White Dwarfs

Reading assignment
THURSDAY 12/3: 16.6, 16.7

Homework Assignment #5 due by:
TUESDAY 12/15 before 9AM.

Massive stars: $M>8\ M_{Sun}$

For a star with $M>8\ M_{Sun}$, the temperature in the core can get high enough for C and O fusion, ending its life as a CORE-COLLAPSE SUPERNOVA (type Ib, Ic, II).

The He-fusing shell adds ash to the C/O core, the core continues to contract, T rises until C fusion ignites, generating ^{16}O, ^{20}Ne, ^{23}Na, ^{23}Mg, ^{24}Mg (very dependent on mass of the star) ==> onion-like shell structure.

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Following C fusion, the O in the resulting Ne/O core will fuse, producing new core dominated by ^{28}Si. At T~3x10^9 K, Si fusion begins:

$$Si^{28} + He^4 \rightleftharpoons S^{32} + \gamma$$
$$S^{32} + He^4 \rightleftharpoons Ar^{36} + \gamma$$
$$\vdots$$
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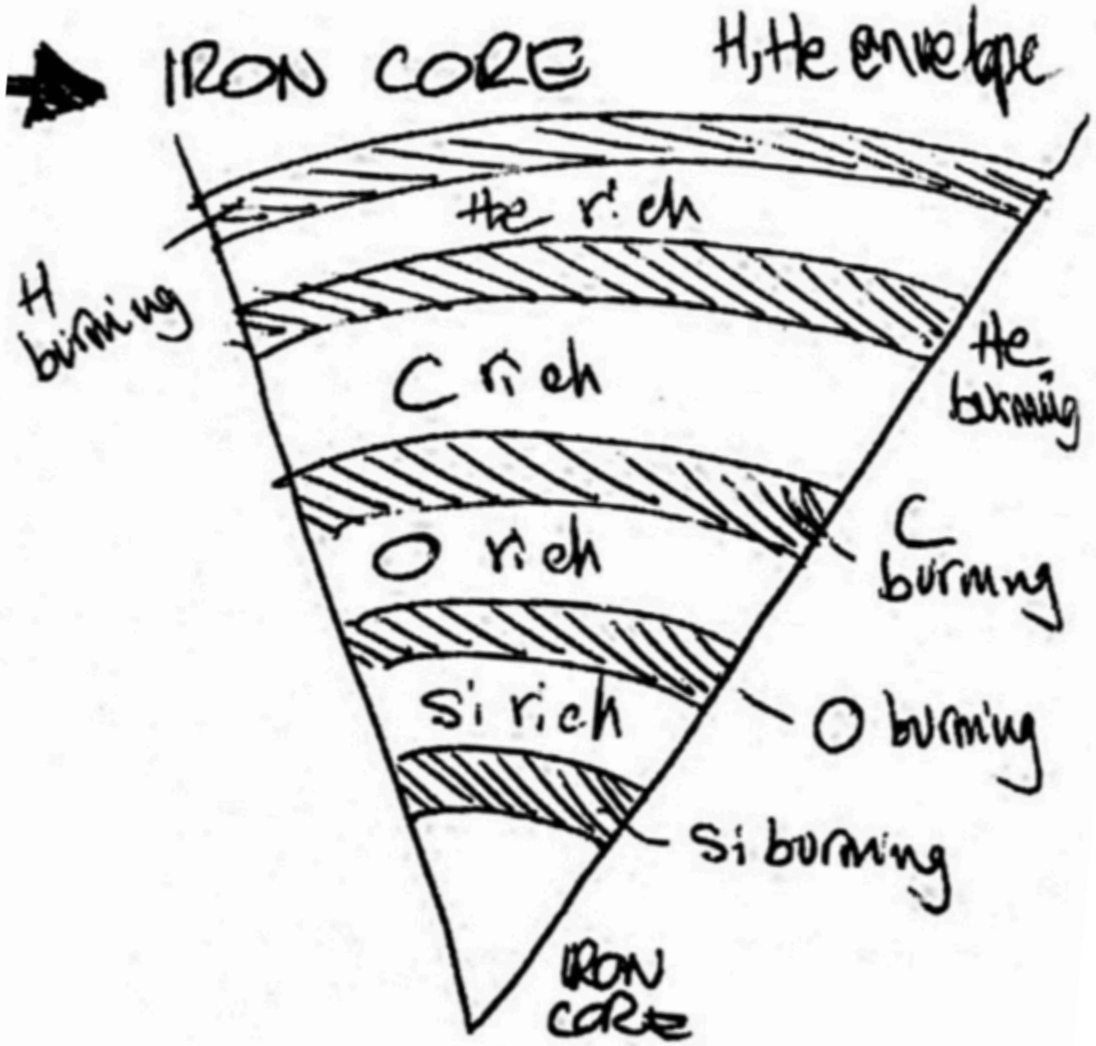
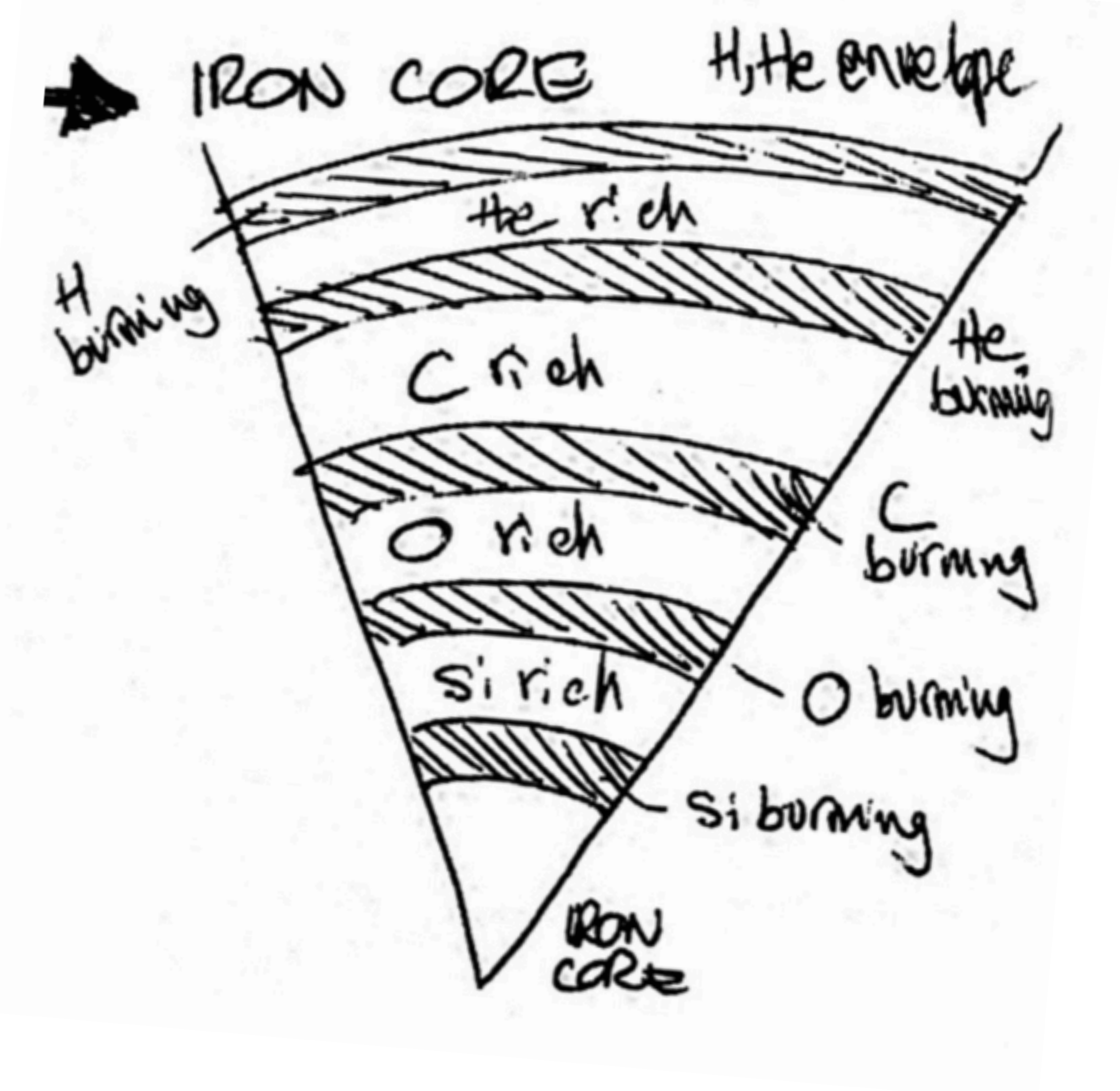
Si fusion produces nuclei centered near ^{56}Fe, most abundant being ^{54}Fe, ^{56}Fe, ^{56}Ni.

IRON CORE


H, He envelope
He rich
H burning
C rich
He burning
O rich
C burning
Si rich
O burning
Si burning
IRON CORE



