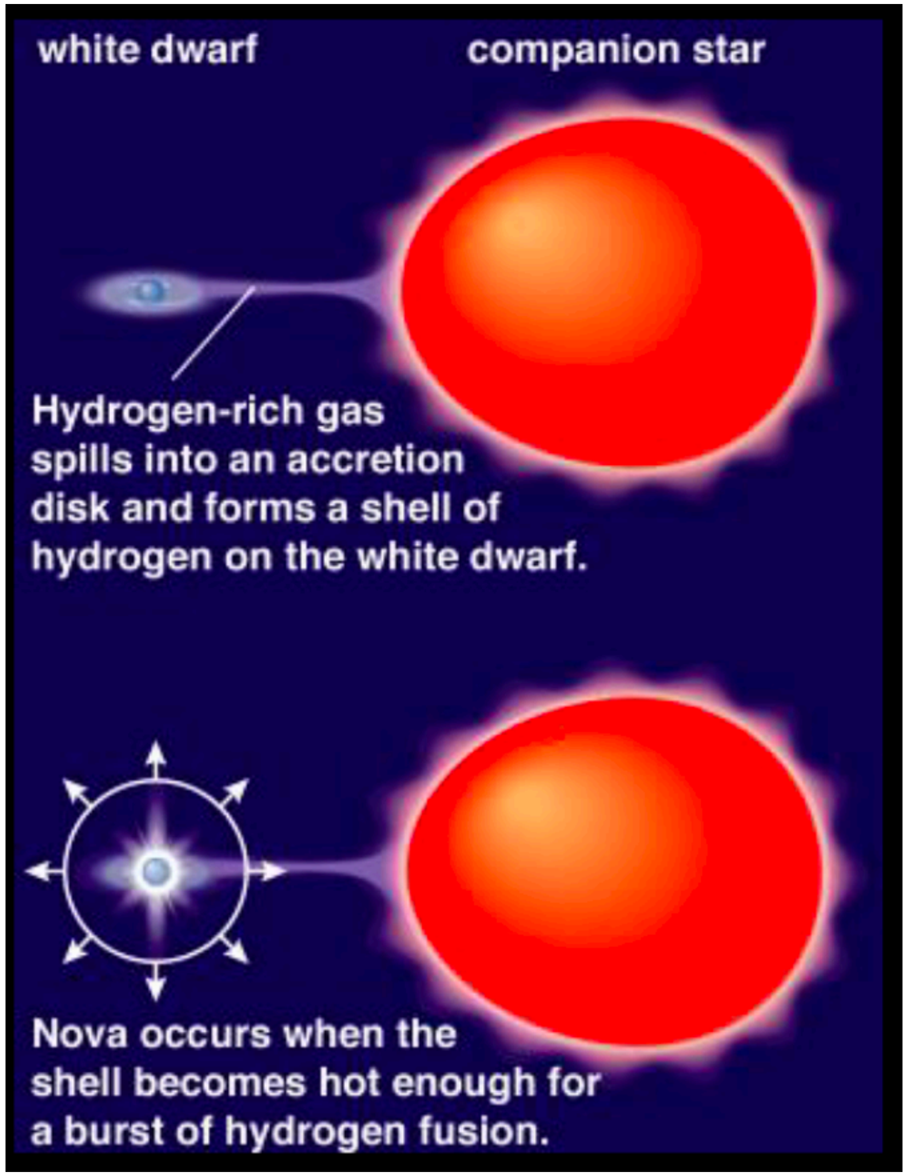
Nova is a long quiescent WD with outbursts with increase in luminosity by a factor of 10 (dwarf nova) to 10^6 (classical nova). Mass is transferred from the main sequence companion of type G or later to the WD forming an accretion disk.



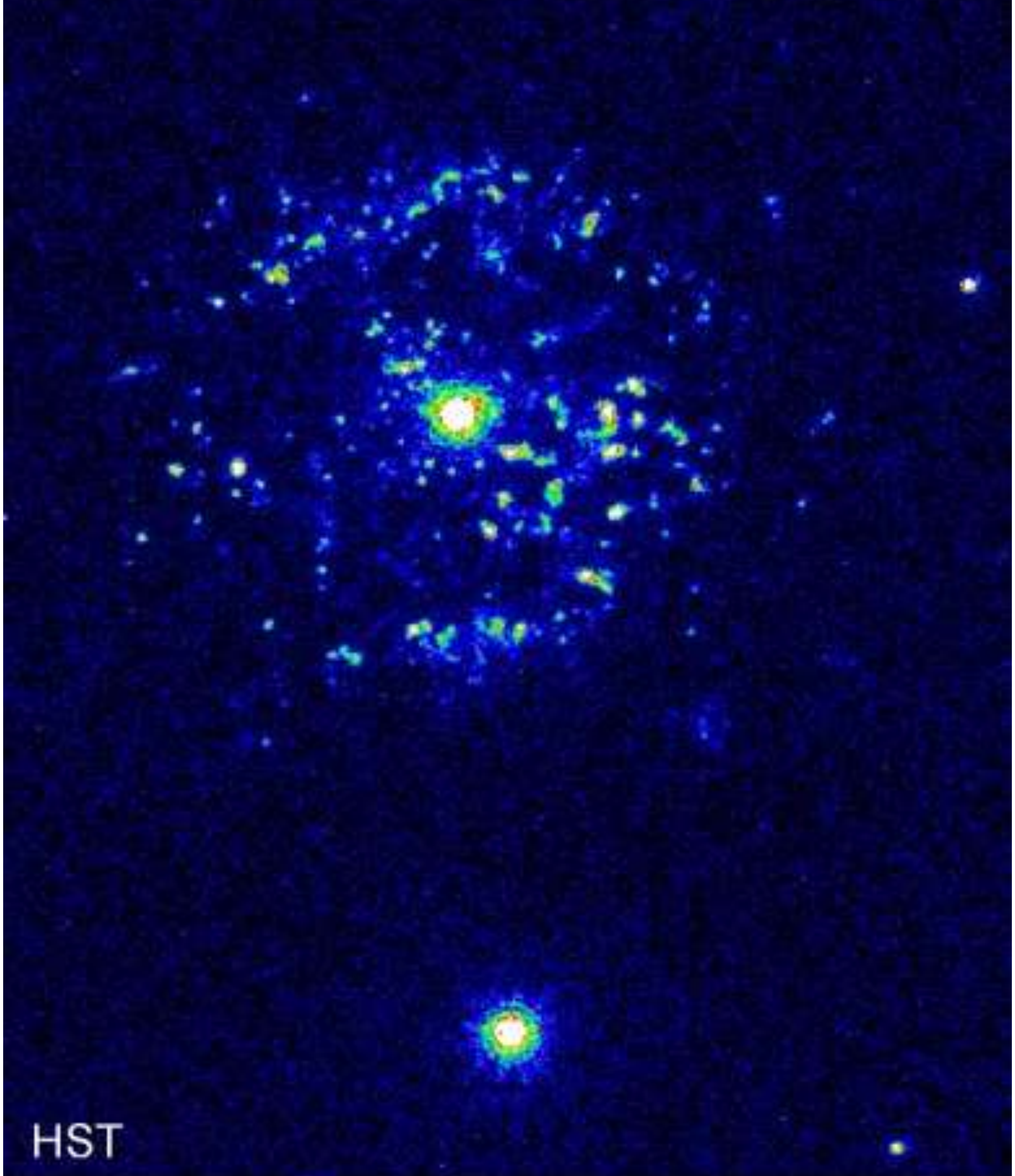
When a WD is the primary component of a semi-detached binary system:

- Dwarf nova
- Classical nova
- Supernova (type Ia) [single-degenerate]

Nova is a long quiescent WD with outbursts with increase in luminosity by a factor of 10 (dwarf nova) to 10^6 (classical nova). Mass is transferred from the main sequence companion of type G or later to the WD forming an accretion disk.

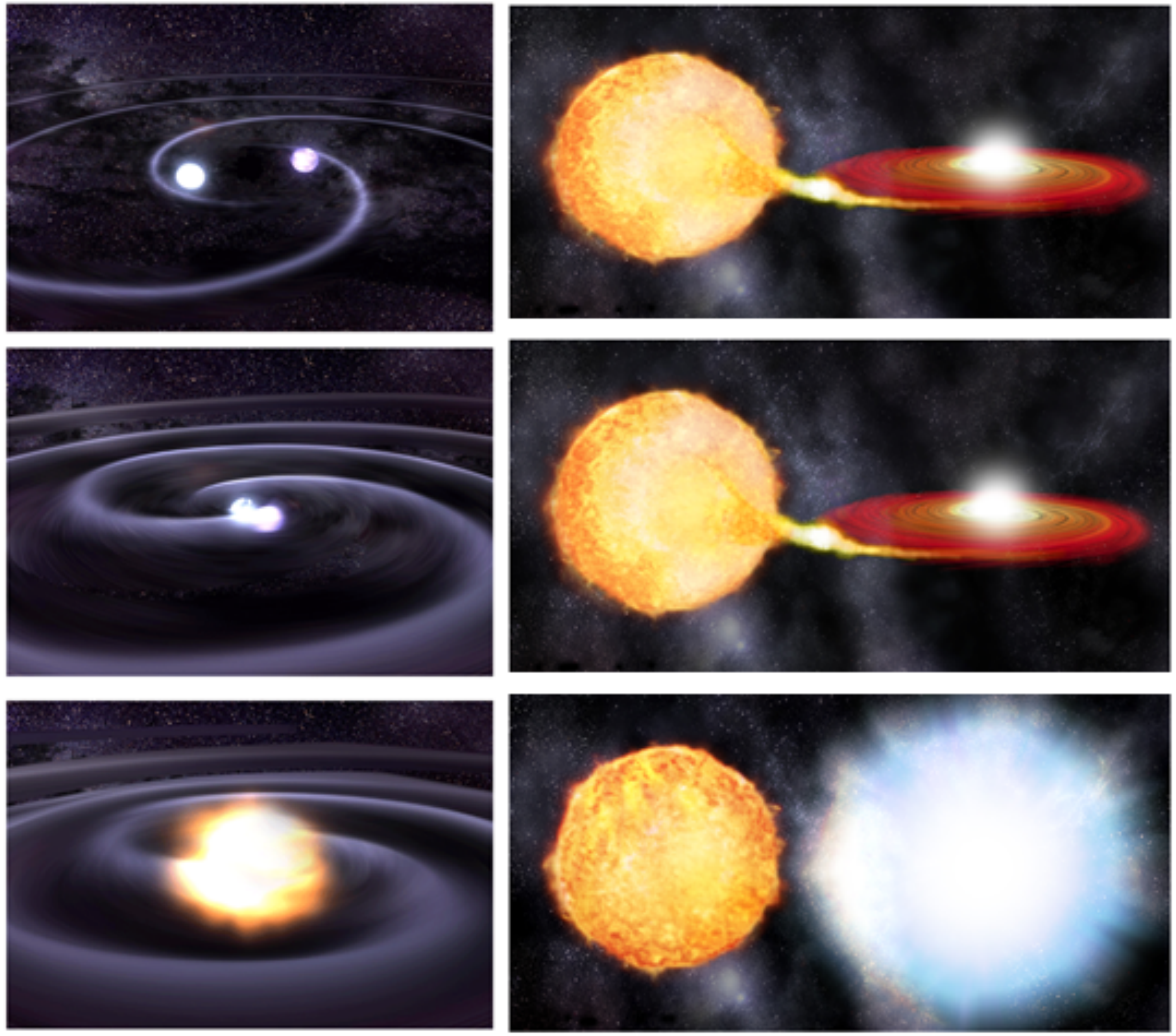


Classical nova: high accretion rates (10^{-8}-10^{-9} M_{Sun}/yr) from the companion
The H-rich gas accumulates on the surface of the WD, where it compresses and heats. At the base of the H layer, H is mixed with C, O, N and supported by electron degenerate pressure. When 10^{-4}-10^{-5} M_{Sun} of H has accumulated and T~ a few 10^6 K, explosive (because degenerate matter) H fusion ignites through the CNO cycle in the shell, resulting in the ejection of material. This phenomenon is recurrent, with a period of 10^4-10^5 yr.


HST

There are two principle avenues for WD mass growth

Double Degenerate   Single Degenerate



In either case, if the mass exceeds the Chandrasekhar mass, then the electron degeneracy pressure cannot support the star and it collapses under gravity resulting in a supernova — ***no remnant***!

Accretion disks

Accretion disks

Type Ia (white dwarf) SN

The Center for Astrophysical Thermonuclear Flashes

Simulation of the Deflagration and Detonation Phases of a Type Ia Supernovae

30 initial bubbles in 100 km radius.
Ignition occurs 80 km from the center of the star.
Hot material is shown in color and stellar surface in green.

This work was supported in part at the University of Chicago by the DOE NNSA ASC ASAP and by the NSF. This work also used computational resources at LBNL NERSC awarded under the INCITE program, which is supported by the DOE Office of Science.

An Advanced Simulation and Computation (ASC)
Academic Strategic Alliances Program (ASAP) Center
at The University of Chicago

ASC

Type Ia (white dwarf) SN

The Center for Astrophysical Thermonuclear Flashes

Simulation of the Deflagration and Detonation Phases of a Type Ia Supernovae

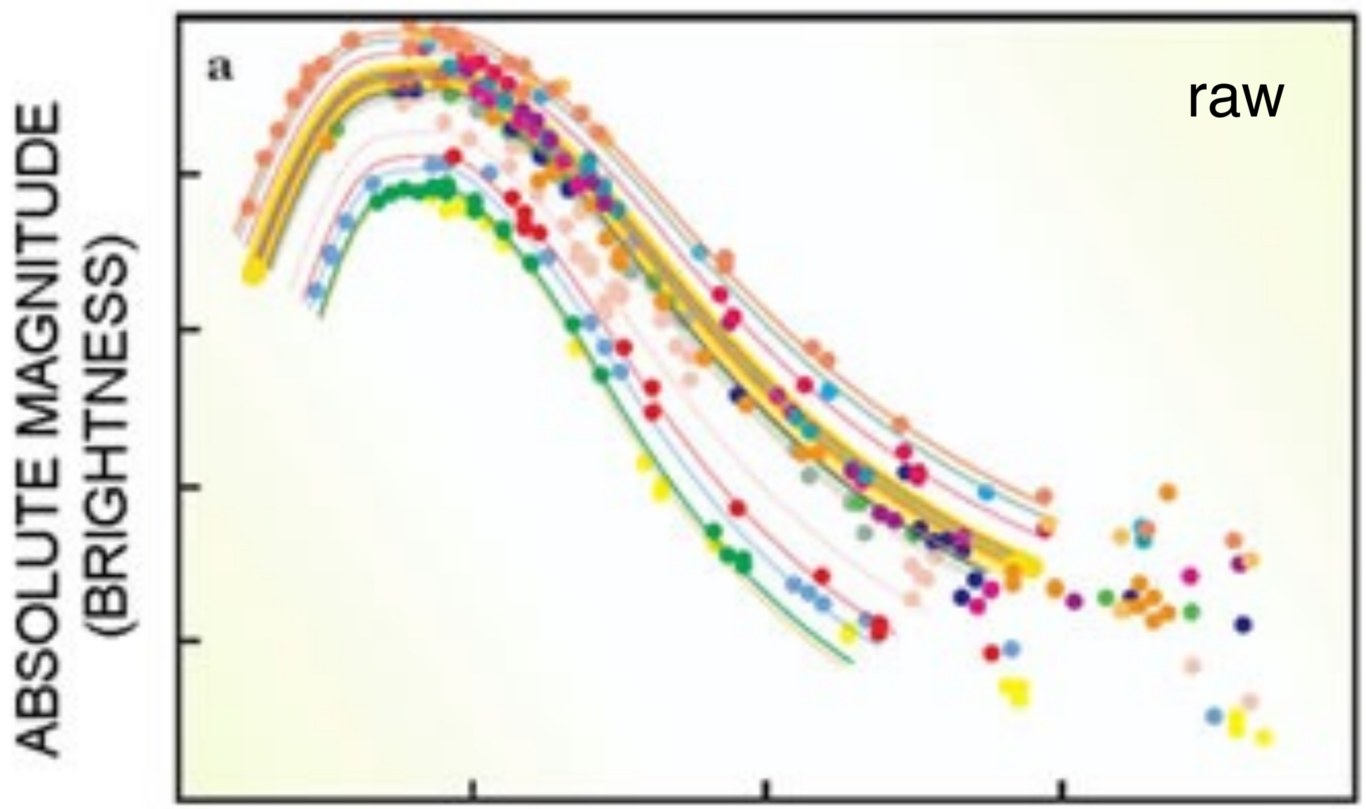
30 initial bubbles in 100 km radius.
Ignition occurs 80 km from the center of the star.
Hot material is shown in color and stellar surface in green.