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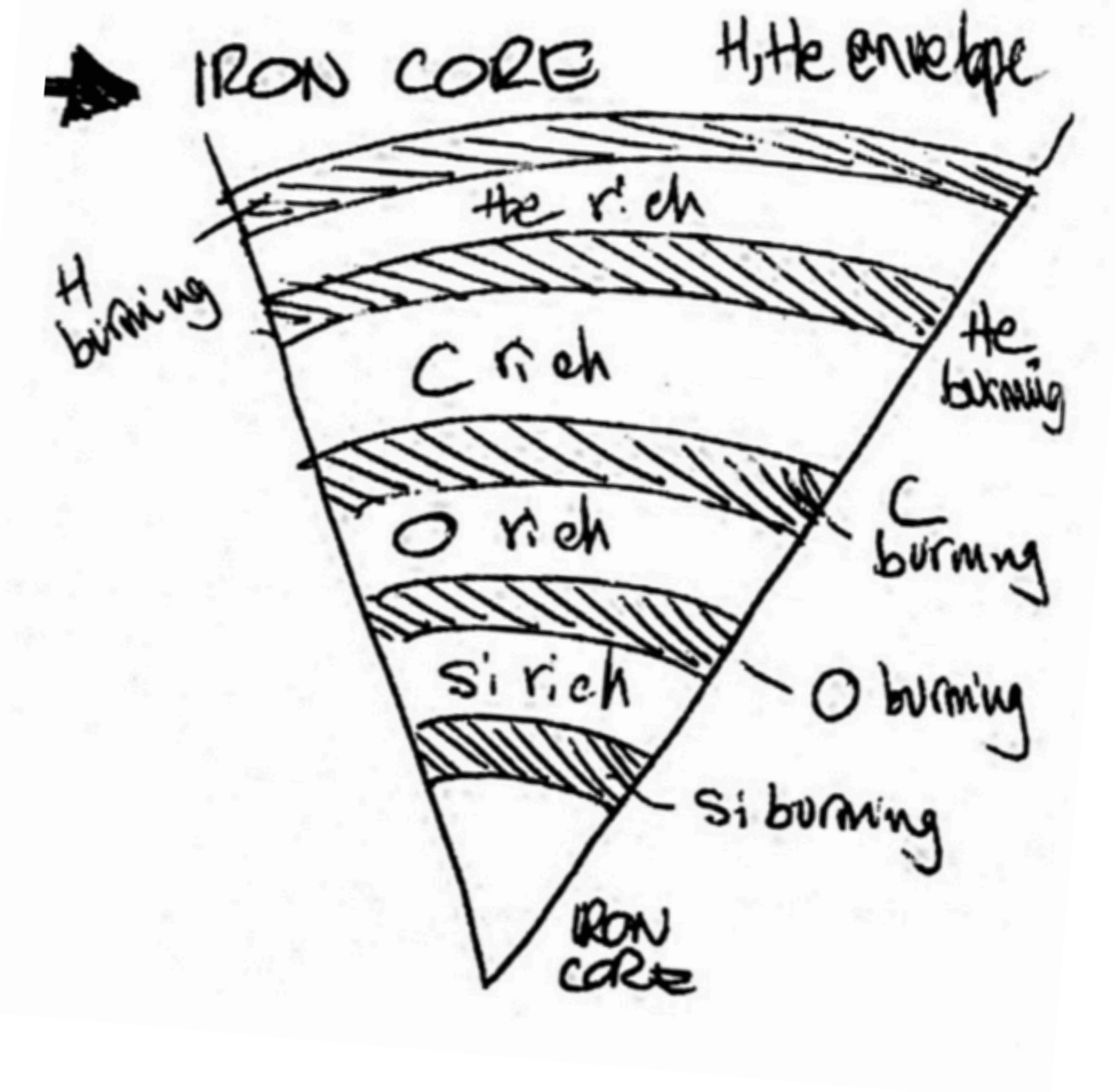

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For M~20 M_{Sun}:
MS ~ 10^7 yr
He core fusion ~ 10^6 yr
C core fusion ~ 300 yr
O core fusion ~ 200 days
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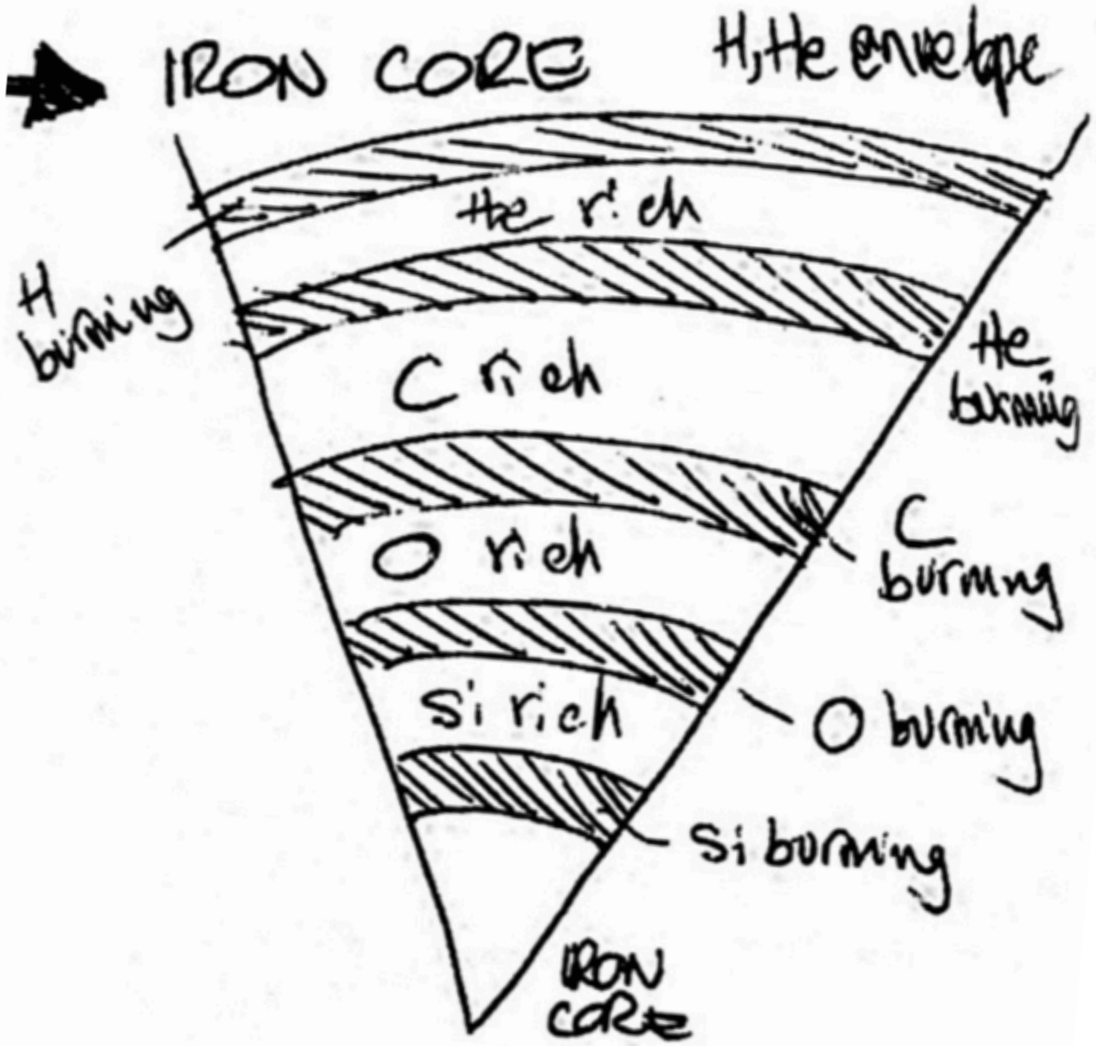

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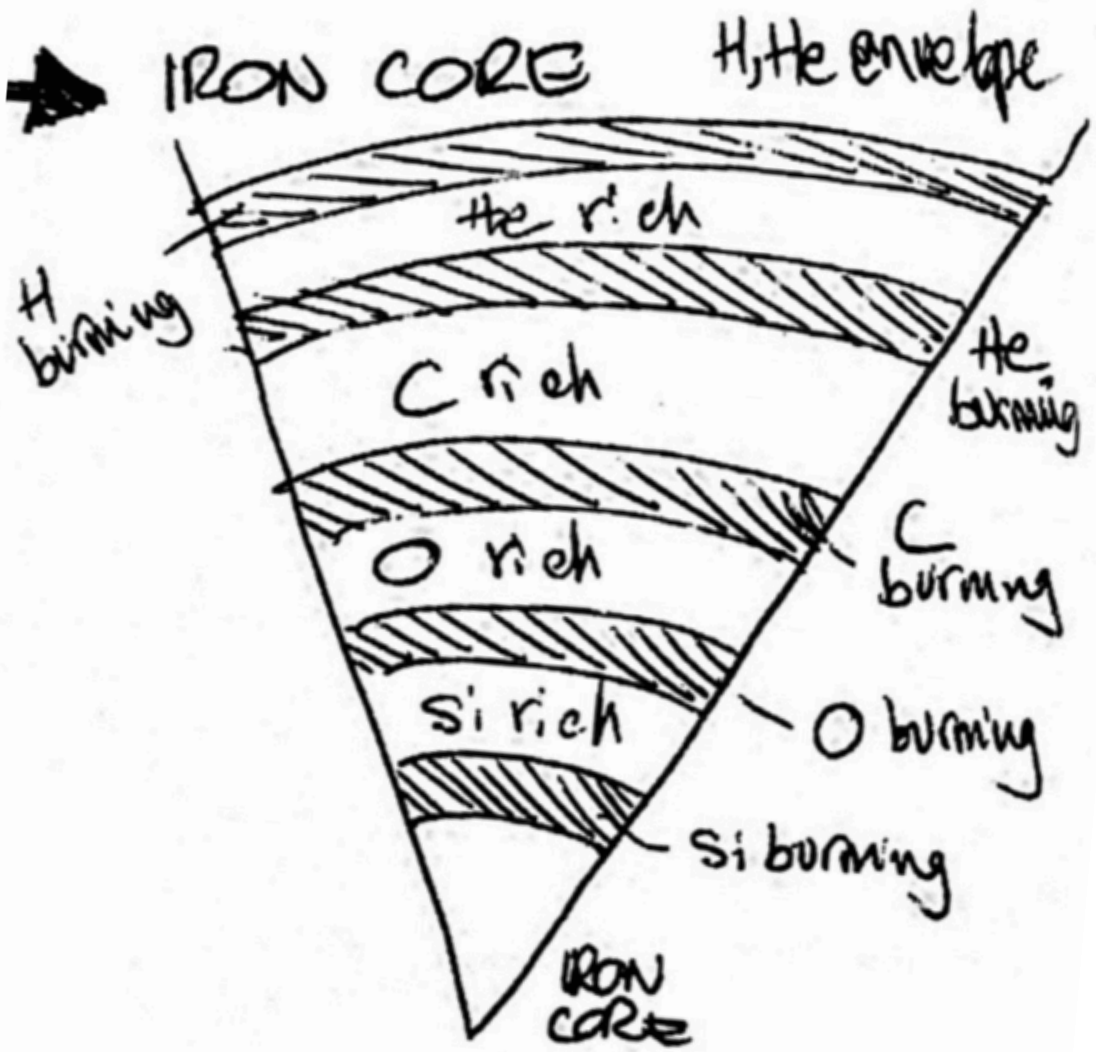
$$Fe^{56} + \gamma \rightarrow 13 He^4 + 4n$$
$$He^4 + \gamma \rightarrow 2p^+ + 2n$$



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This is a highly endothermic process (energy is needed), hence thermal energy is removed from the gas that would have otherwise provided the pressure to support the core. Moreover, at density~10^{10} g/cm^3 and T~8x10^9 K for a M=15 M_{Sun}, electrons (which were providing support via electron degeneracy pressure) are captures by nuclei:

$$p^+ + e^- \rightarrow n + \nu_e$$

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The collapse of the inner core continues until: $\rho \sim 3 \times 10^{14}\ g/cm^3 \approx 3\ \rho_{atomic\ nucleus}$