This work was supported in part at the University of Chicago by the DOE NNSA ASC ASAP and by the NSF. This work also used computational resources at LBNL NERSC awarded under the INCITE program, which is supported by the DOE Office of Science.

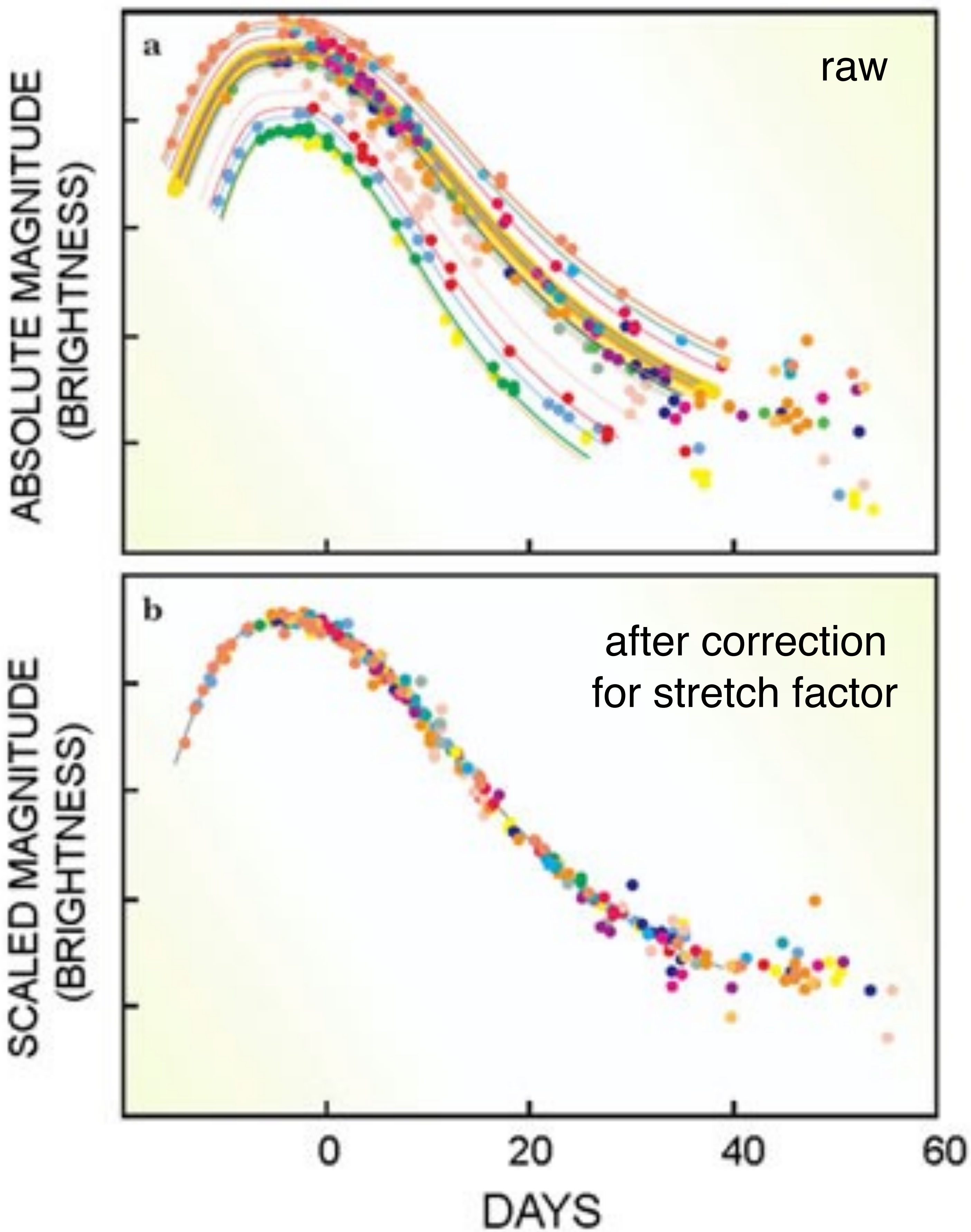
An Advanced Simulation and Computation (ASC)
Academic Strategic Alliances Program (ASAP) Center
at The University of Chicago

ASC

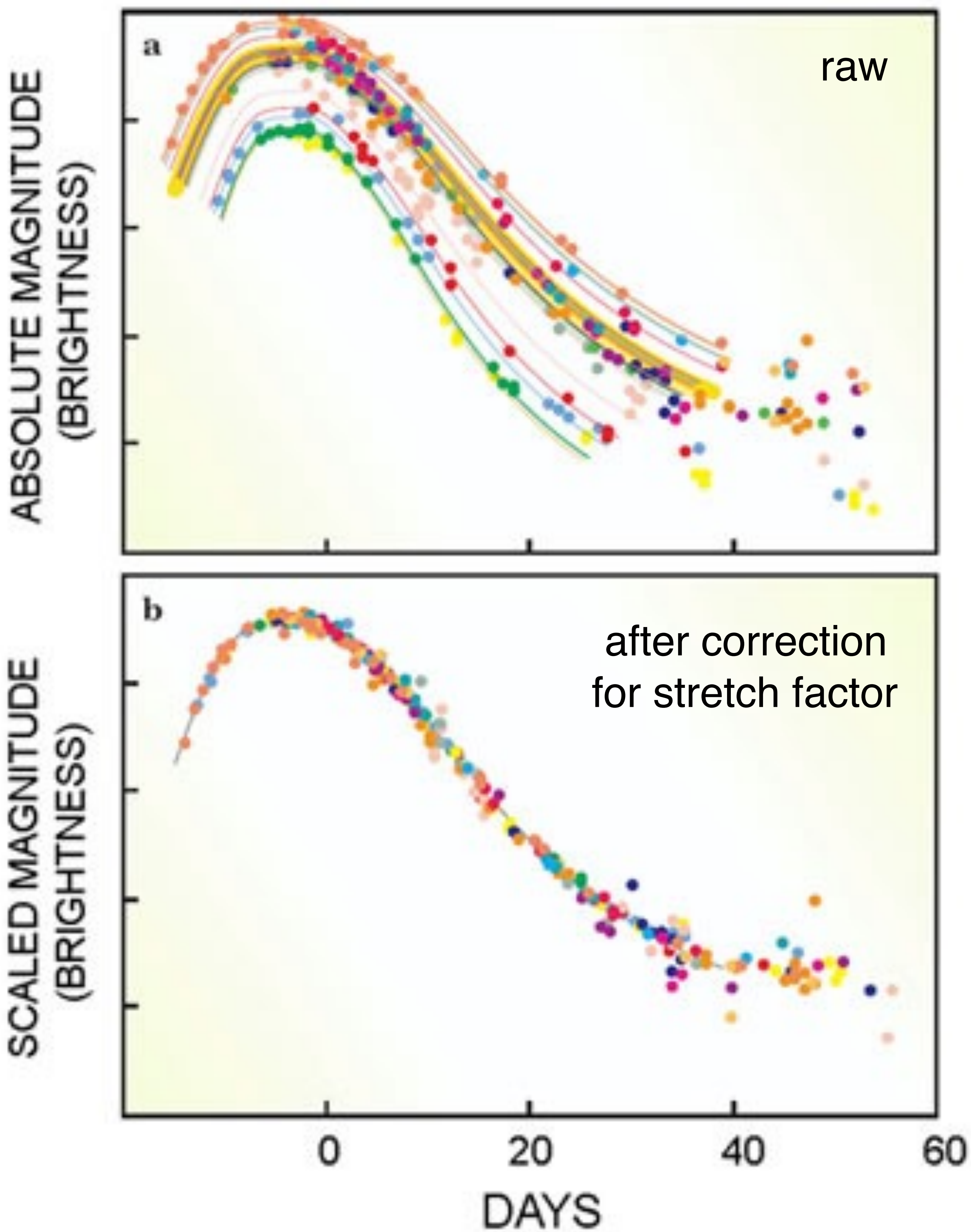
Type Ia supernovae light curves:



Type Ia supernovae light curves:

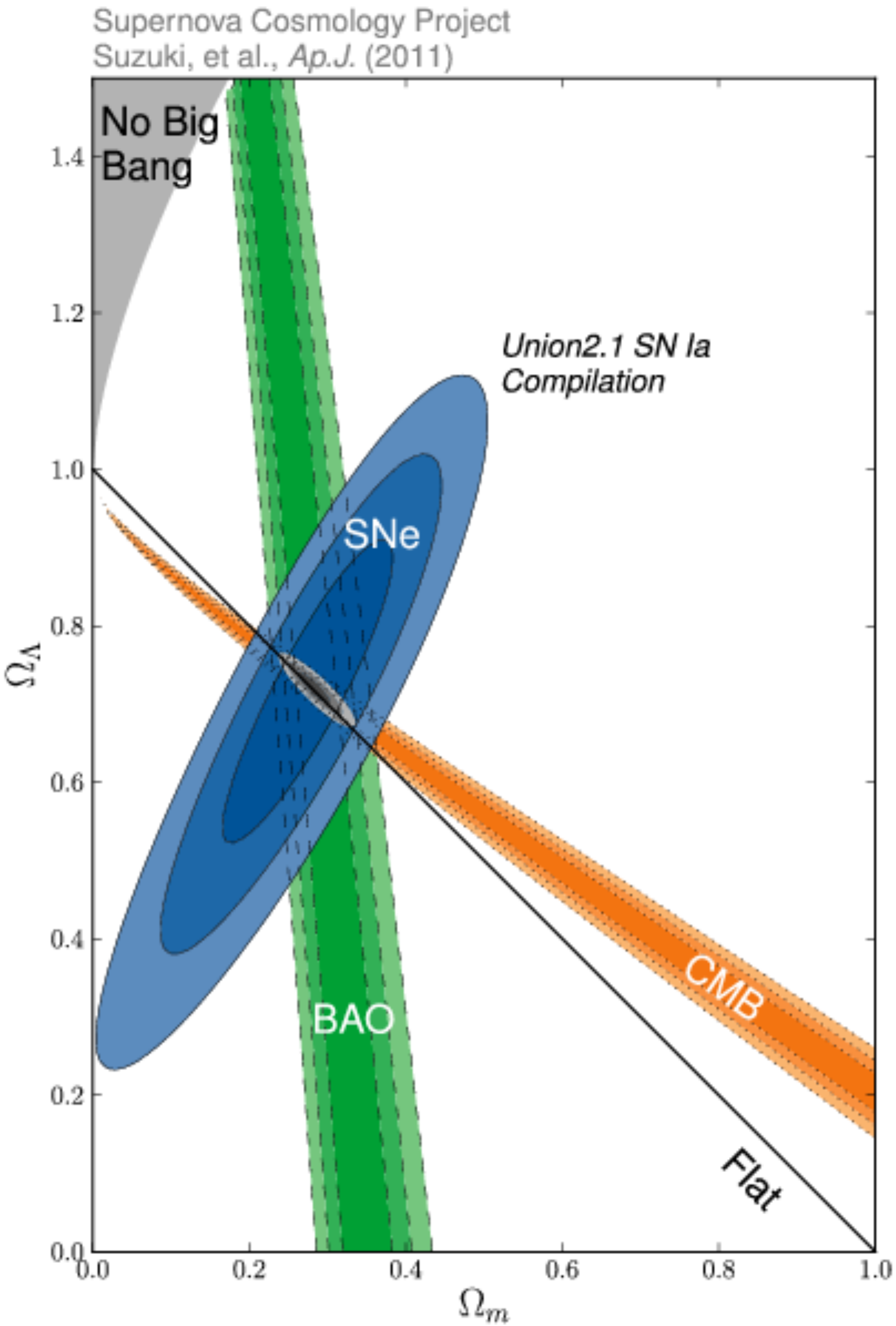


Type Ia supernovae light curves:



Empirical adjustment can be made so all have the same absolute magnitude, i.e. luminosity.

The same luminosity means that if you identify that something is a Type Ia supernova, it can be used as a **standard candle to get distance**.



This is used to constrain cosmological models specifically the fact that we are dominated by dark energy (here Ω_Λ)

(more on this in Astro 32!)

Neutron Stars

A neutron star (NS) is supported by neutron degeneracy pressure (which can be derived using the mass of the neutron instead of the electron mass and the mean molecular weight of 1).

$$R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left(\frac{1}{m_H} \right)^{8/3}$$

Neutron Stars

A neutron star (NS) is supported by neutron degeneracy pressure (which can be derived using the mass of the neutron instead of the electron mass and the mean molecular weight of 1).

$$R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left(\frac{1}{m_H} \right)^{8/3}$$

Compare with the radius of a WD: $R_{wd} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e M_{wd}^{1/3}} \left[\left(\frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}$

Neutron Stars

A neutron star (NS) is supported by neutron degeneracy pressure (which can be derived using the mass of the neutron instead of the electron mass and the mean molecular weight of 1).

$$R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left(\frac{1}{m_H} \right)^{8/3}$$

Compare with the radius of a WD: $R_{wd} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e M_{wd}^{1/3}} \left[\left(\frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}$

Neglecting factors of ~2 which include Z/A and mass difference: $\frac{R_{wd}}{R_{ns}} \approx \frac{m_H}{m_e}$ **~10^3**

Neutron Stars

A neutron star (NS) is supported by neutron degeneracy pressure (which can be derived using the mass of the neutron instead of the electron mass and the mean molecular weight of 1).

$$R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left(\frac{1}{m_H} \right)^{8/3}$$

Compare with the radius of a WD: $R_{wd} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e M_{wd}^{1/3}} \left[\left(\frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}$

Neglecting factors of ~2 which include Z/A and mass difference: $\frac{R_{wd}}{R_{ns}} \approx \frac{m_H}{m_e}$ **~10^3**

If R_{WD} is roughly the radius of Earth (~10,000 km), this implies R_{NS} ~10 km

$$R_{NS} \approx \frac{5 \times 10^{5}\ \mathrm{cm}}{(M_{NS}/M_\odot)^{1/3}} \underset{\substack{| \\ M_{NS} \approx 1.4 M_\odot}}{\approx} 4\ \mathrm{km}$$

$$R_{NS} \approx \frac{5 \times 10^{5}\ \mathrm{cm}}{(M_{NS}/M_\odot)^{1/3}} \underset{M_{NS} \approx 1.4 M_\odot}{\approx} 4\ \mathrm{km}$$

A neutron star (NS) is extremely compact and dense:

$$M_{NS}/m_n = \frac{1.4\, M_\odot}{m_n} \approx 10^{57} \text{ neutrons !}$$

$$\langle \rho \rangle_{NS} \simeq 6.65 \times 10^{14}\ \mathrm{g/cm^3}$$

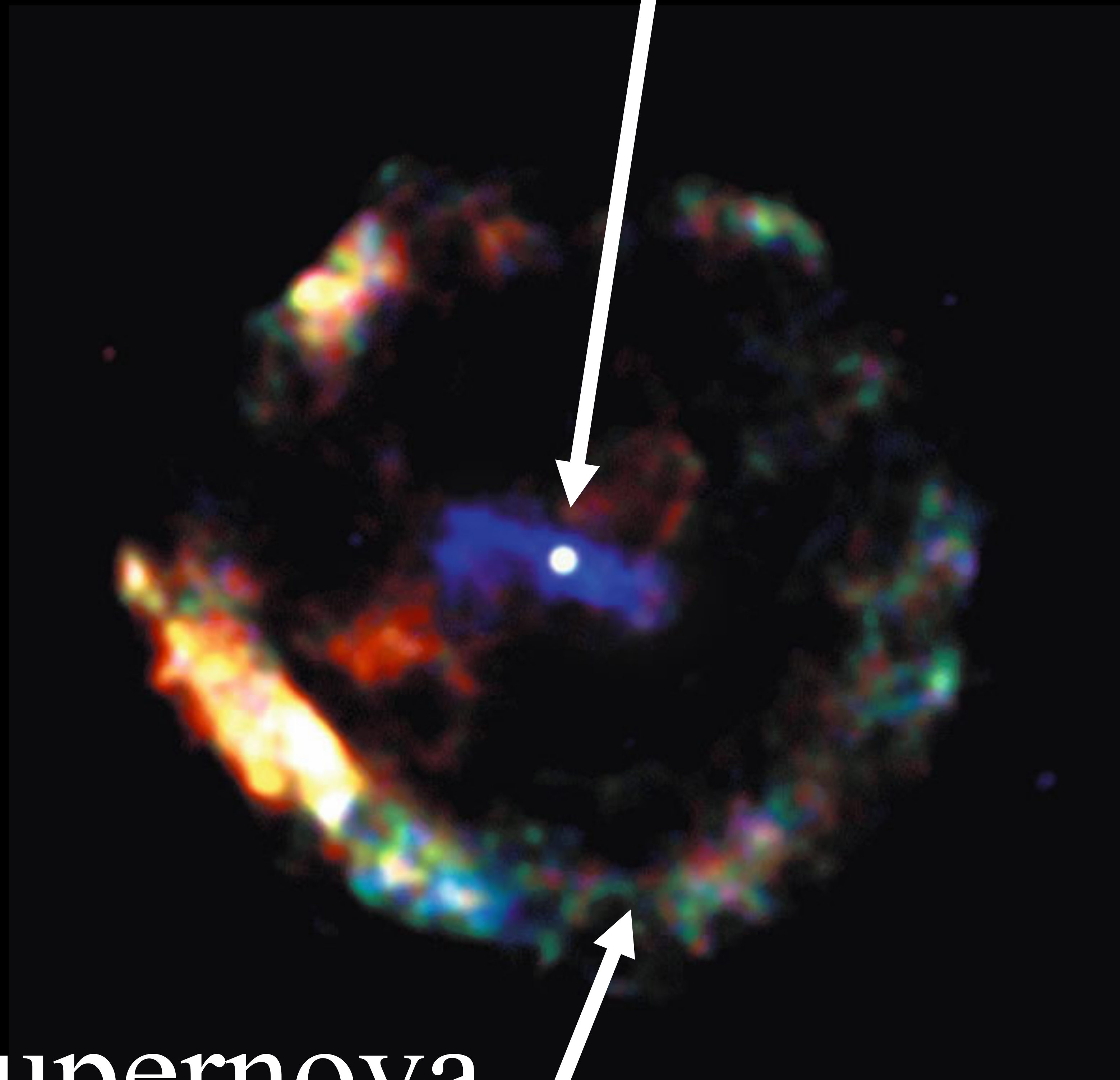
$$R_{NS} \approx \frac{5\times 10^{5}\ \mathrm{cm}}{(M_{NS}/M_\odot)^{1/3}} \underset{M_{NS}\approx 1.4 M_\odot}{\approx} 4\ \mathrm{km}$$

A neutron star (NS) is extremely compact and dense:

$$M_{NS}/m_n = \frac{1.4\, M_\odot}{m_n} \approx 10^{57} \text{ neutrons !}$$

$$\langle \rho \rangle_{NS} \simeq 6.65 \times 10^{14}\ \mathrm{g/cm^3}$$

Effects of general relativity must be included for an accurate description of NS.


Neutron Star
Supernova

At densities of ~10^9 g cm^{-3}:

$p^+ + e^-$ —> n + neutrino_e, i.e., the protons in the ^{56}Fe core capture electrons

At densities of ~10^9 g cm^{-3}:
$p^+ + e^-$ —> n + $neutrino_e$, i.e., the protons in the ^{56}Fe core capture electrons

When the density reaches ~4×10^{11} g cm^{-3}:
The minimum-energy arrangement is with some of the neutrons outside of the nuclei. This is call **neutron drip**, with the appearance of free neutrons, with a mixture of a lattice of neutron-rich nuclei (e.g., ^{118}Kr), non-relativistic degenerate neutrons, and relativistic degenerate electrons (for charge neutrality). A fluid with free neutrons has no viscosity —> **superfluid**, flowing without resistance.

At densities of ~10^9 g cm^{-3}:
$p^+ + e^-$ —> n + neutrino$_e$, i.e., the protons in the ^{56}Fe core capture electrons

When the density reaches ~4×10^{11} g cm^{-3}:
The minimum-energy arrangement is with some of the neutrons outside of the nuclei. This is call **neutron drip**, with the appearance of free neutrons, with a mixture of a lattice of neutron-rich nuclei (e.g., ^{118}Kr), non-relativistic degenerate neutrons, and relativistic degenerate electrons (for charge neutrality). A fluid with free neutrons has no viscosity —> **superfluid**, flowing without resistance.

Everything is extreme with NS, their interior state (simulating a huge nucleus), the velocity of sound (~60% of the speed of light), their rotation (frequency of 1-1000 Hz), and their magnetic field (up to 10^{12} Gauss; the magnetic field at Earth's surface is <1 Gauss).

At densities of ~10^9 g cm^{-3}:
$p^+ + e^-$ —> n + neutrino$_e$, i.e., the protons in the ^{56}Fe core capture electrons

When the density reaches ~4×10^{11} g cm^{-3}:
The minimum-energy arrangement is with some of the neutrons outside of the nuclei. This is call **neutron drip**, with the appearance of free neutrons, with a mixture of a lattice of neutron-rich nuclei (e.g., ^{118}Kr), non-relativistic degenerate neutrons, and relativistic degenerate electrons (for charge neutrality). A fluid with free neutrons has no viscosity —> **superfluid**, flowing without resistance.

Everything is extreme with NS, their interior state (simulating a huge nucleus), the velocity of sound (~60% of the speed of light), their rotation (frequency of 1-1000 Hz), and their magnetic field (up to 10^{12} Gauss; the magnetic field at Earth's surface is <1 Gauss).

As the density increases further, number of free neutrons increases as the number of electrons decreases. The neutron degeneracy pressure exceeds the electron degeneracy pressure when density is **~4×10^{12} g cm^{-3}**.

At densities of ~10^9 g cm^{-3}:
$p^+ + e^-$ —> n + neutrino$_e$, i.e., the protons in the ^{56}Fe core capture electrons

When the density reaches ~4×10^{11} g cm^{-3}:
The minimum-energy arrangement is with some of the neutrons outside of the nuclei. This is call **neutron drip**, with the appearance of free neutrons, with a mixture of a lattice of neutron-rich nuclei (e.g., ^{118}Kr), non-relativistic degenerate neutrons, and relativistic degenerate electrons (for charge neutrality). A fluid with free neutrons has no viscosity —> **superfluid**, flowing without resistance.

Everything is extreme with NS, their interior state (simulating a huge nucleus), the velocity of sound (~60% of the speed of light), their rotation (frequency of 1-1000 Hz), and their magnetic field (up to 10^{12} Gauss; the magnetic field at Earth's surface is <1 Gauss).

As the density increases further, number of free neutrons increases as the number of electrons decreases. The neutron degeneracy pressure exceeds the electron degeneracy pressure when density is **~4×10^{12} g cm^{-3}**.

As the density approaches the density of the nucleus (**~2.4×10^{14} g cm^{-3}**), the nuclei effectively dissolve, resulting in a degenerate fluid mixture of free neutrons, protons and electrons dominated by neutron degeneracy pressure, with neutrons and protons (number of protons <1%) paired to form superfluids. The fluid of pairs of protons is superconductive.

At densities of ~10^9 g cm^{-3}:
$p^+ + e^-$ —> n + neutrino$_e$, i.e., the protons in the ^{56}Fe core capture electrons

When the density reaches ~4×10^{11} g cm^{-3}:
The minimum-energy arrangement is with some of the neutrons outside of the nuclei. This is call **neutron drip**, with the appearance of free neutrons, with a mixture of a lattice of neutron-rich nuclei (e.g., ^{118}Kr), non-relativistic degenerate neutrons, and relativistic degenerate electrons (for charge neutrality). A fluid with free neutrons has no viscosity —> **superfluid**, flowing without resistance.

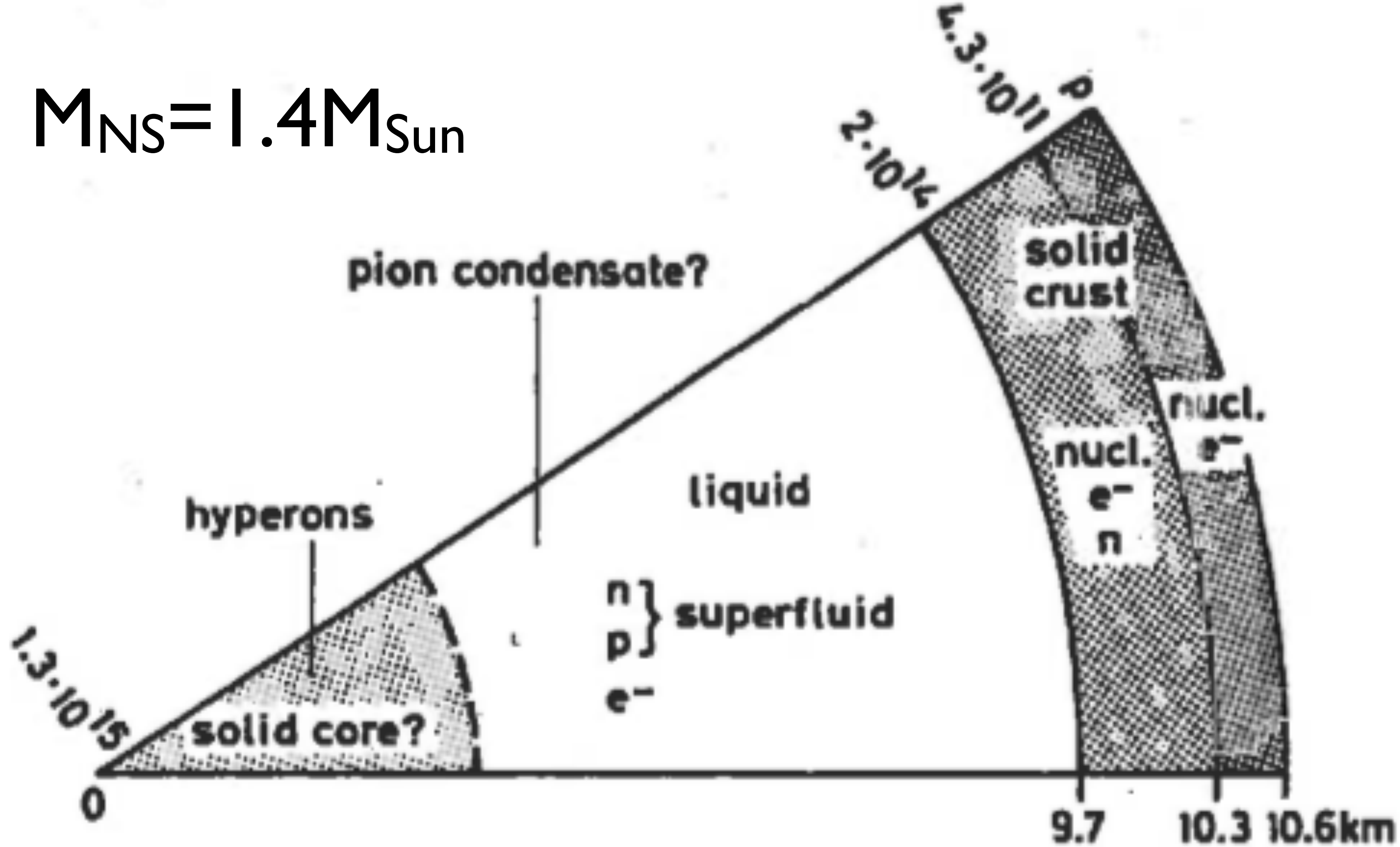
Everything is extreme with NS, their interior state (simulating a huge nucleus), the velocity of sound (~60% of the speed of light), their rotation (frequency of 1-1000 Hz), and their magnetic field (up to 10^{12} Gauss; the magnetic field at Earth's surface is <1 Gauss).

As the density increases further, number of free neutrons increases as the number of electrons decreases. The neutron degeneracy pressure exceeds the electron degeneracy pressure when density is **~4×10^{12} g cm^{-3}**.

As the density approaches the density of the nucleus (**~2.4×10^{14} g cm^{-3}**), the nuclei effectively dissolve, resulting in a degenerate fluid mixture of free neutrons, protons and electrons dominated by neutron degeneracy pressure, with neutrons and protons (number of protons <1%) paired to form superfluids. The fluid of pairs of protons is superconductive.