At this point, neutron degeneracy pressure dominates, halting the collapse; the core rebounds, sending pressure waves outward into the falling material from the outer core —> shock wave moving outward. The shock will drive the envelope and the remainder of the nuclear-processed matter in front of it. The total kinetic energy in the expanding material is ~10^{51} erg, roughly 1% of the energy released in neutrinos. At r~100 AU, the material becomes optically thin, with ~10^{49} erg of energy released as photons with peak luminosity nearly ~10^{43} erg/s, or $3 \times 10^9\ L_{Sun}$ ==> **CORE-COLLAPSE SUPERNOVA**

NOTE:

If $M<25\ M_{Sun}$ —> neutron star, supported by degenerate neutron pressure

If $M>>25\ M_{Sun}$ —> black hole

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s- and r- process nucleosynthesis:

$$ {}^{A}_{Z}X + n \longrightarrow {}^{A+1}_{Z}X + \gamma $$

$$ \xrightarrow{\text{beta decay}} {}^{A+1}_{Z+1}X + e^{-} + \bar{\nu}_e + \gamma $$

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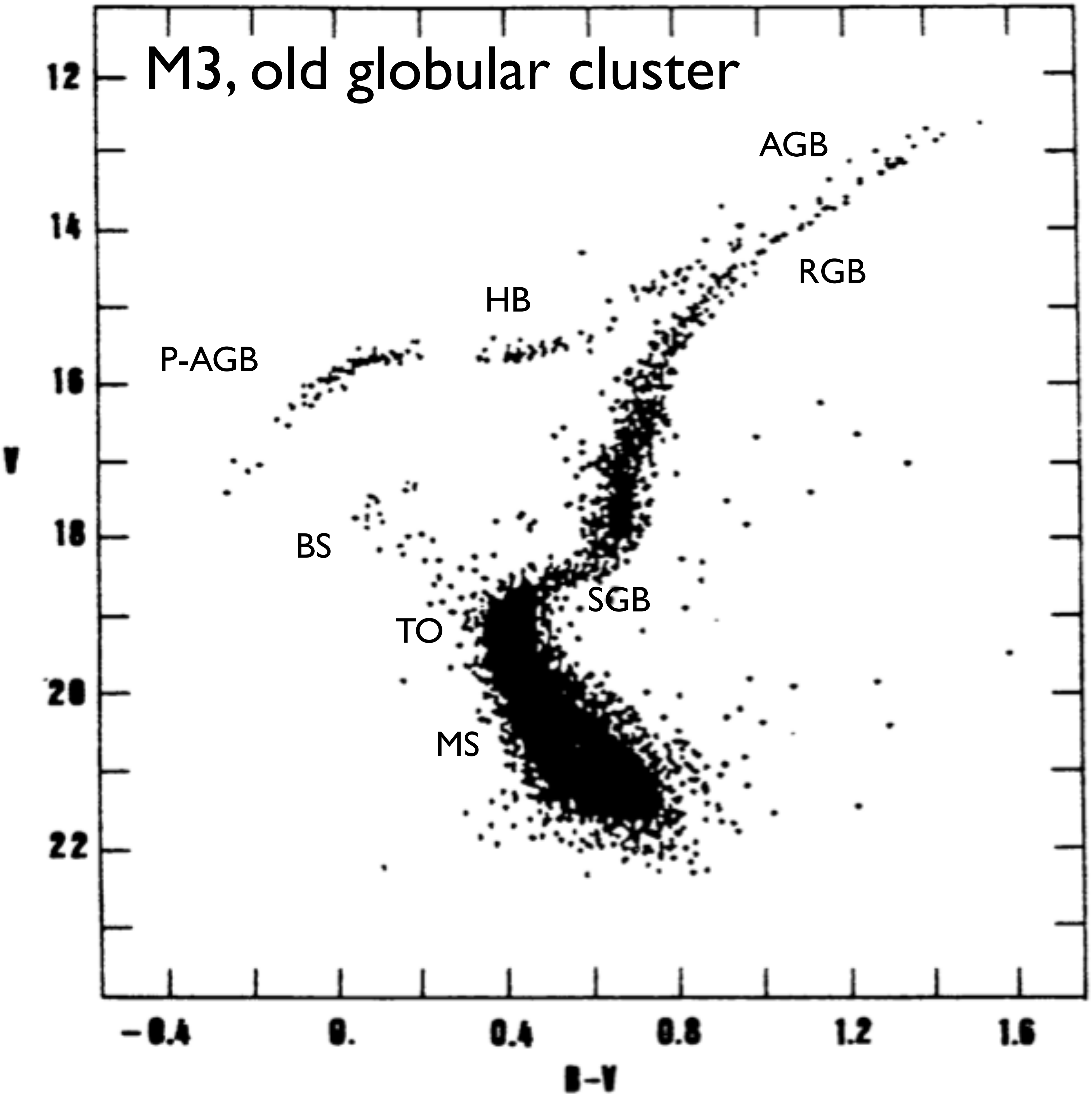
If the beta-decay half-life is short compared to the timescale for neutron capture, then SLOW PROCESS (“s”). s-process reactions tend to yield stable nuclei.

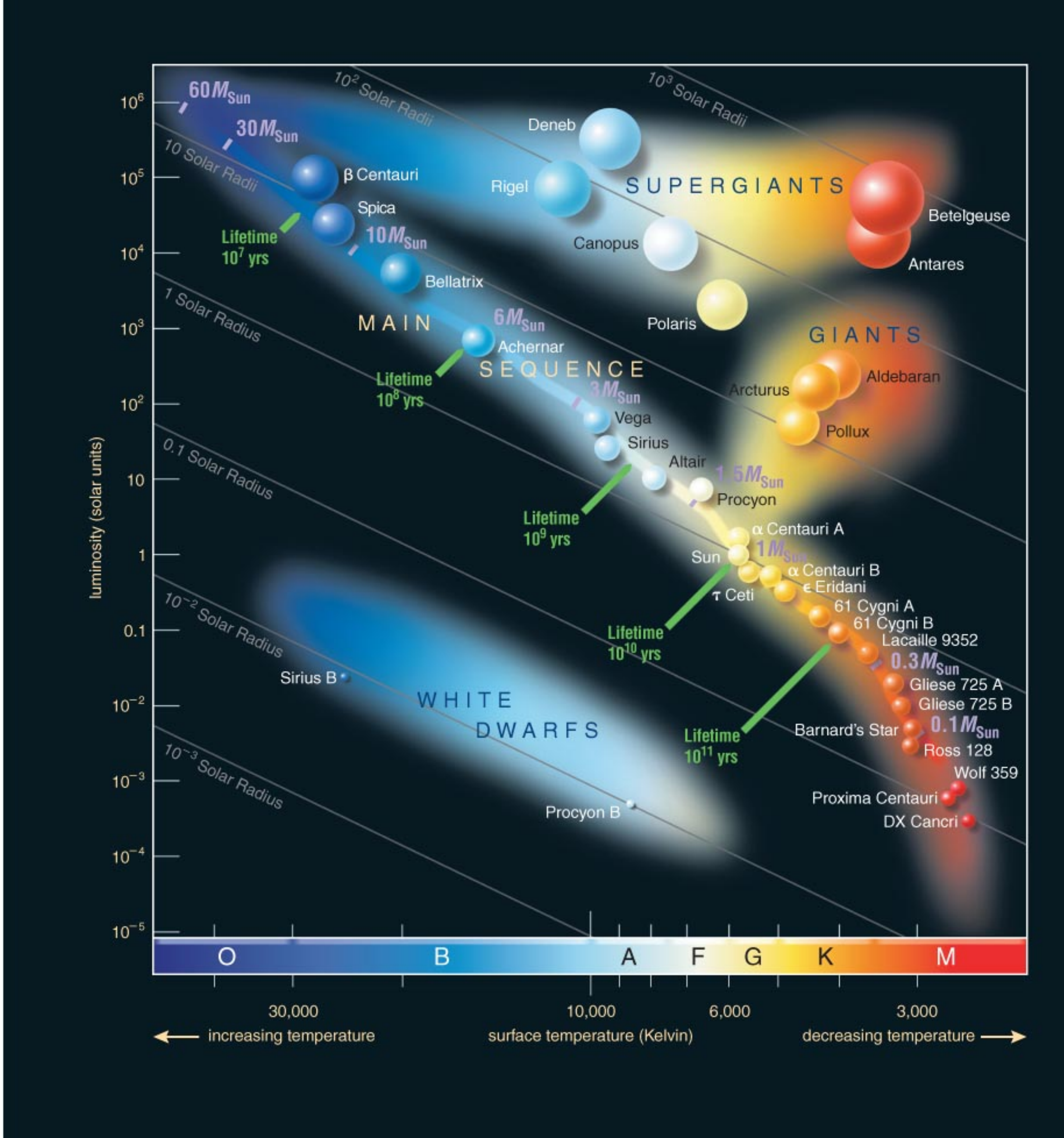
If the beta-decay half-life is long compared to the timescale for neutron capture, then RAPID PROCESS (“r”). r-process reactions result in n-rich nuclei.