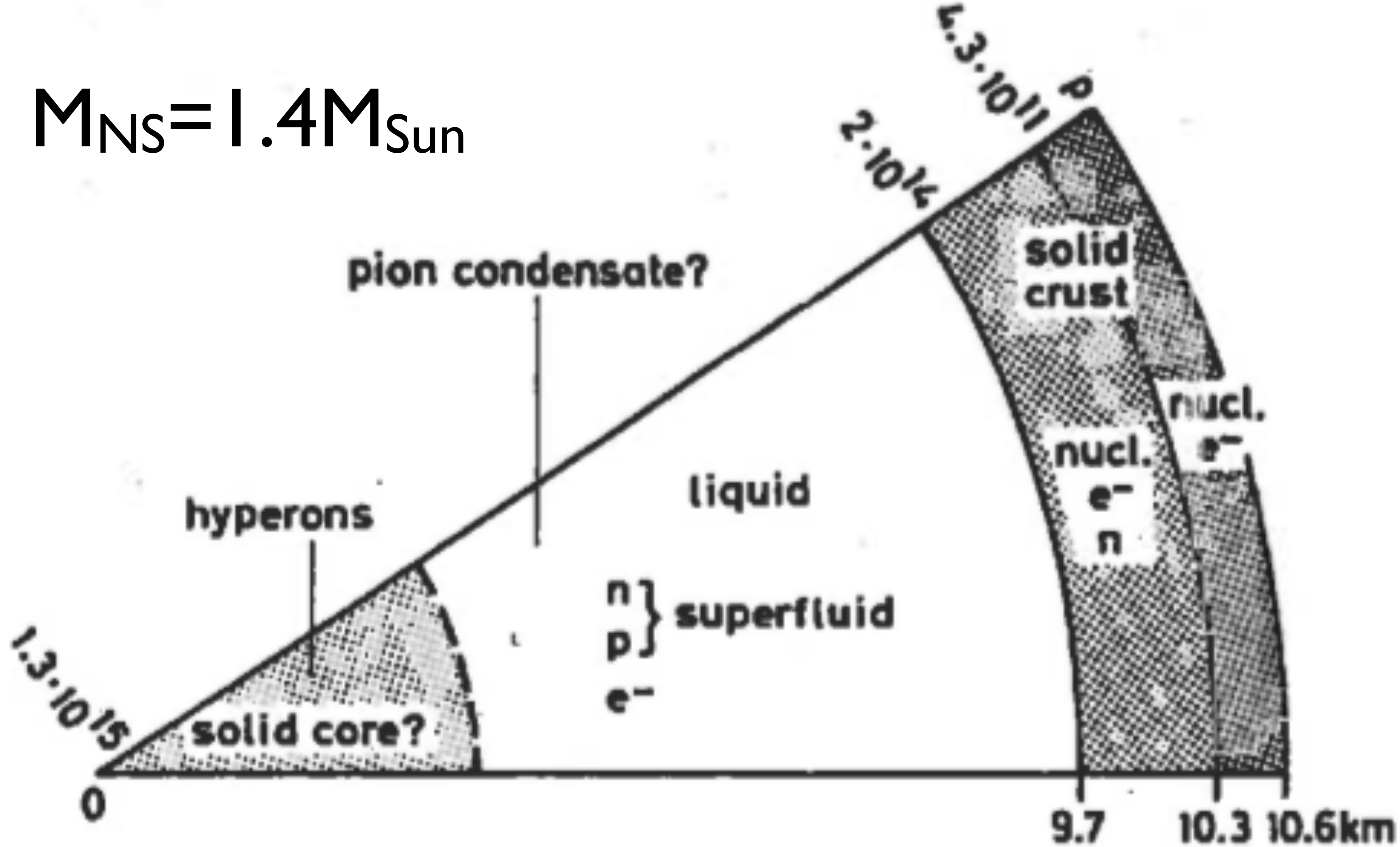
As density further increases, the limit n:p:e = 8:1:1 is reached.

$M_{NS}=1.4M_{Sun}$



Outer crust: heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, and relativistic degenerate electrons; near the surface, nuclei likely ^{56}Fe; at greater depth, n-rich nuclei, with neutron drip beginning at the bottom of the outer crust.

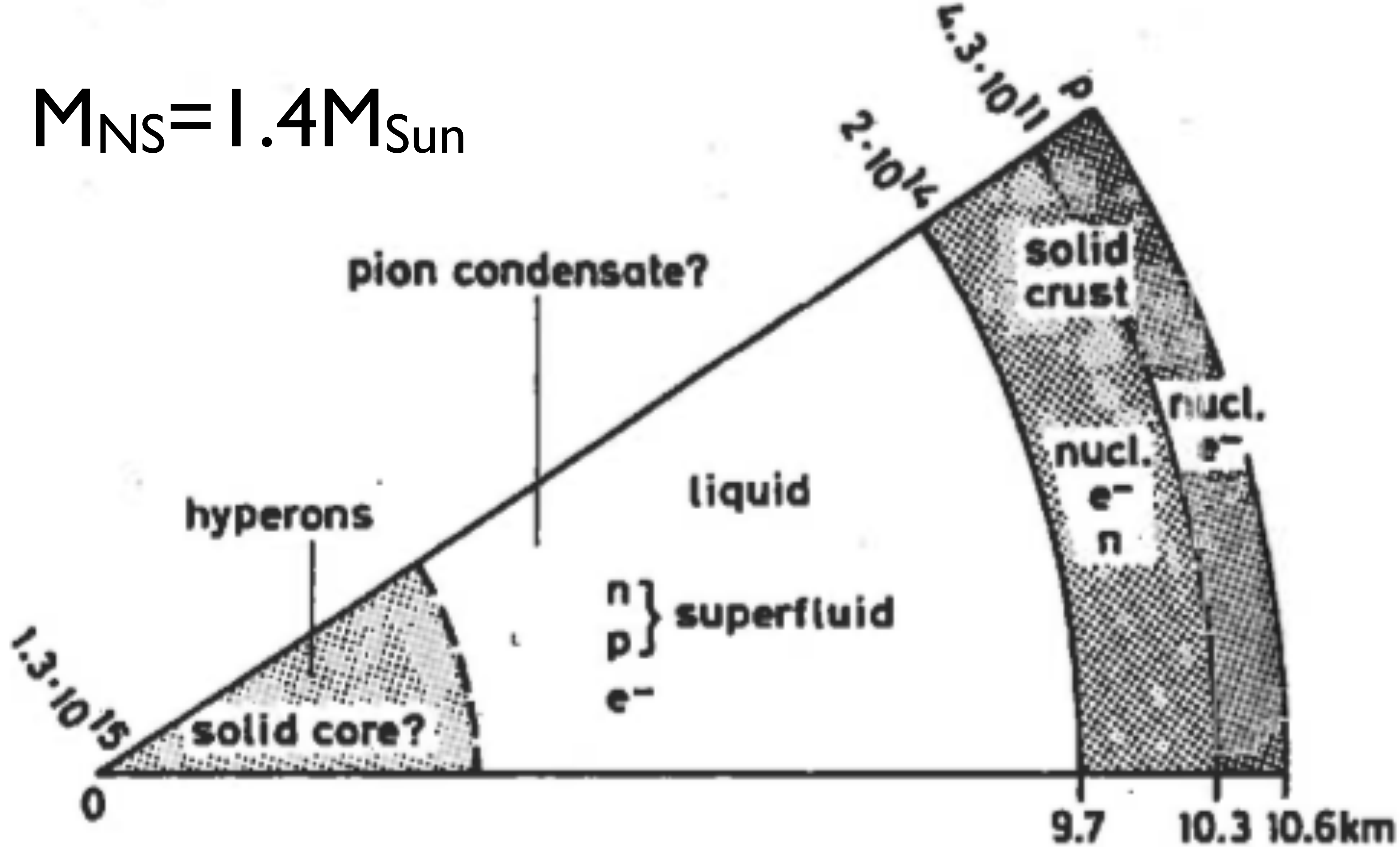
$M_{NS}=1.4M_{Sun}$



Outer crust: heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, and relativistic degenerate electrons; near the surface, nuclei likely ^{56}Fe; at greater depth, n-rich nuclei, with neutron drip beginning at the bottom of the outer crust.

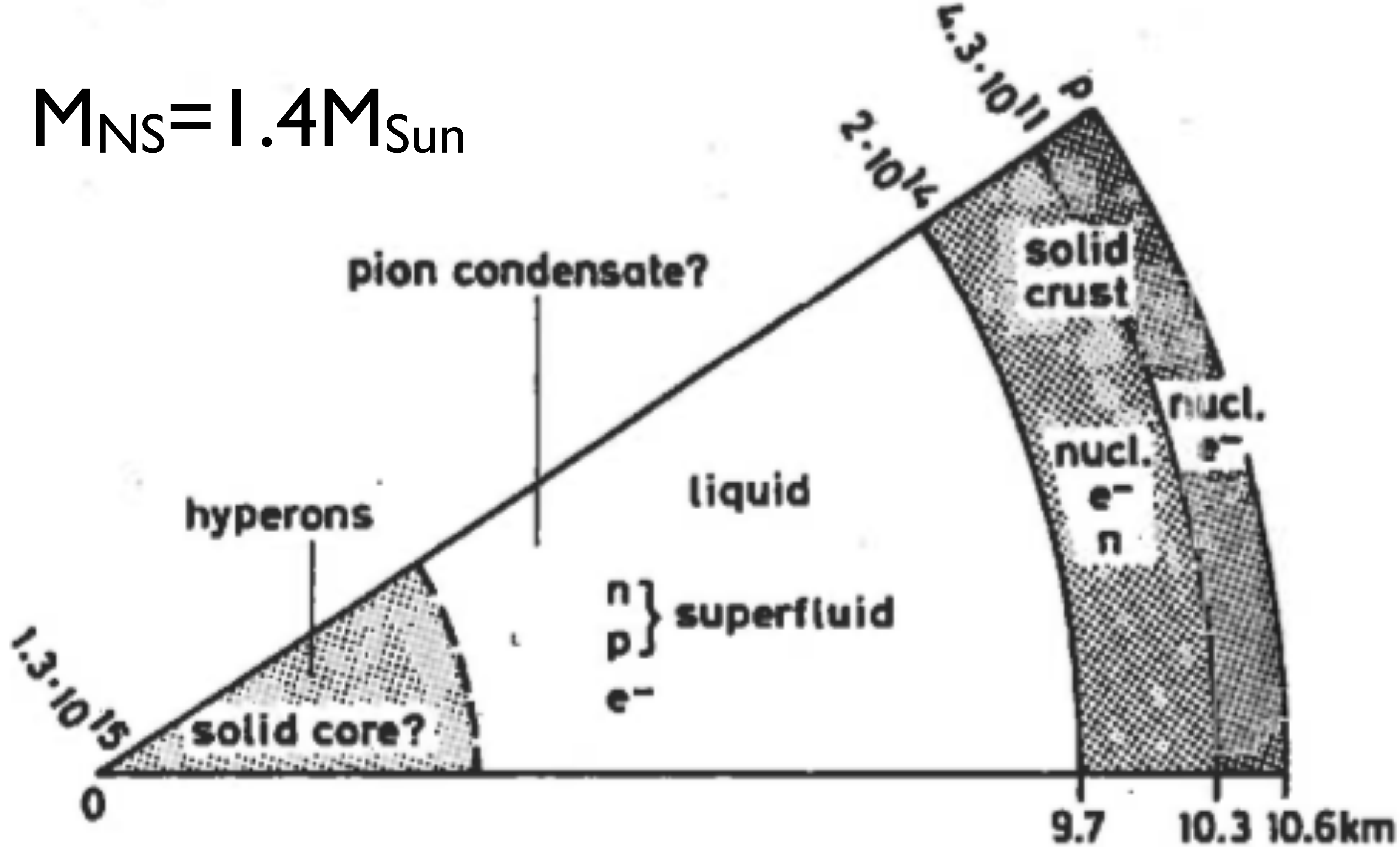
Inner crust: lattice of nuclei (e.g., ^{118}Kr), a superfluid of free neutrons and relativistic degenerate electrons. Bottom of inner crust, the density is similar ot the density of the nucleus, and the nuclei dissolve.

$M_{NS}=1.4M_{Sun}$



Outer crust: heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, and relativistic degenerate electrons; near the surface, nuclei likely ^{56}Fe; at greater depth, n-rich nuclei, with neutron drip beginning at the bottom of the outer crust.
Inner crust: lattice of nuclei (e.g., ^{118}Kr), a superfluid of free neutrons and relativistic degenerate electrons. Bottom of inner crust, the density is similar ot the density of the nucleus, and the nuclei dissolve.
Interior: superfluid of neutrons, with a small number of superfluid, superconducting, protons and relativistic degenerate electrons.

$M_{NS}=1.4M_{Sun}$



Outer crust: heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, and relativistic degenerate electrons; near the surface, nuclei likely ^{56}Fe; at greater depth, n-rich nuclei, with neutron drip beginning at the bottom of the outer crust.
Inner crust: lattice of nuclei (e.g., ^{118}Kr), a superfluid of free neutrons and relativistic degenerate electrons. Bottom of inner crust, the density is similar ot the density of the nucleus, and the nuclei dissolve.
Interior: superfluid of neutrons, with a small number of superfluid, superconducting, protons and relativistic degenerate electrons.
Core?: made of pions or other sub-nuclear particles. The Fermi energy of the degenerate neutrons gradually exceed the rest-mass of hyperons of lowest mass, and these particles will then appear, beginning the hyperonization of the core.

NOTES

With the considered densities, the interaction between nucleons is far from being negligible, and it dominates the behavior:

- **uncertainties due to the absence of a rigorous theory and of experiments at such high densities of how particles interact**.
- **uncertainty connected with the appearance of new particles when the density increases**. When Hyperons occur in sufficient number, they contribute to the density but scarcely to the pressure —> hyperonization makes the gas even more compressible!

==> Extremely uncertain equation of state

Like WDs, NS also obey $\mathbf{M_{NS}\, V_{NS} = constant}$, i.e., NS become smaller and more dense with increasing mass.

Like WDs, NS also obey **M_{NS} V_{NS} = constant**, i.e., NS become smaller and more dense with increasing mass.

Max M_{NS} ~ 2.2 M_{sun} if the NS is static
Max M_{NS} ~ 2.9 M_{sun} if the NS is rapidly rotating
If M_{NS} larger than this limit —> BLACK HOLE.

NOTE: M_{NS} sets constraints on the equation of state, and so on the particle physics determining it.

NOTE: M_{ch}=5.73 M_{Sun}, but M_{NS}<M_{Ch} because of general relativity effects

Treating a star like a sphere with a moment of inertia:

$$I = CMR^2 \rightarrow I_i \omega_i = I_f \omega_f$$

$$\hookrightarrow \omega_f = \omega_i \left(R_i / R_f \right)^2$$ Angular velocity

$$P_f = P_i \left(R_f / R_i \right)^2$$ Period

Treating a star like a sphere with a moment of inertia:

$$I = CMR^2 \longrightarrow I_i \omega_i = I_f \omega_f$$

$$\omega_f = \omega_i \left(R_i / R_f \right)^2$$ Angular velocity

$$P_f = P_i \left(R_f / R_i \right)^2$$ Period

$$R_{core} / R_{NS} \simeq \frac{m_N}{m_{e^-}} \left(\frac{Z}{A} \right)^{5/3} \sim 512$$

Treating a star like a sphere with a moment of inertia:

$$I = CMR^2 \rightarrow I_i \omega_i = I_f \omega_f$$

$$\hookrightarrow \omega_f = \omega_i \left(R_i / R_f \right)^2$$ Angular velocity

$$P_f = P_i \left(R_f / R_i \right)^2$$ Period

$$R_{core} / R_{NS} \simeq \frac{m_N}{m_{\bar{e}}} \left(\frac{Z}{A} \right)^{5/3} \sim 512$$

$$\text{FD} \quad P_{NS} \approx 3.8 \times 10^{-6} P_{core} \approx 5 \times 10^{-3}\,\text{s} \left(200/\text{s} \right)$$

NS rotates very rapidly when they are formed, with period of a few milliseconds.

The magnetic field flux through the surface of a WD is conserved as it collapses to form a NS:

$$B_i \, 4\pi R_i^2 = B_f \, 4\pi R_f^2$$

The magnetic field flux through the surface of a WD is conserved as it collapses to form a NS:

$$B_i \, 4\pi R_i^2 = B_f \, 4\pi R_f^2$$

$$\Rightarrow \quad B_{NS} \simeq B_{WD} \left(\frac{R_{WD}}{R_{NS}} \right)^2 \simeq 1.3 \times 10^{10} \, T$$

Huge magnetic field

$B_{WD} \sim 5 \times 10^4$ T (extreme case)
$B_{Sun} \sim 1 \times 10^{-4}$ T

The temperature T of NS when formed out of the SN explosion is extremely high, T~10^{11} K.

During the first day, the NS cools by emitting neutrinos via:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

$$p^+ + e^- \rightarrow n + \nu_e$$

The temperature T of NS when formed out of the SN explosion is extremely high, $T \sim 10^{11}$ K.

During the first day, the NS cools by emitting neutrinos via:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$
$$p^+ + e^- \rightarrow n + \nu_e$$

When $T \sim 10^9$ K, protons and neutrons become degenerate (after ~1 day) and those processes stop. Other neutrino-dominated processes dominate the cooling in the following $\sim 10^3$ years, and then photons from the surface take over. After $\sim 10^2$ years, internal temperature is $T_{internal} \sim 10^8$ K (hence the neutrons are degenerate, nearly relativistic), with the surface temperature of $T_{surface} \sim 10^6$ K. Therefore:

The temperature T of NS when formed out of the SN explosion is extremely high, $T \sim 10^{11}$ K.

During the first day, the NS cools by emitting neutrinos via:

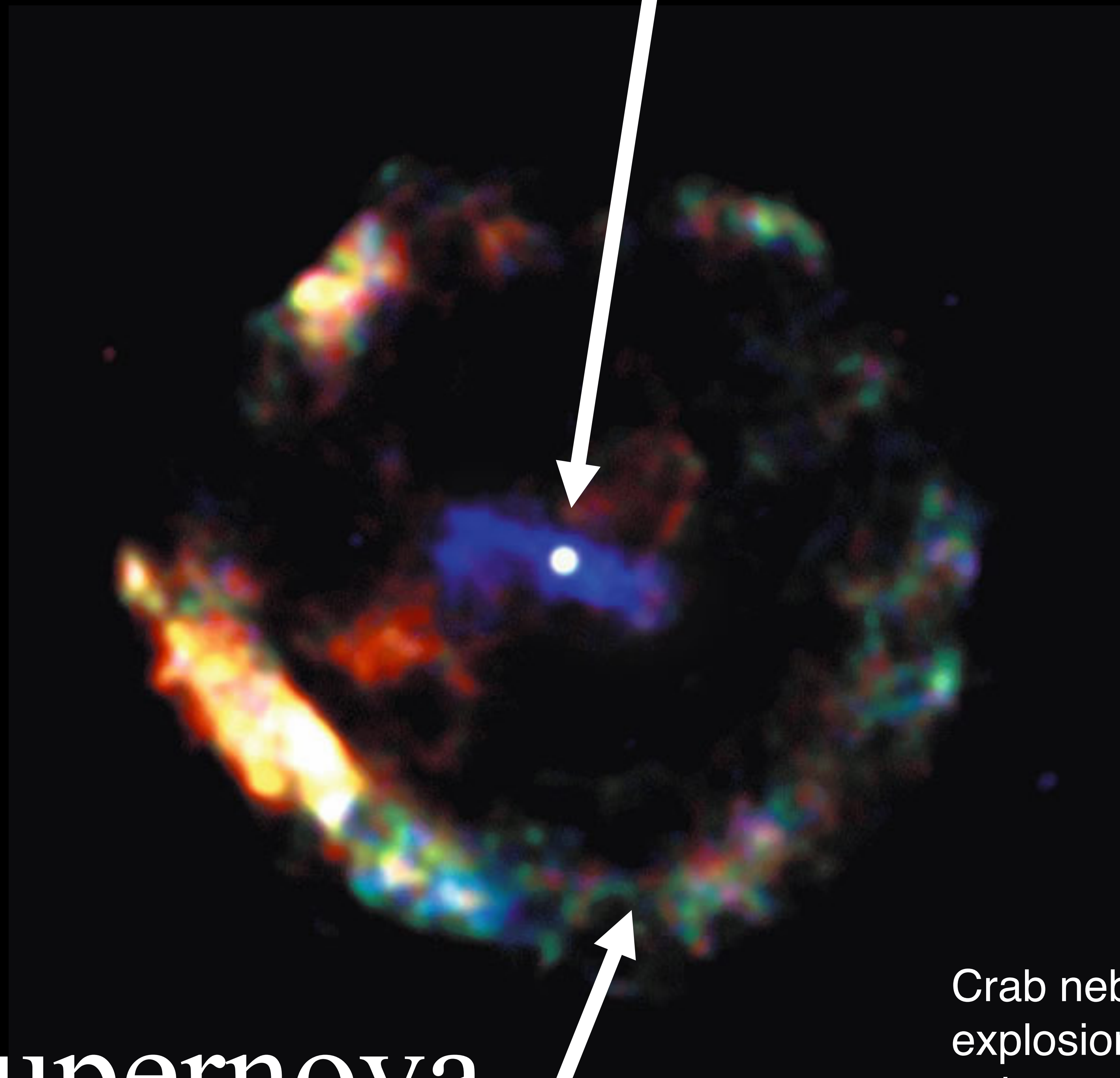
$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$
$$p^+ + e^- \rightarrow n + \nu_e$$

When $T \sim 10^9$ K, protons and neutrons become degenerate (after ~1 day) and those processes stop. Other neutrino-dominated processes dominate the cooling in the following $\sim 10^3$ years, and then photons from the surface take over. After $\sim 10^2$ years, internal temperature is $T_{internal} \sim 10^8$ K (hence the neutrons are degenerate, nearly relativistic), with the surface temperature of $T_{surface} \sim 10^6$ K. Therefore:

$$L = 4\pi R^2 \sigma T_e^4 \simeq 7.13 \times 10^{32} \text{ erg/s} \approx L_\odot$$

But in the X-rays (~2.9 nm) region of the electron-magnetic spectrum

Neutron Star

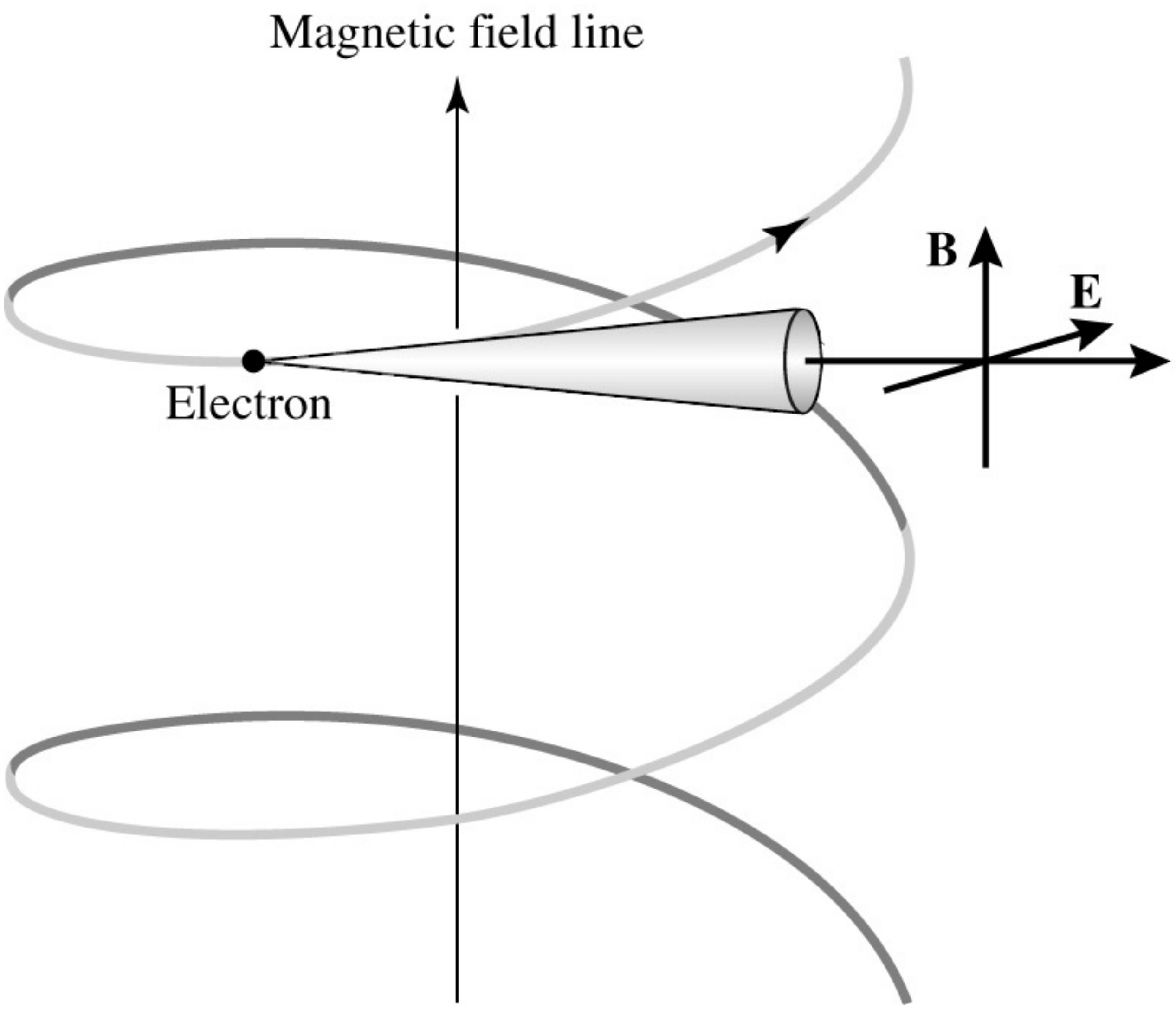


Supernova

Crab nebula: SN explosion is AD 1054 + pulsar at its center (synchrotron radiation)

Pulsar: rapidly rotating NS

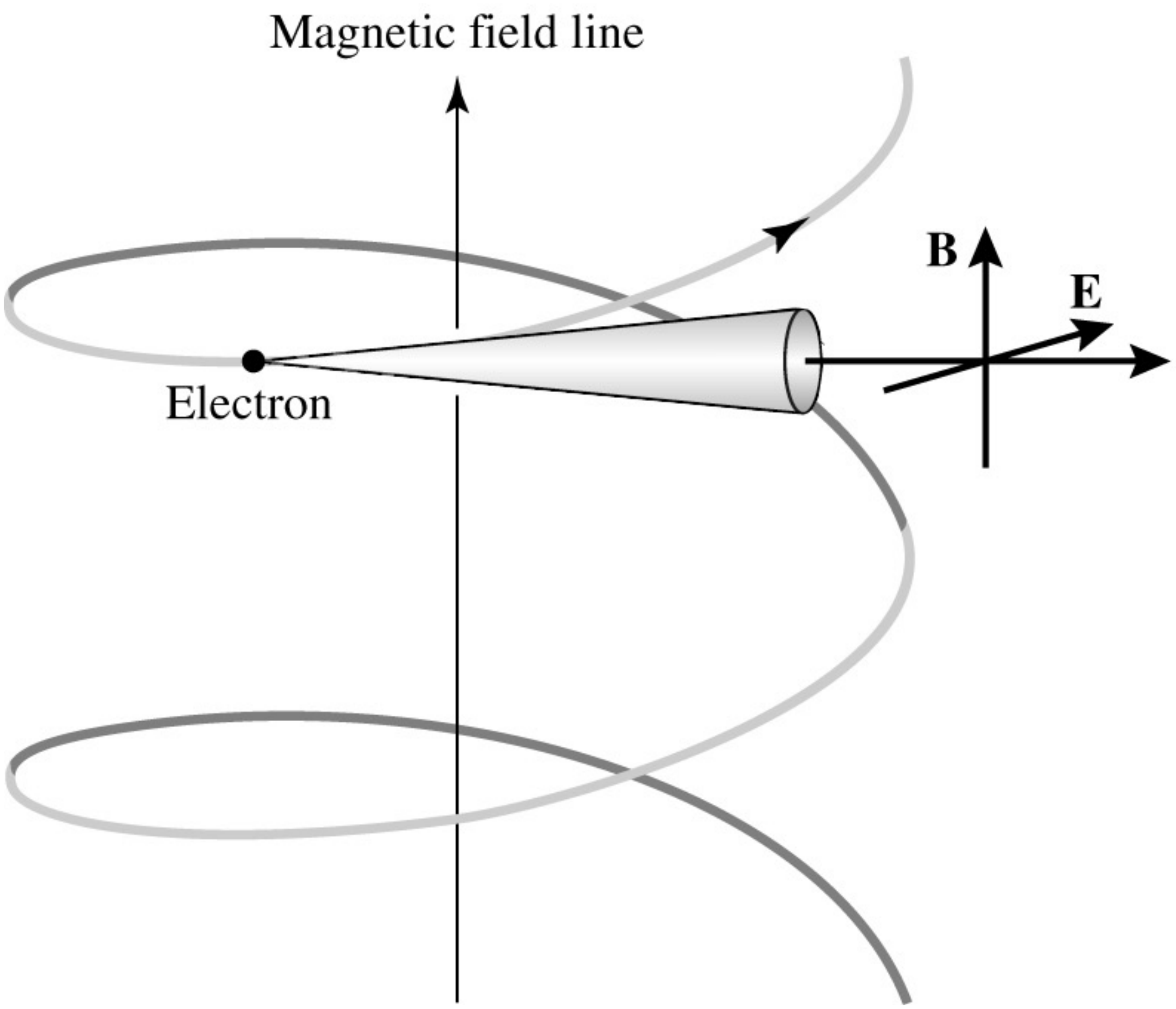
Synchrotron radiation



Synchrotron radiation is produced when relativistic electrons spiral along the magnetic field lines.