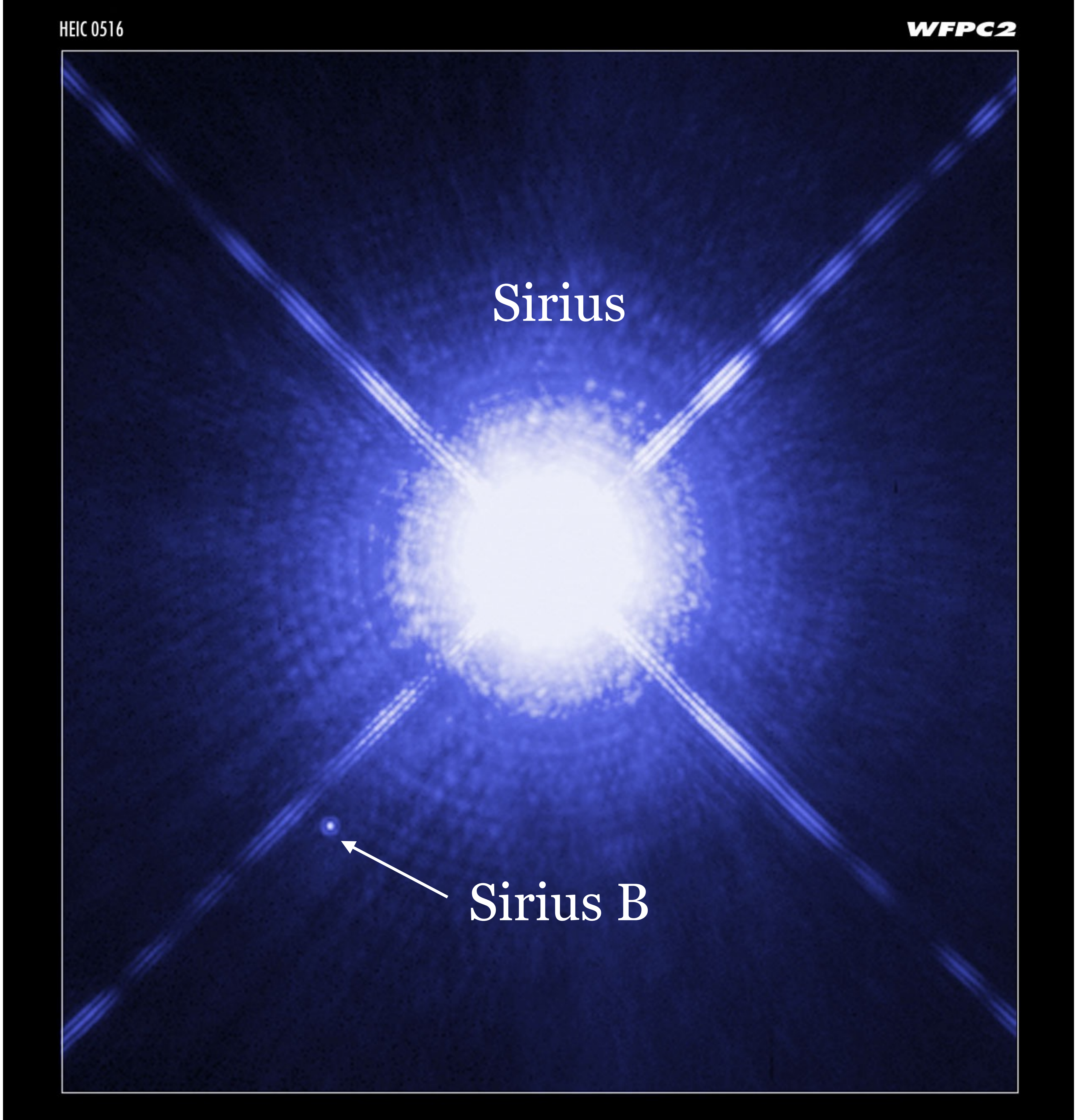
“s” processes tend to happen in normal phases of stellar evolution, whereas “r” processes occur during supernova explosions when large flux of neutrons exists. These processes account for the abundance ratios of nuclei with A>60.

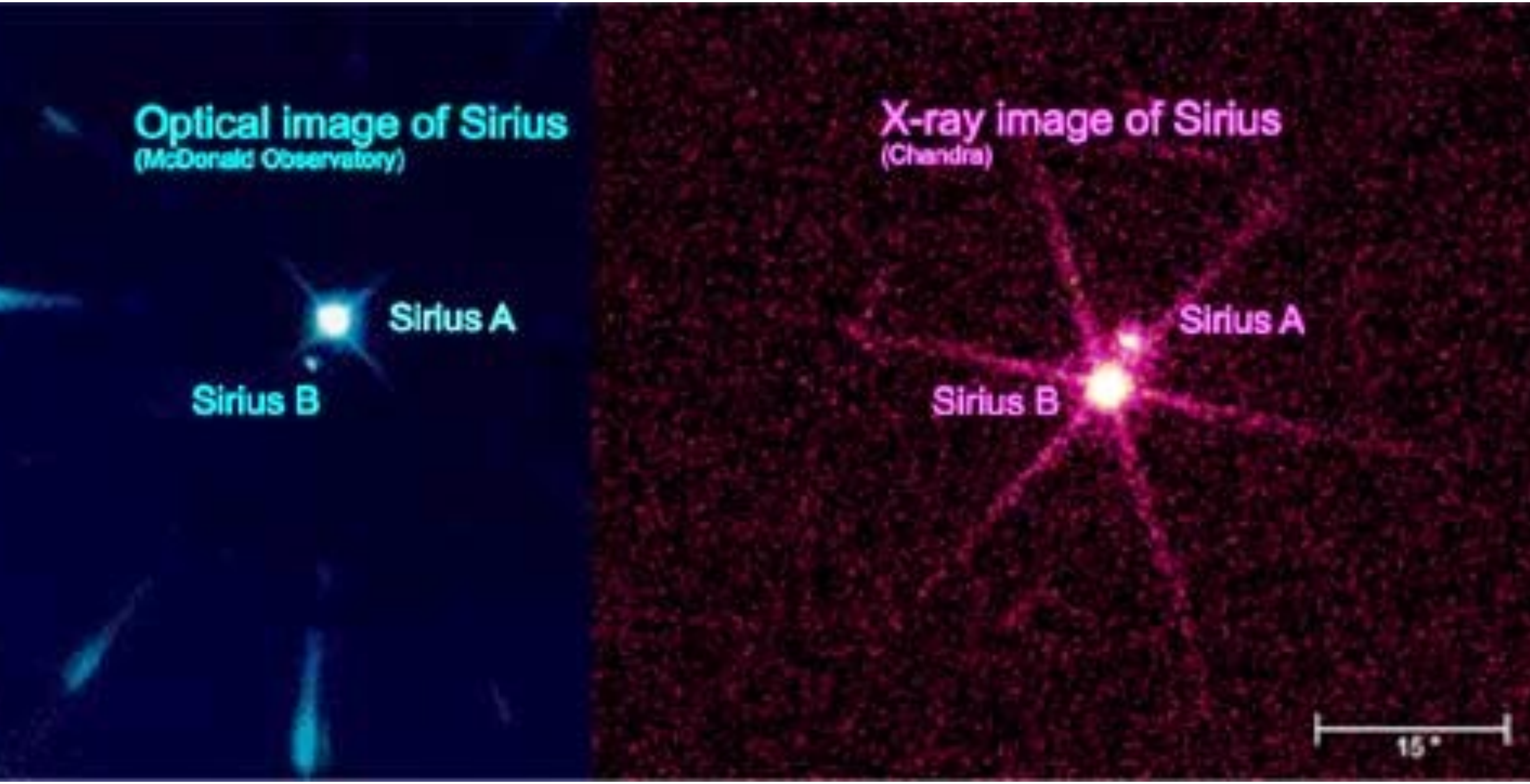

M3, old globular cluster
AGB
RGB
HB
P-AGB
BS
SGB
TO
MS
V
B−V
12
14
16
18
20
22
−0.4
0.
0.4
0.8
1.2
1.6

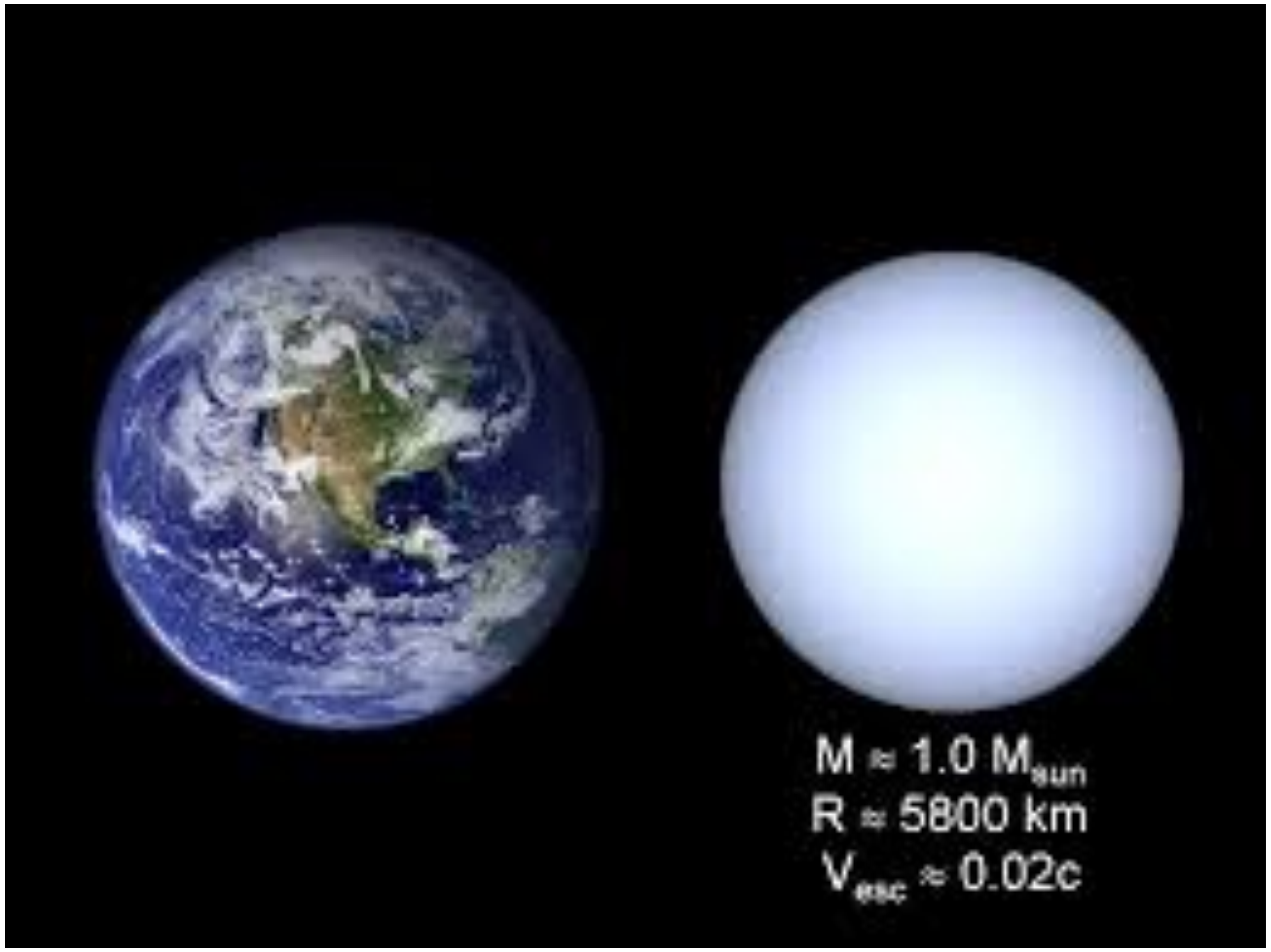
WHITE DWARFS

$T_{eff} \sim 5{,}000 - 80{,}000$ K

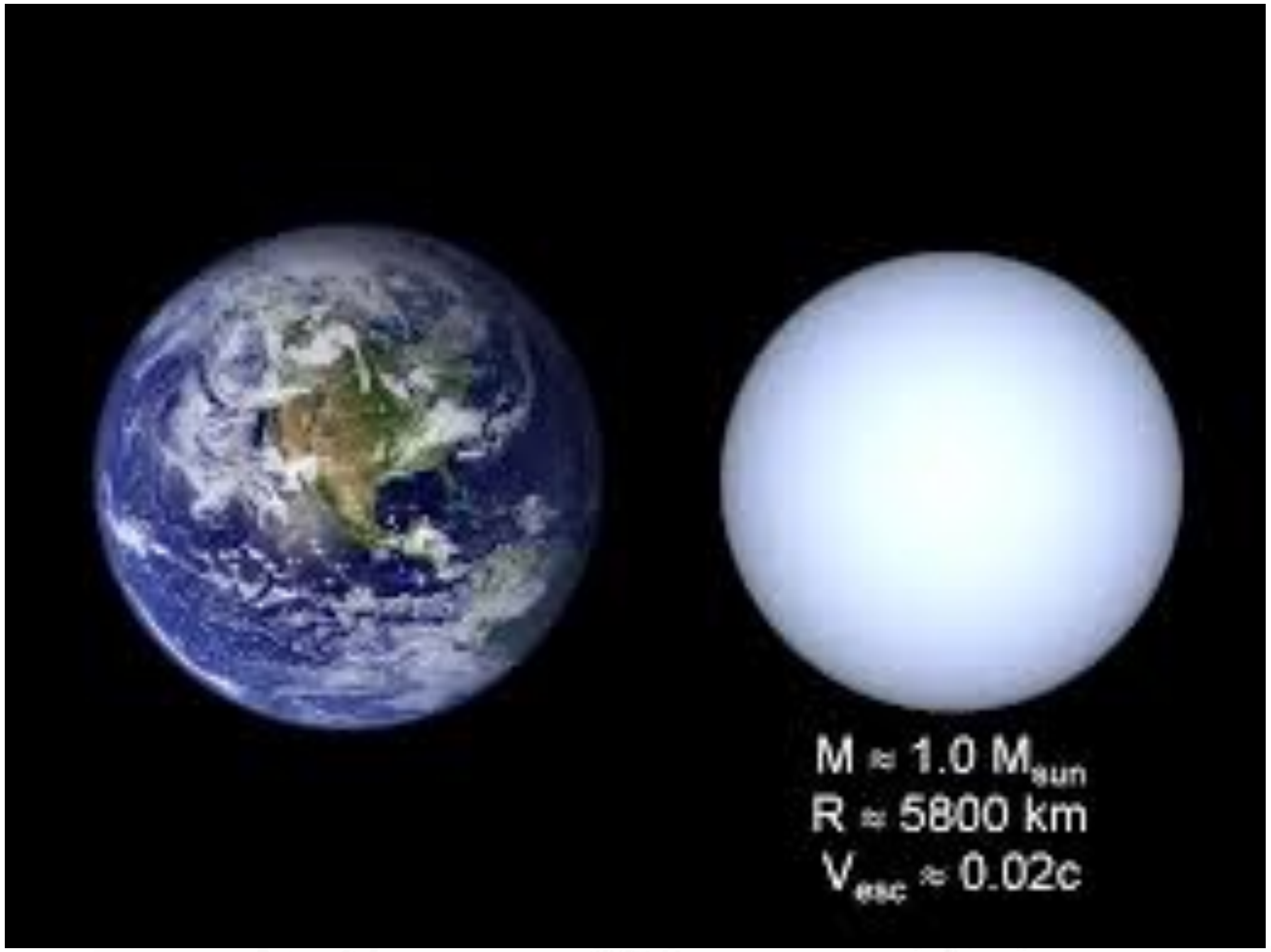



HEIC 0516
WFPC2
Sirius
Sirius B


Optical image of Sirius
(McDonald Observatory)
Sirius A
Sirius B
X-ray image of Sirius
(Chandra)
Sirius A
Sirius B
15 "



A typical white dwarf has a radius comparable to the Earth! But the mass is comparable to the Sun (here show 1 M_{Sun}, but really the average is ~0.5 M_{Sun}).



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Sirius B: $R \sim 5.5\times10^{8}\ \mathrm{cm} \approx 0.008\, R_\odot < R_E$

$\langle\rho\rangle \sim 3\times10^{6}\ \mathrm{g/cm^3}$

$P_c \simeq \frac{2}{3}\pi G \rho^2 R_{WD}^2 \sim 3.8\times10^{23}\ \frac{\mathrm{dine}}{\mathrm{cm^2}} \approx \boxed{2\times10^{6}\, P_{c,\odot}}$

$P(r) = \frac{2}{3}\pi G\rho^2 (R^2 - r^2)$

from eq. Hydrostatic eq.

$\frac{dP}{dr} = -\frac{4}{3}\pi G \rho^2 r$

$P_{c,\odot} \approx 2\times10^{17}\ \frac{\mathrm{dine}}{\mathrm{cm^2}}$

From $$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{T_{WD} - T_c}{R_{WD} - 0} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T_c^3}\frac{L_{WD}}{4\pi R_{WD}^2}$$

Using $\bar{\kappa} = 0.2\,\frac{cm^2}{g}$ (using $X=0$ for the e^- scattering opacity)

$$\Rightarrow T_c \simeq \left[\frac{3\bar{\kappa}\rho}{4ac}\frac{L_{WD}}{4\pi R_{WD}}\right]^{1/4} \simeq 7.6 \times 10^7\ K$$

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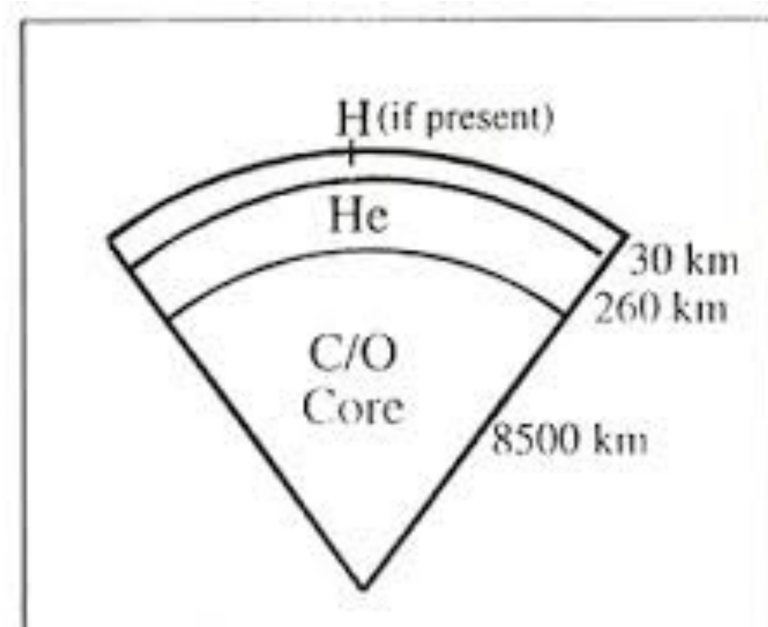
White dwarfs are very hot ==> **they cannot be made of H**, otherwise nuclear reactions (pp chains and CNO cycle) would make these objects much more luminous. WDs have a thin layer of H covering a layer of He on the top of the C/O core.



Cross Section of a Typical White Dwarf

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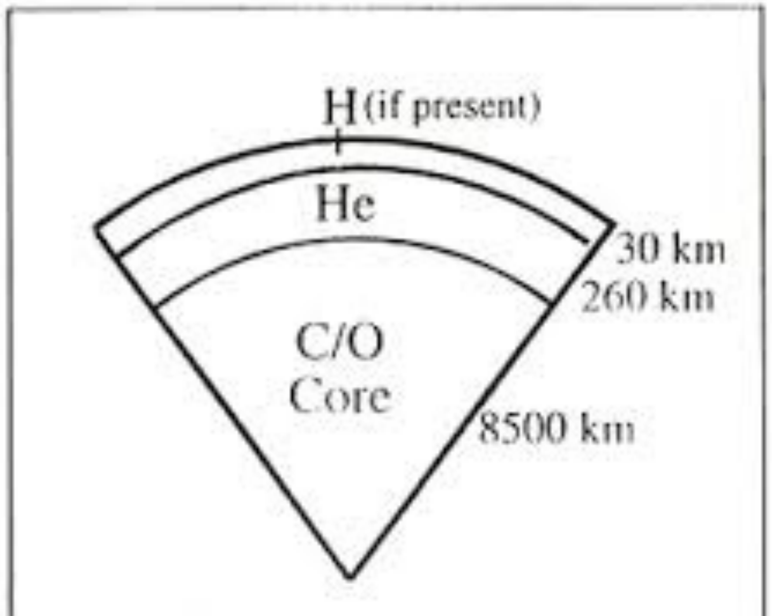

Cross Section of a Typical White Dwarf

Thermonuclear reactions are not responsible in producing the energy radiated by WDs. WDs are made of cores from stars with $M<8$-$9\ M_{Sun}$, mostly made of completely ionized C and O. Typical mass is $0.56\ M_{Sun}$, with 80% in $[0.42, 0.70]\ M_{Sun}$. Significant mass loss occurred while on the AGB, through thermal pulses and superwinds.

For Sirius B, complete degeneracy in the centre:

$$P_{e,nr} = 1.004 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \frac{\text{dynes}}{\text{cm}^2} \underset{\mu_e \simeq 2}{\approx} 2 \times 10^{23} \frac{\text{dynes}}{\text{cm}^2}$$

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By placing $P_{e,nr} = \frac{2}{3}\pi G \rho^2 R_{WD}^2$ w/ $\rho = M_{WD} / \frac{4}{3}\pi R_{WD}^3$

$$\Rightarrow R_{WD} = \frac{7.723 \times 10^{12} \text{ cm}}{(\mu_e)^{5/3} (M_{WD})^{1/3} G} \underset{\underset{\substack{M_{WD} \sim 1 M_\odot \\ \mu_e \sim 2.}}{|}}{\approx} 2.9 \times 10^{8} \text{ cm}$$