Pulsar: rapidly rotating NS

Synchrotron radiation



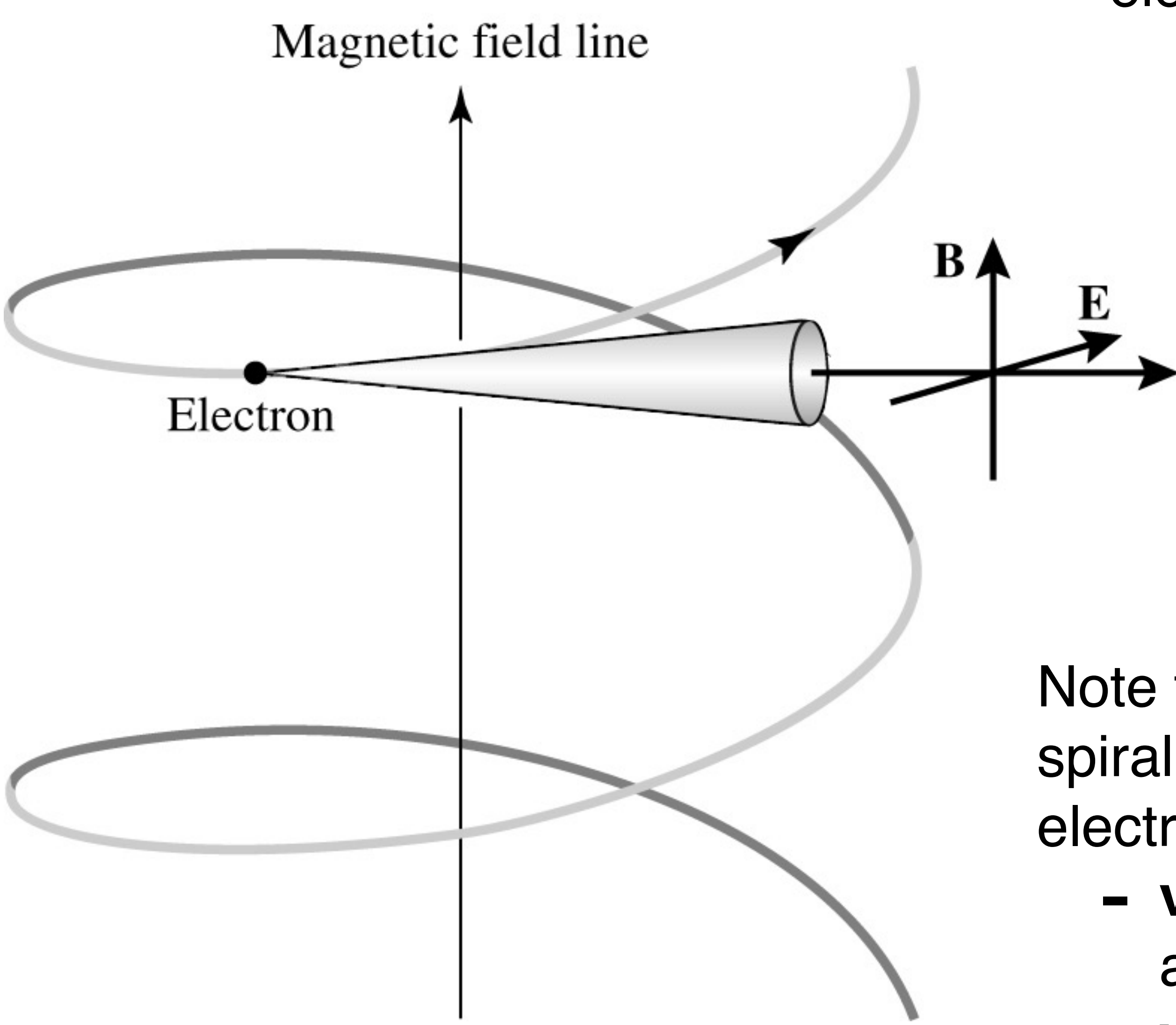
Synchrotron radiation is produced when relativistic electrons spiral along the magnetic field lines.

Electrons pass near a magnetic field line and are accelerated following the Lorentz force:

$$\boxed{\mathbf{F} = q(\mathbf{v} \times \mathbf{B})}$$

Pulsar: rapidly rotating NS

Synchrotron radiation



Synchrotron radiation is produced when relativistic electrons spiral along the magnetic field lines.

Electrons pass near a magnetic field line and are accelerated following the Lorentz force:

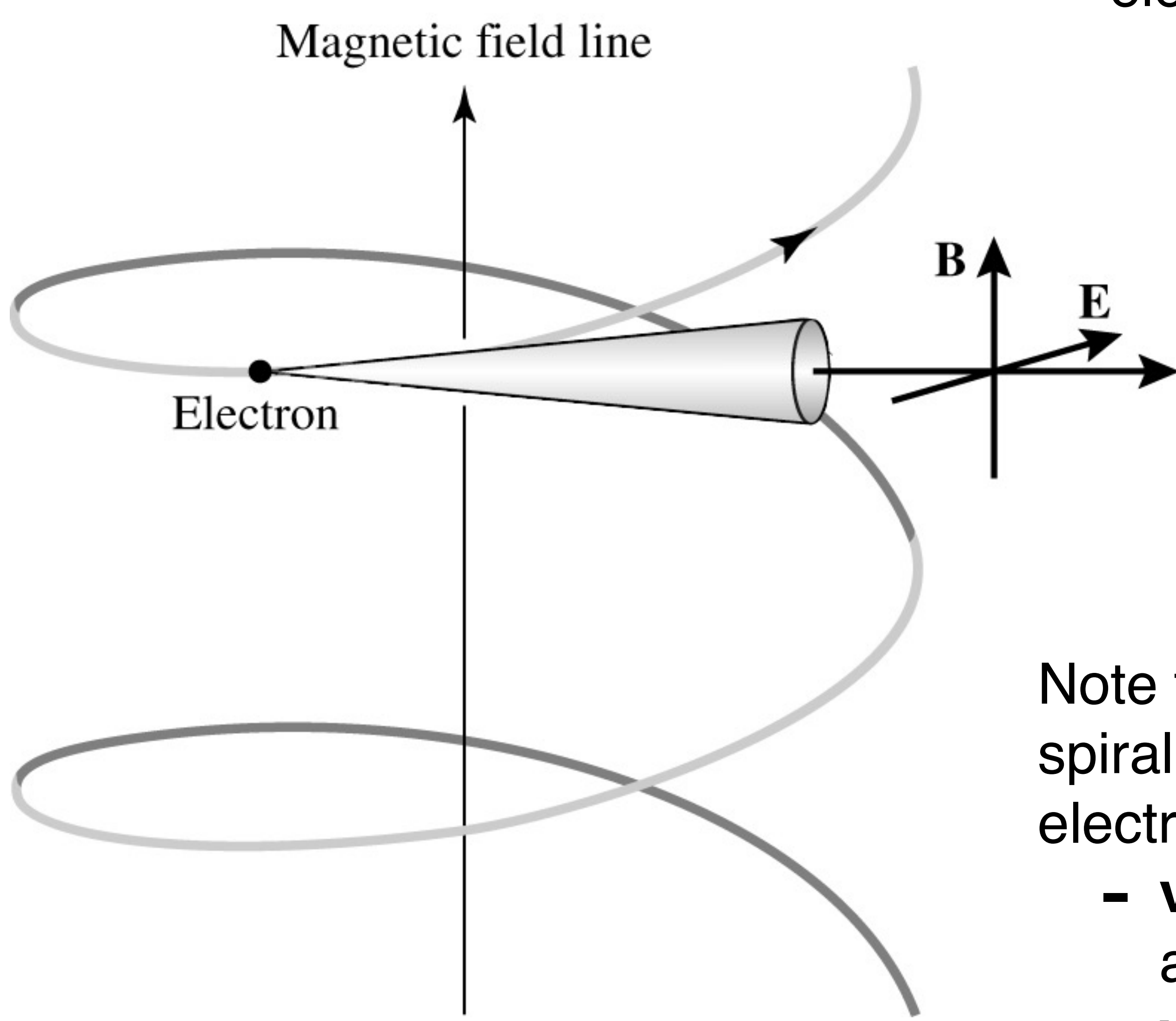
$$\boxed{\mathbf{F} = q(\mathbf{v} \times \mathbf{B})}$$

Note that the cross-product results in the electron spiraling around the magnetic field line, and emitting electro-magnetic radiation:

- **v** perpendicular to **B** produces a circular motion around the magnetic field lines
- **v** parallel to **B** is not affected

Pulsar: rapidly rotating NS

Synchrotron radiation



Synchrotron radiation is produced when relativistic electrons spiral along the magnetic field lines.

Electrons pass near a magnetic field line and are accelerated following the Lorentz force:

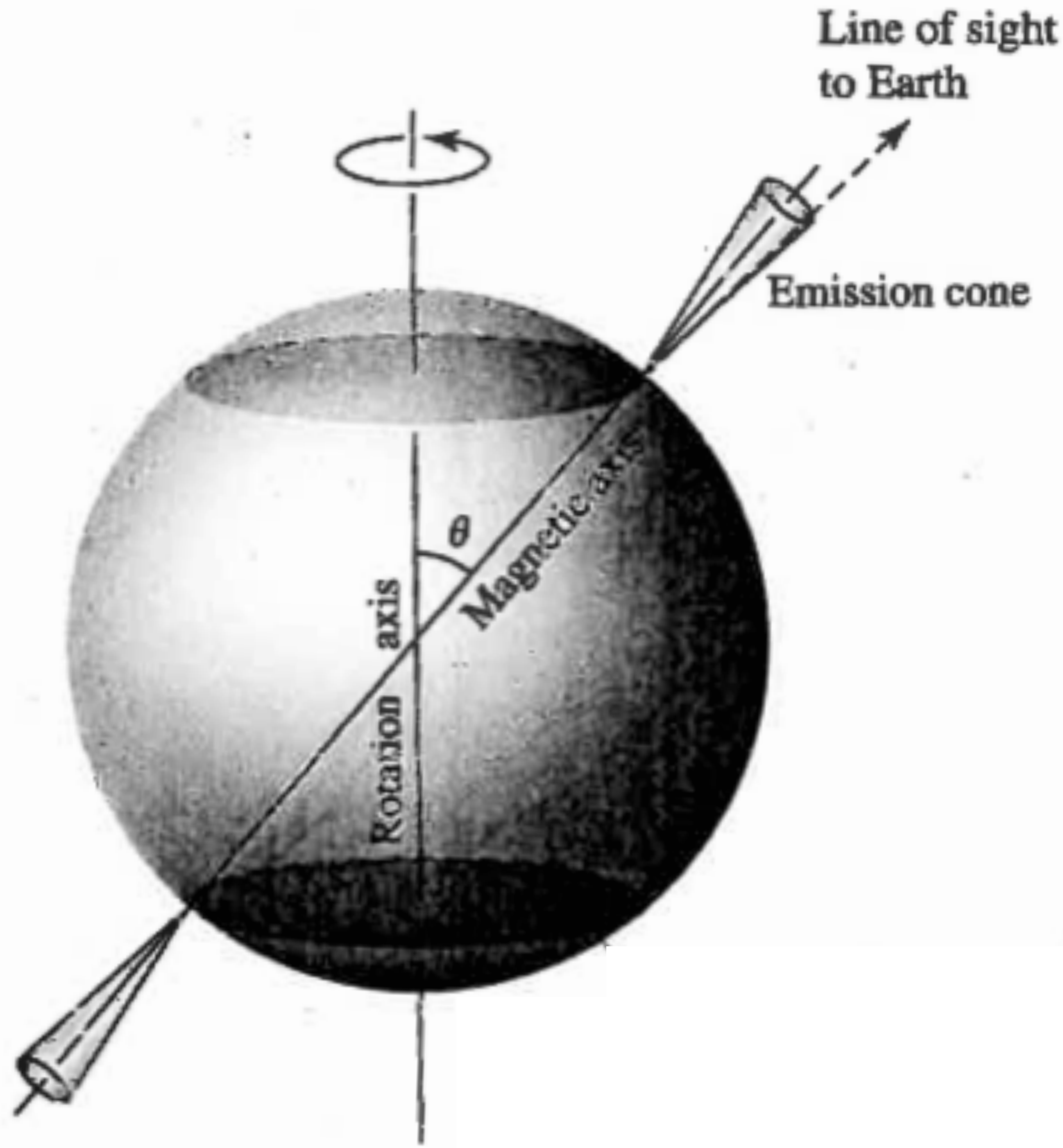
$$\boxed{\mathbf{F} = q(\mathbf{v} \times \mathbf{B})}$$

Note that the cross-product results in the electron spiraling around the magnetic field line, and emitting electro-magnetic radiation:

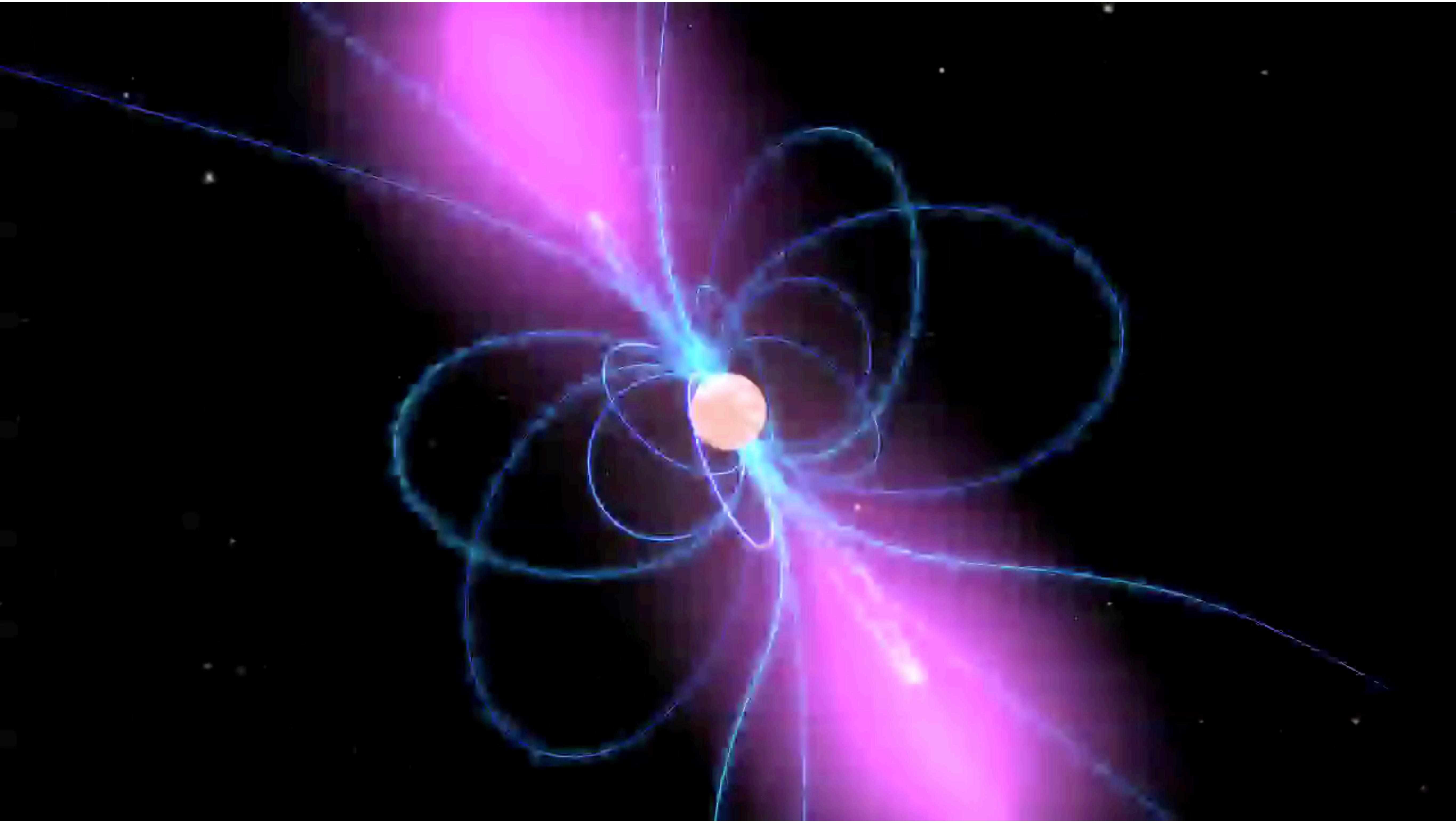
- **v** perpendicular to **B** produces a circular motion around the magnetic field lines
- **v** parallel to **B** is not affected

The rotting NS provides the required energy to accelerate the expansion of the nebula, the relativistic electrons, and the magnetic field.

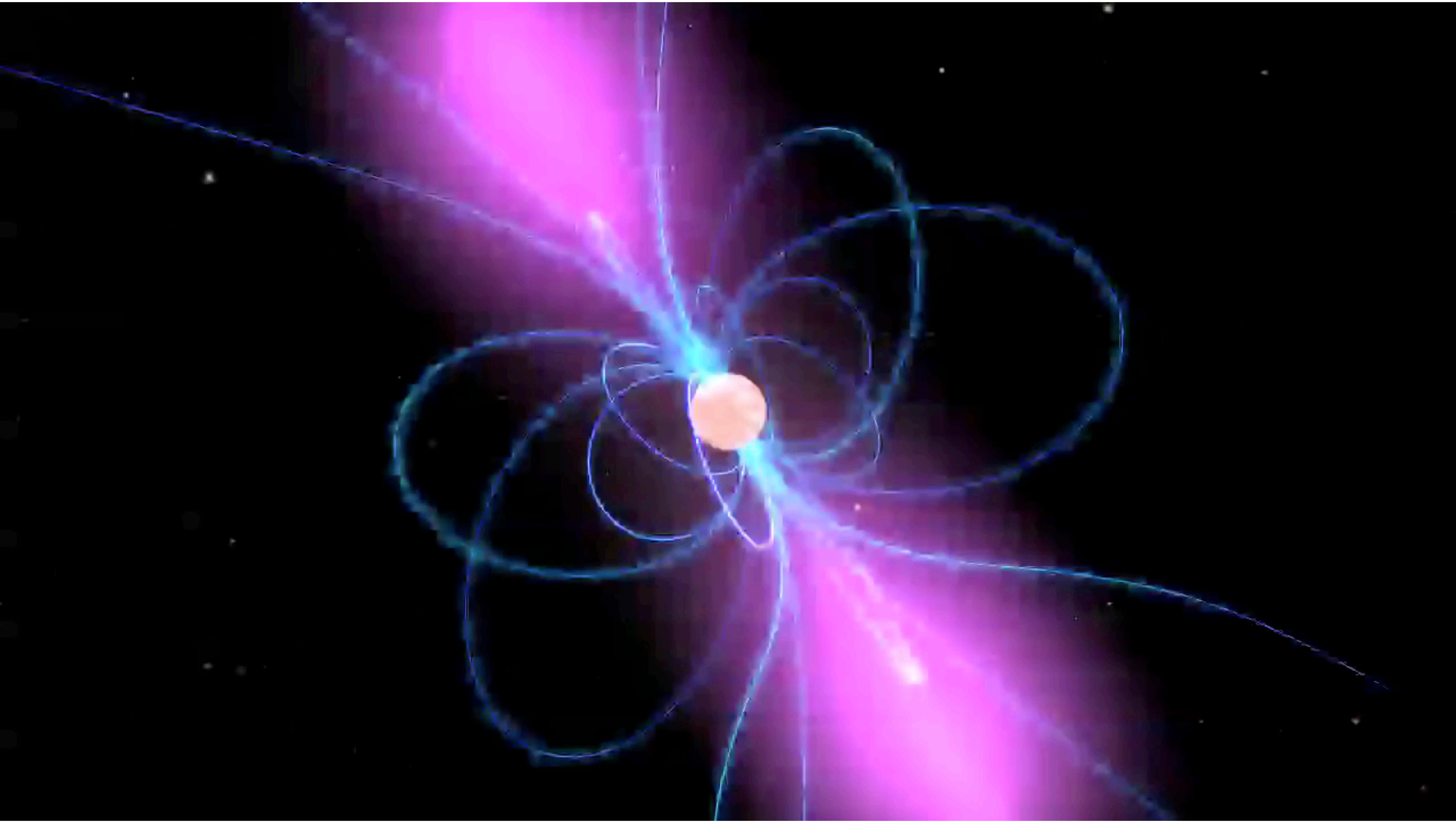
Pulsar: **rapidly rotating** NS with **strong** dipole **magnetic field**, inclined with respect to the rotation axis at an angle



Pulsar as rapidly rotating NS



Pulsar as rapidly rotating NS




Jocelyn Bell in 1967

Pulsars were **discovered in 1967 by then graduate student Jocelyn Bell** and her advisor Anthony Hewish.

Example pulsar light curve (radio).

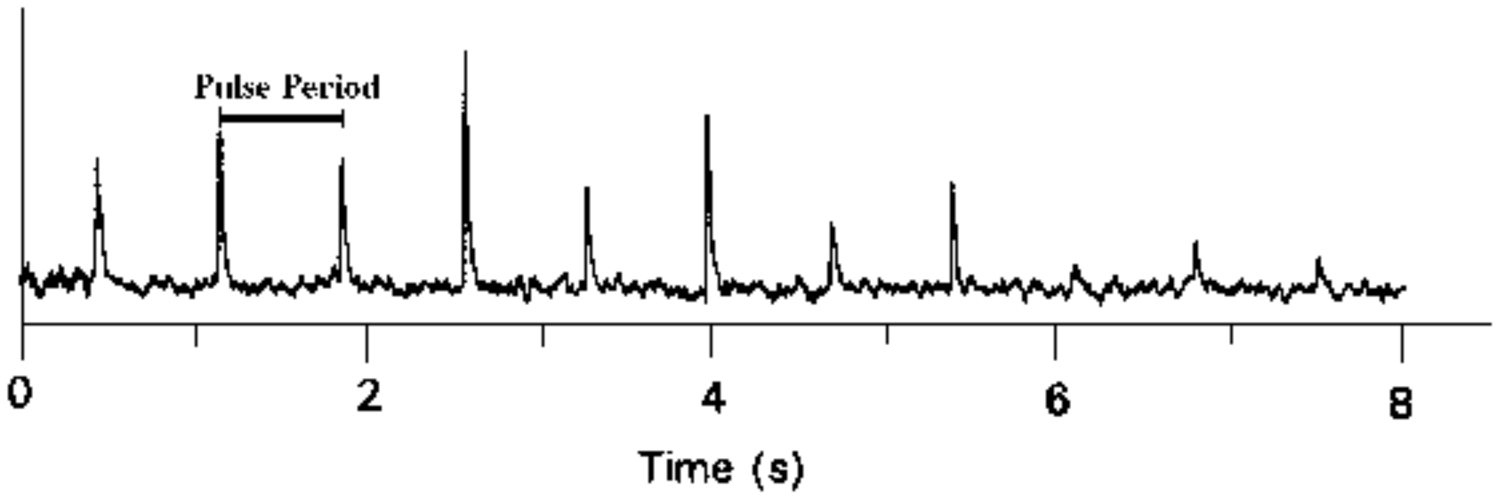


Image Credit: Manchester, R.N. and Taylor, J.H., Pulsars, Freeman, 1977.

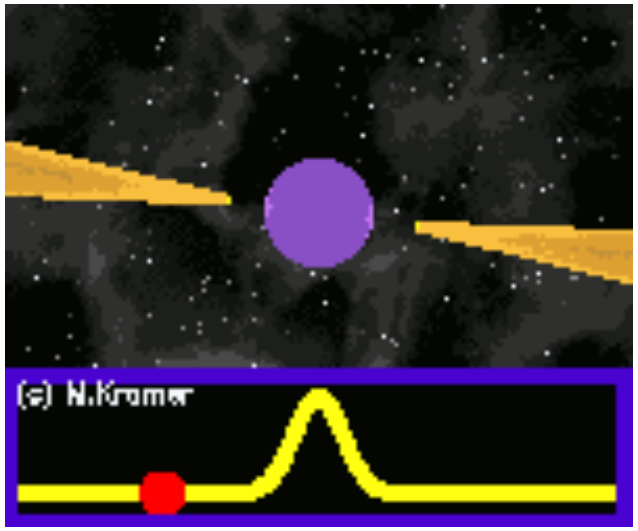


Image Credit: Michael Kramer (University of Manchester)

Example pulsar light curve (radio).

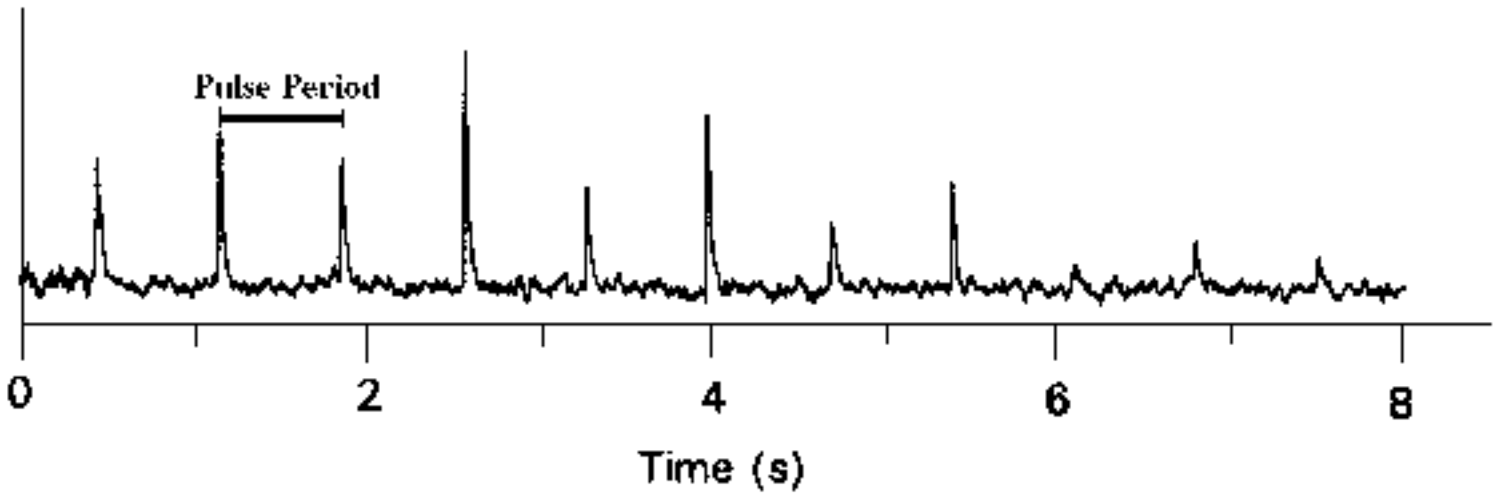


Image Credit: Manchester, R.N. and Taylor, J.H., Pulsars, Freeman, 1977.

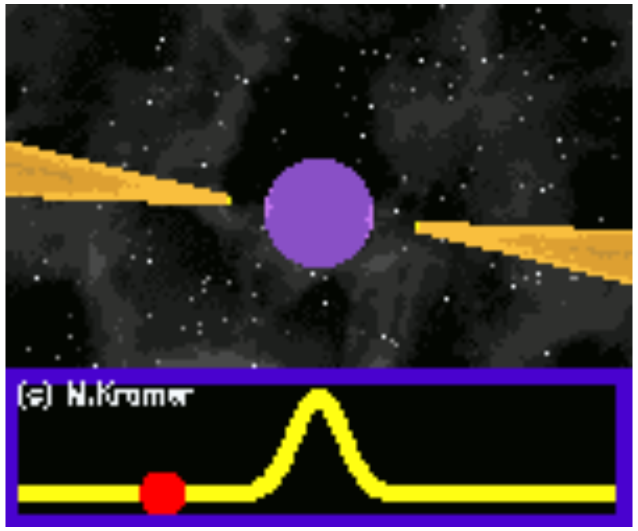
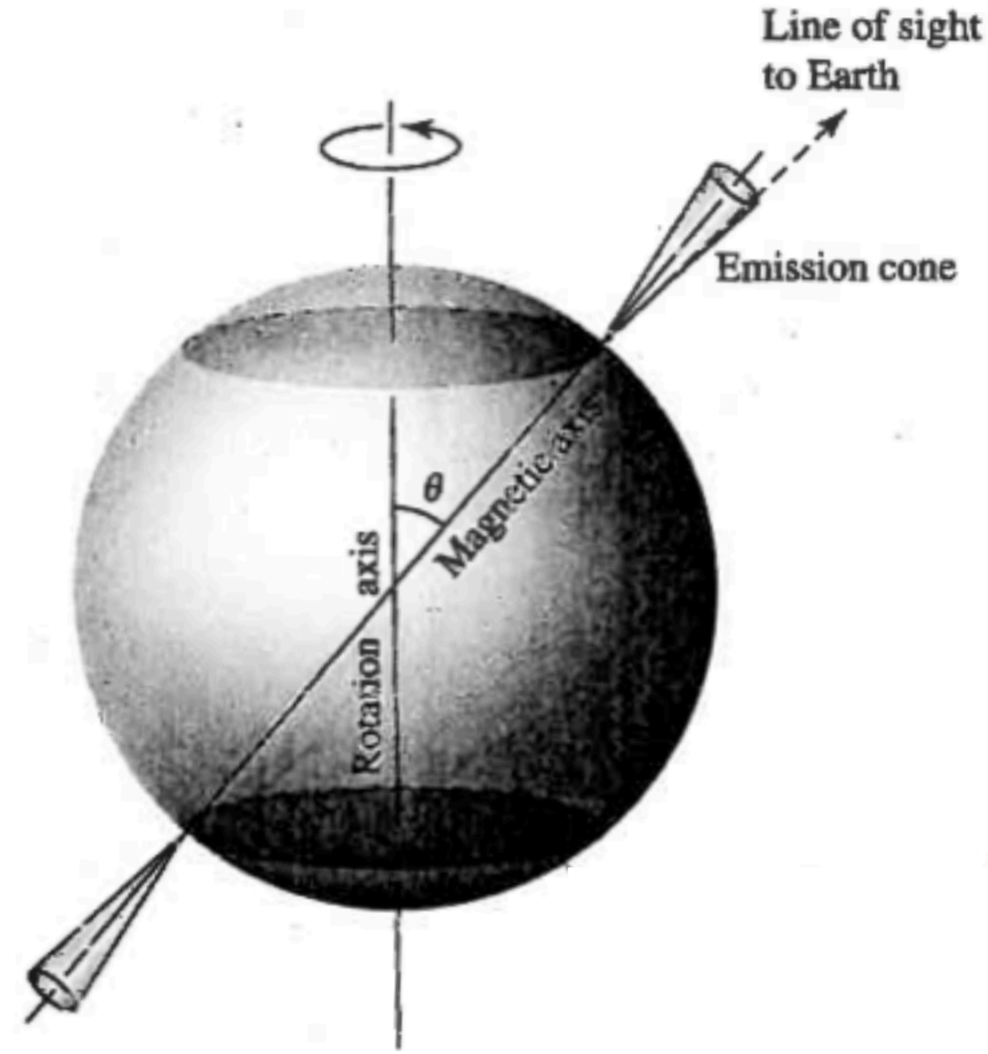