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$M_{WD} \sim 1 M_\odot$

$\mu_e \sim 2.$

$$M_{WD} \cdot R_{WD}^3 = \text{const}$$

$$\Updownarrow$$

$$\boxed{M_{WD} V_{WD} = \text{const}}$$

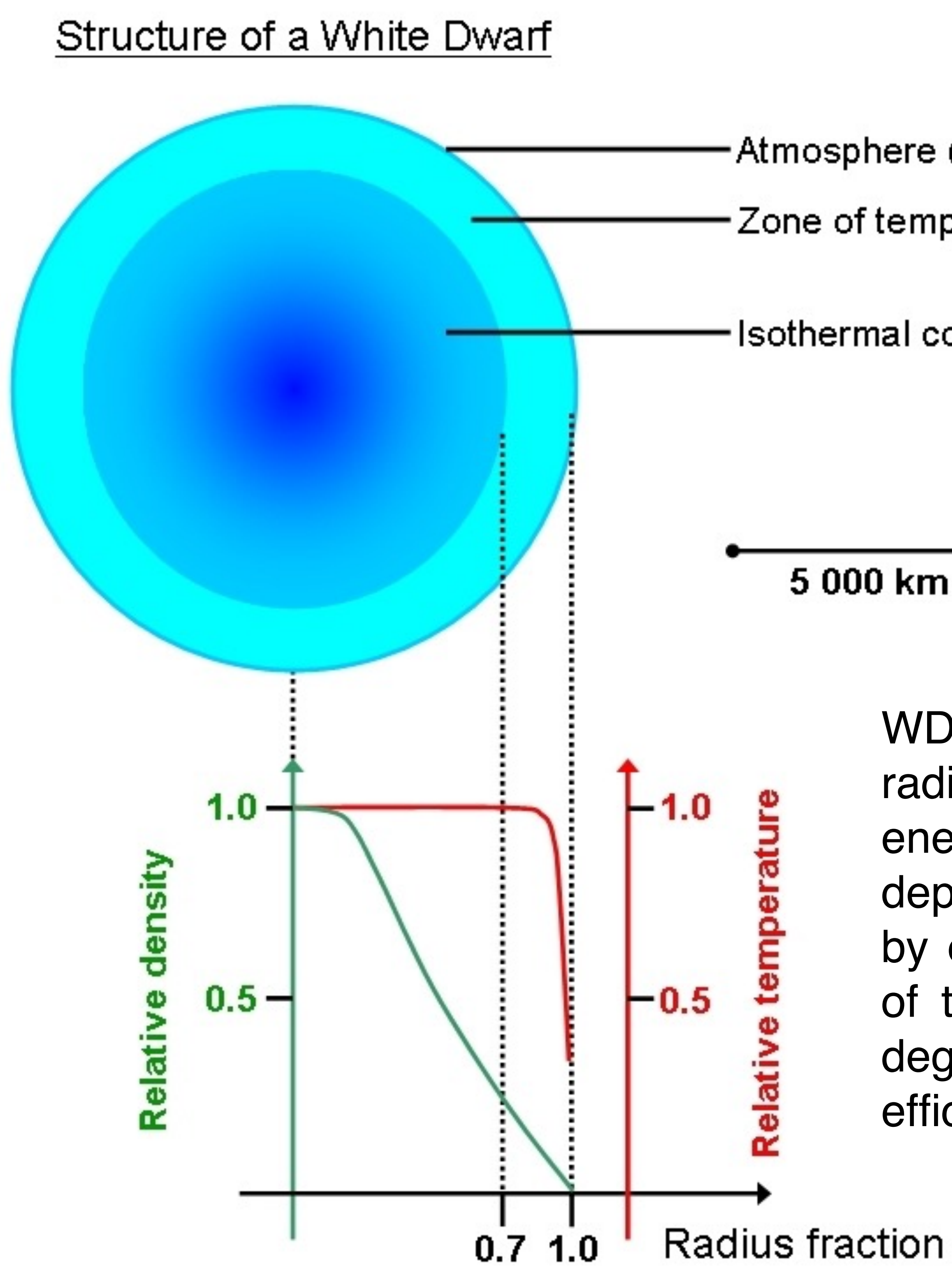
i.e., **more massive WDs are smaller**

The mass-volume relation comes from the star deriving its support from electron degeneracy pressure.

Chandrasekhar limit:

$$M_{WD} = \frac{5.83}{\mu_e^2} M_\odot \approx \boxed{1.45 M_\odot}$$

$\mu_e \sim 2$ for C/O WDs.



WDs simply cool off at an essentially constant radius as they slowly deplete their supply of thermal energy (degenerate electron pressure does not depend on T). The energy inside the WD is carried by electron conduction, so efficient that the interior of the WD is almost isothermal. Only at the non-degenerate surface, the heat is transferred less efficiently (through convection).

From Eqs. 2 and 5A of stellar structure with the mass as the independent variable:

$$\frac{dP}{dM} = -\frac{GM}{4\pi r^4} \; ; \; \frac{dT}{dM} = -\frac{3}{4ac}\frac{\tilde{\kappa}}{T^3}\frac{L}{16\pi^2 r^4}$$

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$$\frac{dP}{dT} = \frac{16\pi}{3}\frac{GM}{L}\frac{ac\,k}{A\,\mu m_H}\frac{T^{7.5}}{P} \quad \text{w/ } A = A_{bf} + A_{ff}$$

$$P = \frac{\rho k T}{\mu m_H}$$

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$$P = \frac{\rho k T}{\mu m_H}$$

integrating w/ respect to T & solving for P $\Longrightarrow$

$$P = \sqrt{\frac{4}{17}\frac{16\pi}{3}\frac{GM}{L}\frac{ack}{A\mu m_H}}\; T^{17/4}$$

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$$\Rightarrow \quad \rho = \left(\frac{4}{17} \frac{16\pi a c}{3} \frac{G M_{WD}}{L_{WD}} \frac{\mu m_H}{A K} \right)^{1/2} T^{13/4} \quad (\sim)$$

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At the transition between the non-degenerate surface layer and its isothermal degenerate interior: $\frac{\rho}{\mu_e} = 2.4 \times 10^{-8} \, T_c^{3/2} \quad (\approx)$

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Equating (~) and (~~) and solving for the WD luminosity:

$$L_{WD} = \text{constant} \cdot T_c^{7/2}$$

$$\text{w/ constant} = \frac{4}{17} \frac{16\pi ac}{3} G \frac{\mu m_H}{AK} M_{WD} \frac{1}{(2.4 \times 10^{-8} \mu_e)^2}$$

$$\approx 1.44 \times 10^{4} \left(\frac{M_{WD}}{M_\odot} \right) \frac{\mu}{Z(1+X)}$$

with $[L_{WD}]$=erg/s

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and using $P = \rho kT/\mu m_H$ to replace the pressure P

$$\Rightarrow \rho = \left(\frac{4}{17} \frac{16\pi a c}{3} \frac{G M_{WD}}{L_{WD}} \frac{\mu m_H}{A k}\right)^{1/2} T^{13/4} \quad (\sim)$$

At the transition between the non-degenerate surface layer and its isothermal degenerate interior: $\frac{\rho}{\mu_e} = 2.4 \times 10^{-8} T_c^{3/2} \quad (\approx)$

Equating (~) and (~~) and solving for the WD luminosity:

$$L_{WD} = \text{constant} \cdot T_c^{7/2}$$

$$\text{w/ constant} = \frac{4}{17} \frac{16\pi a c}{3} G \frac{\mu m_H}{A k} M_{WD} \frac{1}{(2.4 \times 10^{-8} \mu_e)^2}$$

$$\approx 1.44 \times 10^4 \left(\frac{M_{WD}}{M_\odot}\right) \frac{\mu}{Z(1+X)}$$

with $[L_{WD}]$=erg/s

For M_{WD}=1 M_{Sun},
X=0, Y=0.9, Z=0.1
—> mean mol. weight = 1.4
L_{WD}=0.03 L_{Sun} ==> T_C=4.4x10^7 K

L_{WD}/L_{Sun} ~ [10^{-4},10^{-2}]
==> T_C ~ 4-20x10^6 K

Cooling of a WD:

You can estimate the cooling timescale with: $\mathbf{t_{cooling} = U/L_{WD}}$

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Number of nuclei in the WD

Average thermal energy per nuclei

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Number of nuclei in the WD

Average thermal energy per nuclei

A better estimate needs to account for the fact that the temperature decreases with cooling:

$$-\frac{d}{dt} U = L_{WD} = C\,(T_C)^{7/2}$$

Depletion of the internal thermal energy providing the luminosity

Integrating: $$T_c(t) = T_0 \left(1 + \frac{5}{2}\frac{t}{\tau_0}\right)^{-7/5}$$

initial temperature

$$\tau_0 = \frac{3 M_{WD} K}{2 A m_H C T_0^{5/2}} \sim 10^9 \text{ yr}$$

A=4, M=M_{Sun}, L=0.001 L_{Sun}

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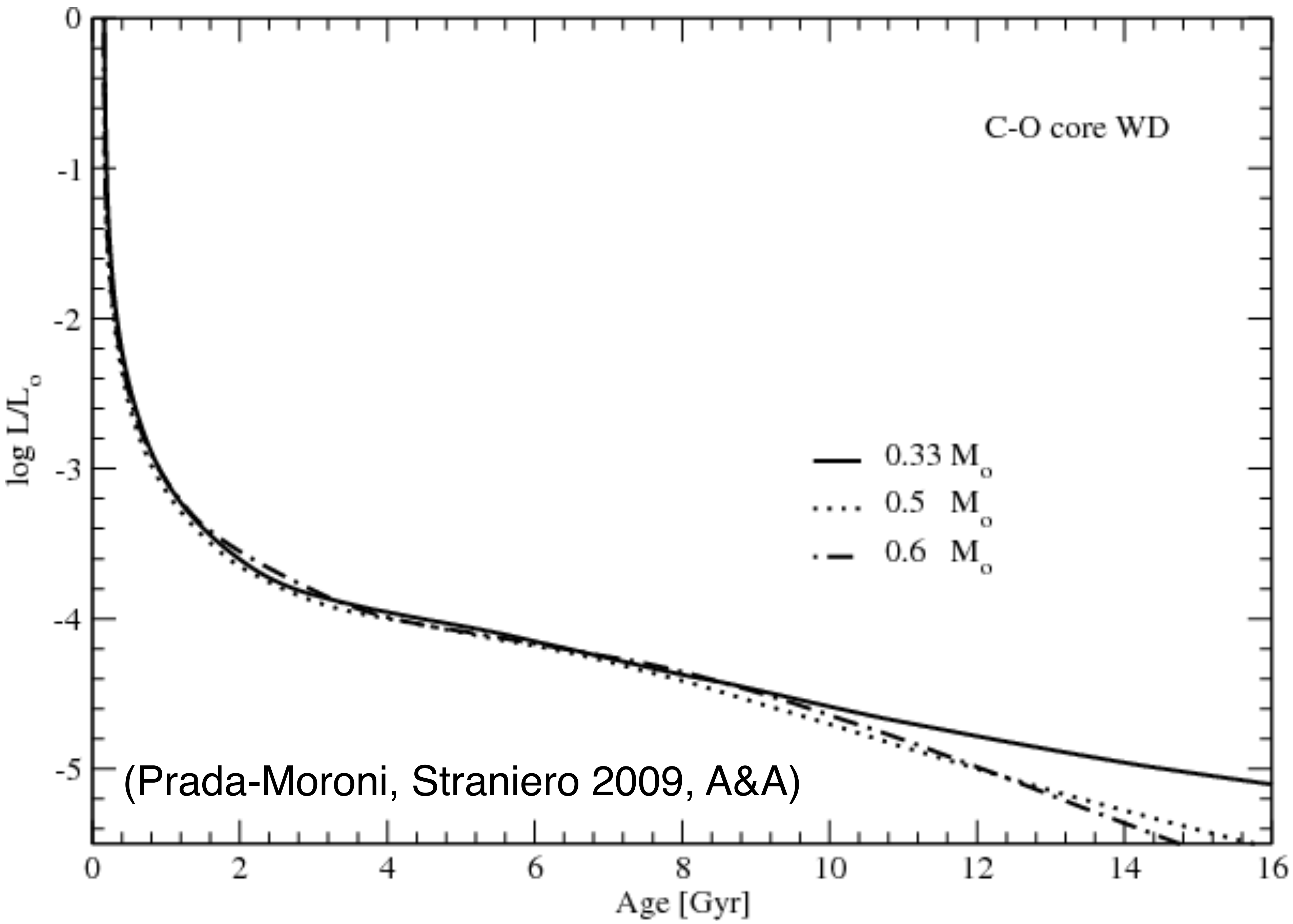

(Prada-Moroni, Straniero 2009, A&A)

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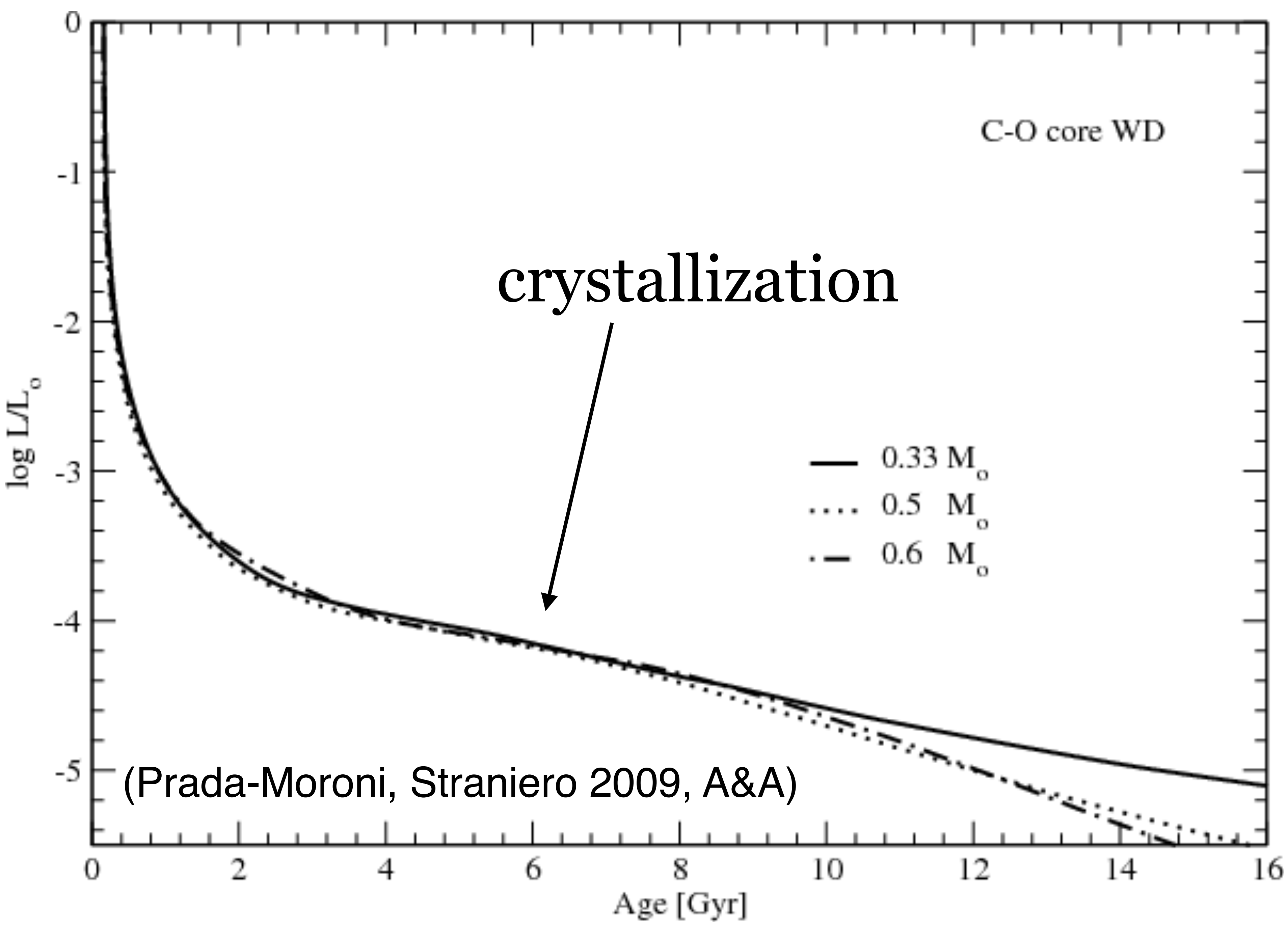

As the WD cools, the nuclei crystallize.
The release of latent heat slightly slows down the cooling process (extra internal source of heat).

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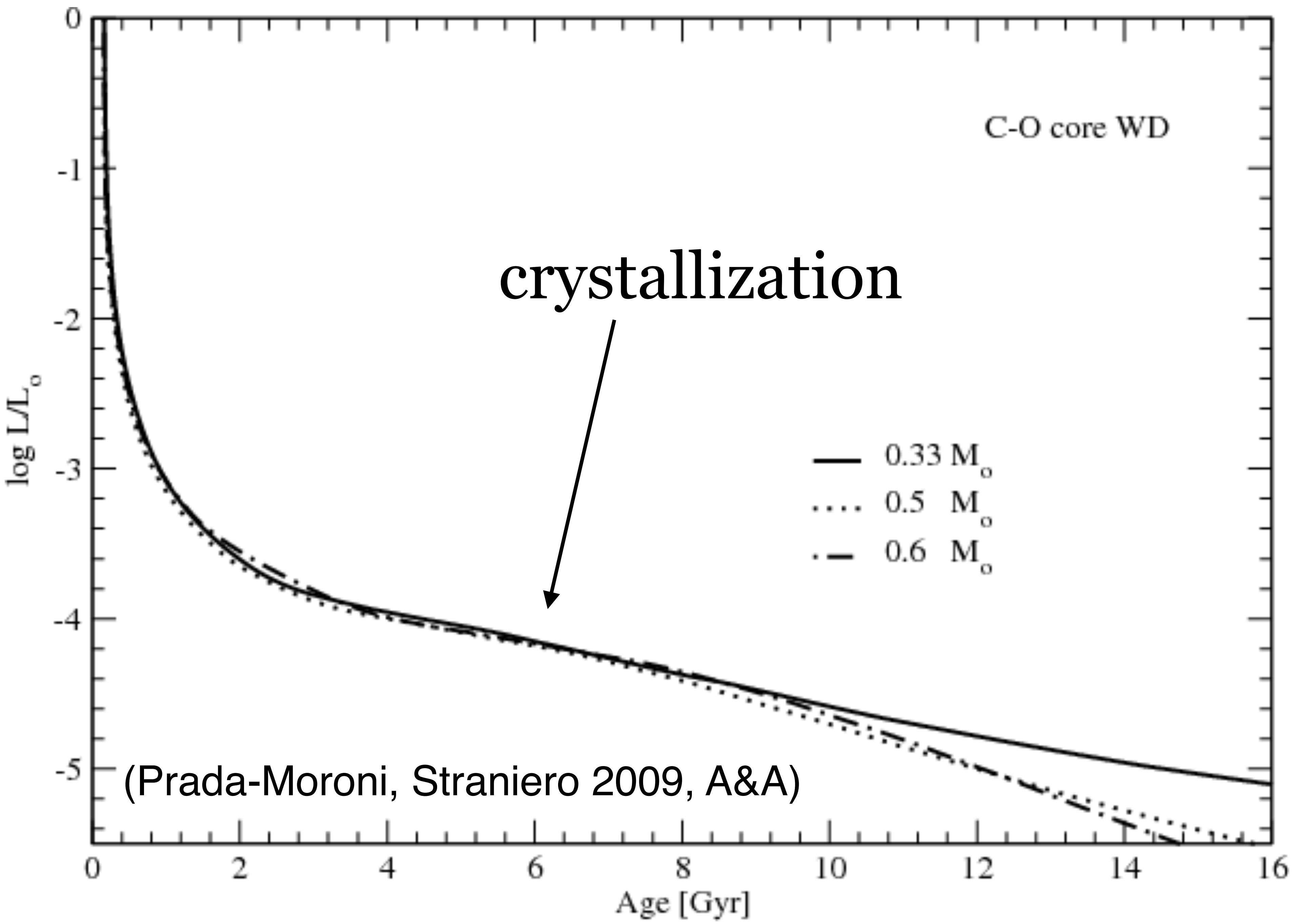
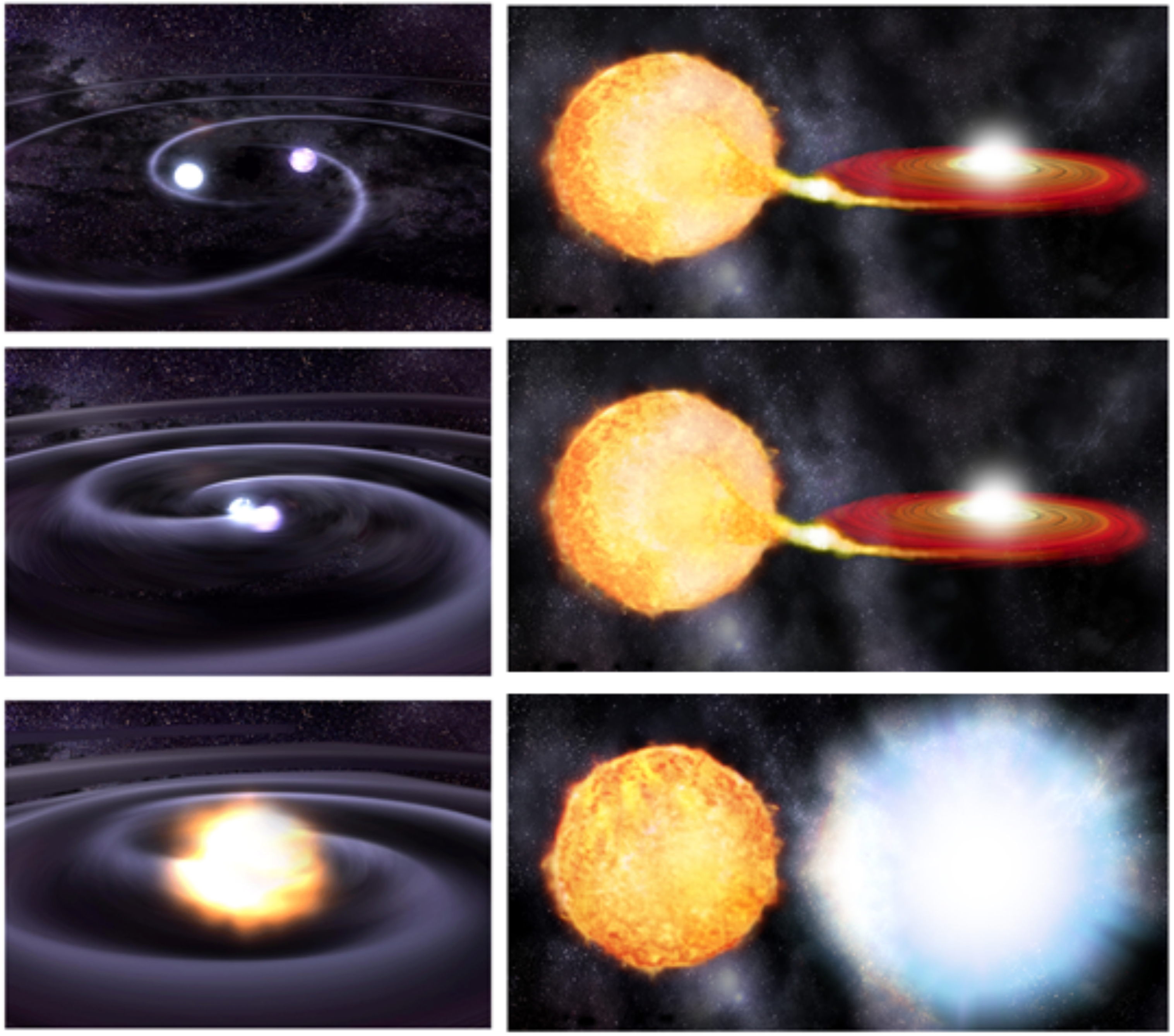