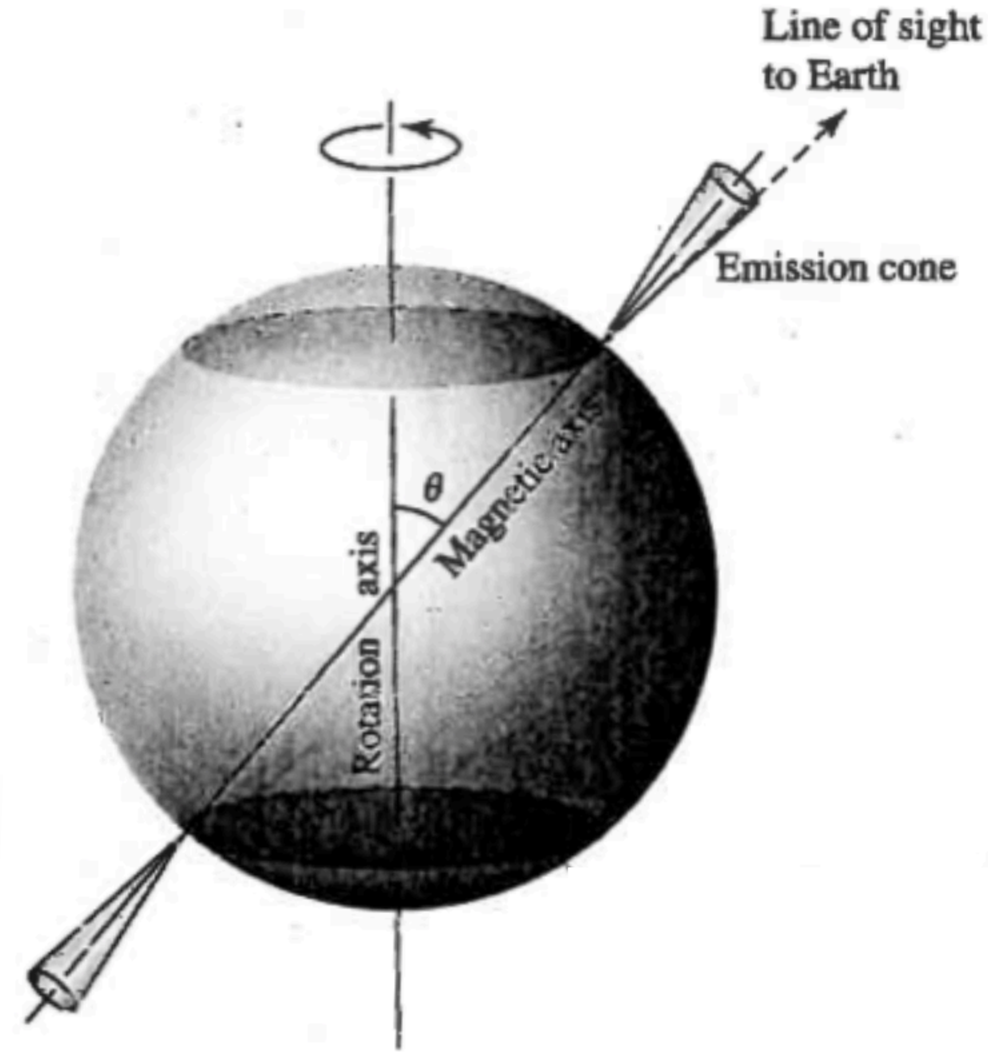
Image Credit: Michael Kramer (University of Manchester)

The magnetic field at any point in space changes rapidly due to the pulsar rotation; this induces an electric field at that point. The time-varying electric and magnetic fields form an electro-magnetic wave that carries energy away (**magnetic dipole radiation**)



$$K = \frac{1}{2} I \omega^2 = \frac{2\pi^2 I}{P^2}$$

The magnetic field at any point in space changes rapidly due to the pulsar rotation; this induces an electric field at that point. The time-varying electric and magnetic fields form an electro-magnetic wave that carries energy away (**magnetic dipole radiation**)



The energy per second (power) emitted by the rotating magnetic dipole is:

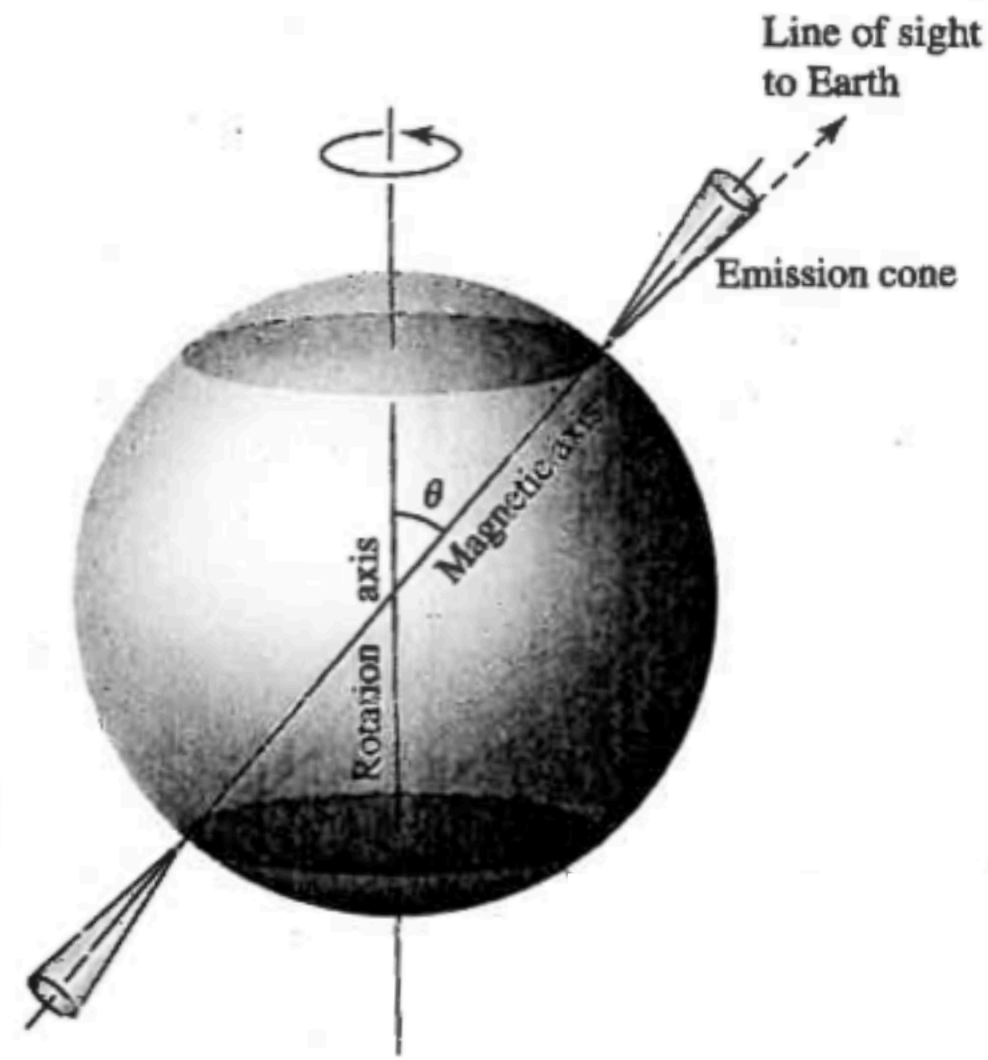
$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3 \mu_0 c^3 P^4}$$

permeability of free space

B is the strength of the field at the magnetic pole of the star with radius R and period or rotation P

$$K = \frac{1}{2} I\omega^2 = \frac{2\pi^2 I}{P^2}$$

The magnetic field at any point in space changes rapidly due to the pulsar rotation; this induces an electric field at that point. The time-varying electric and magnetic fields form an electro-magnetic wave that carries energy away (**magnetic dipole radiation**)



The energy per second (power) emitted by the rotating magnetic dipole is:

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}$$

permeability of free space

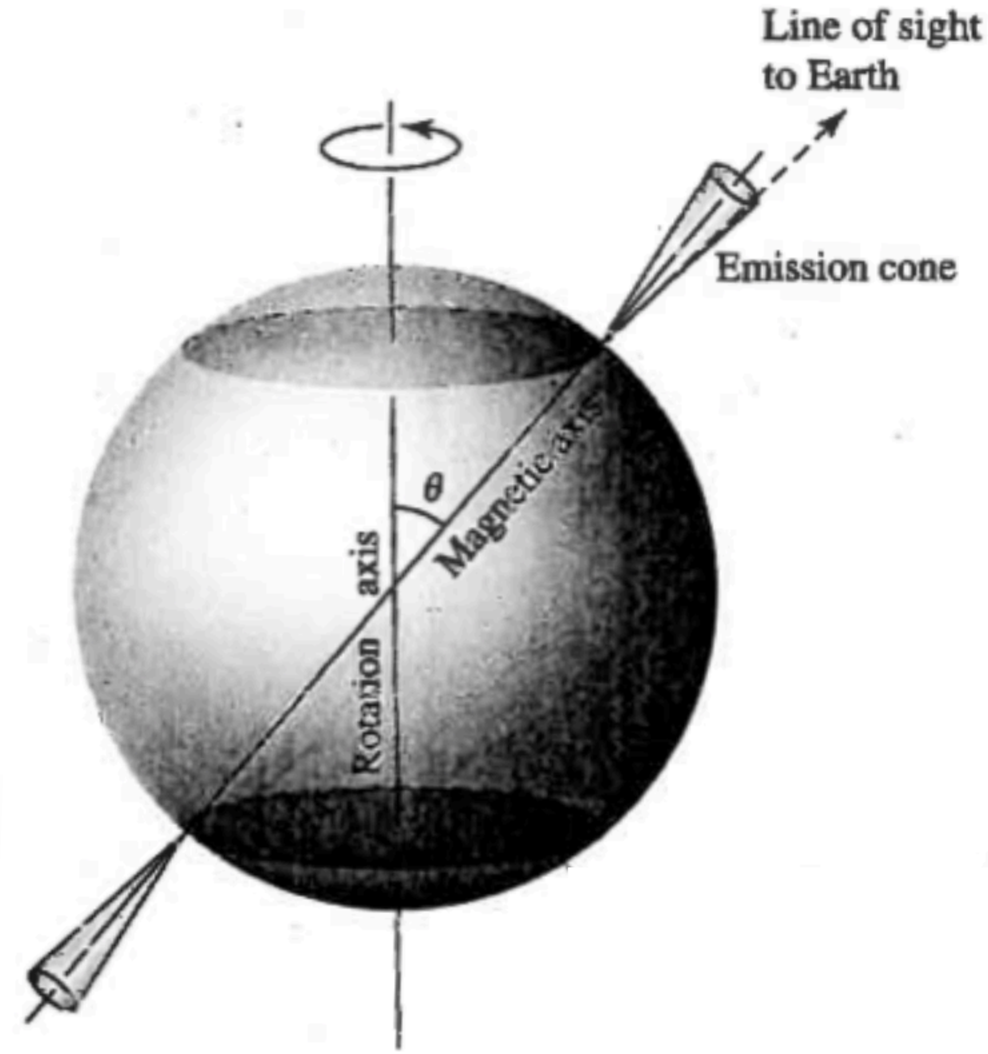
B is the strength of the field at the magnetic pole of the star with radius R and period or rotation P

The energy is taken away at the expense of the rotational kinetic energy (K) of the NS, which will slow down:

$$\frac{dE}{dt} = \frac{dK}{dt}$$

$$K = \frac{1}{2}I\omega^2 = \frac{2\pi^2 I}{P^2}$$

The magnetic field at any point in space changes rapidly due to the pulsar rotation; this induces an electric field at that point. The time-varying electric and magnetic fields form an electro-magnetic wave that carries energy away (**magnetic dipole radiation**)



The energy per second (power) emitted by the rotating magnetic dipole is:

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3 \mu_0 c^3 P^4}$$

permeability of free space

B is the strength of the field at the magnetic pole of the star with radius R and period or rotation P

The energy is taken away at the expense of the rotational kinetic energy (K) of the NS, which will slow down:

$$\frac{dE}{dt} = \frac{dK}{dt}$$

$$K = \frac{1}{2} I\omega^2 = \frac{2\pi^2 I}{P^2}$$

I: moment of inertia
P: period

$$\Rightarrow \quad -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4} = -\frac{4\pi I \dot{P}}{P^3}$$

$$\Rightarrow \quad B = \frac{1}{2\pi R^3 \sin\theta}\sqrt{\frac{3\mu_0 c^3 I P\dot{P}}{2\pi}}$$

For crab nebula $P = 0{,}0333\ s$

$\dot{P} = 4{,}21 \times 10^{-13}$

$\theta = 90°$ (assumed)

$\Rightarrow \quad B = 8 \times 10^{8}\ T$

$$\Rightarrow \quad -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4} = -\frac{4\pi I \dot{P}}{P^3}$$

$$\Rightarrow \quad B = \frac{1}{2\pi R^3 \sin\theta}\sqrt{\frac{3\mu_0 c^3 I P\dot{P}}{2\pi}}$$

For crab nebula $P = 0,0333\ \mathrm{s}$

$\dot{P} = 4,21 \times 10^{-13}$

$\theta = 90°$ (assumed)

$$\Rightarrow \quad B = 8 \times 10^{8}\ \mathrm{T}$$

NOTE: the emission of radiation is one of the mostly poorly known/understood aspects of pulsars

White Dwarfs / Neutron Stars

Reading assignment
TUESDAY 12/8: 15.4 (Gamma ray bursts)

Homework Assignment #5 due by:
TUESDAY 12/15 before 9AM.