(Prada-Moroni, Straniero 2009, A&A)

As the WD cools, the nuclei crystallize.
The release of latent heat slightly slows down the cooling process (extra internal source of heat).

There are two principle avenues for WD mass growth

Double Degenerate | Single Degenerate



In either case, if the mass exceeds the Chandrasekhar mass, then the electron degeneracy pressure cannot support the star and it collapses under gravity resulting in a supernova — ***no remnant***!

Accretion disks

Accretion disks

Type Ia (white dwarf) SN

The Center for Astrophysical Thermonuclear Flashes

Simulation of the Deflagration and Detonation Phases of a Type Ia Supernovae

30 initial bubbles in 100 km radius.
Ignition occurs 80 km from the center of the star.
Hot material is shown in color and stellar surface in green.

This work was supported in part at the University of Chicago by the DOE NNSA ASC ASAP and by the NSF. This work also used computational resources at LBNL NERSC awarded under the INCITE program, which is supported by the DOE Office of Science.

An Advanced Simulation and Computation (ASC)
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ASC

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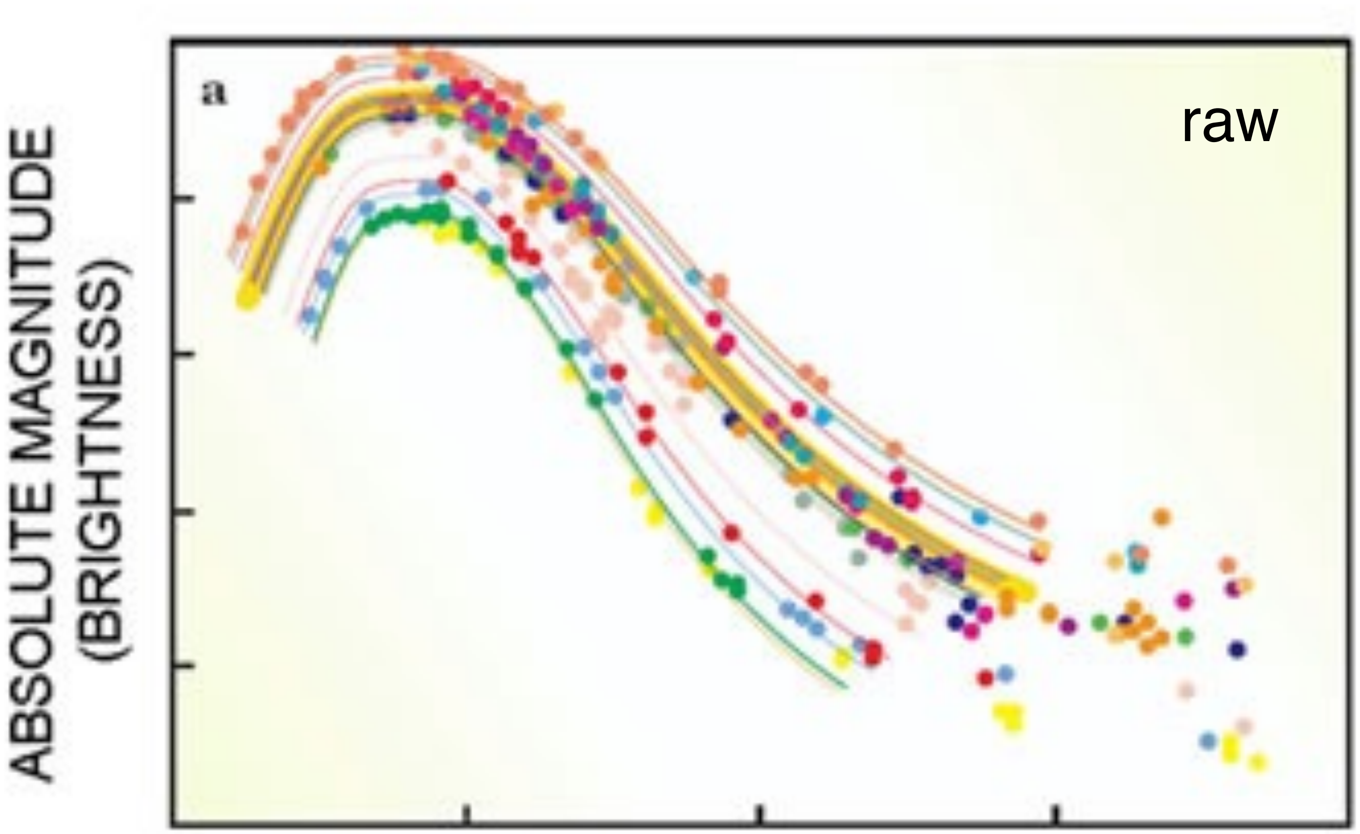
Type Ia supernovae light curves:



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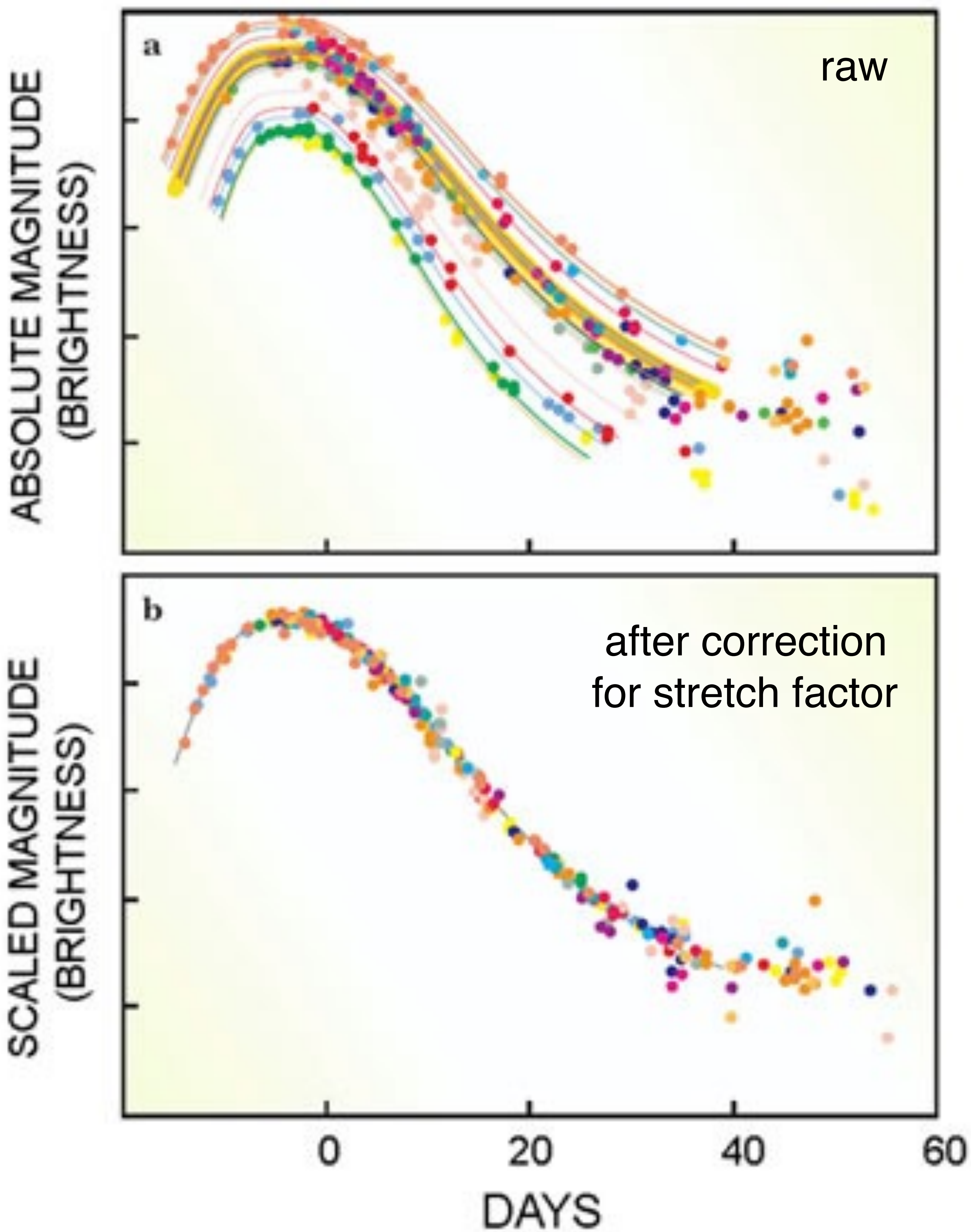

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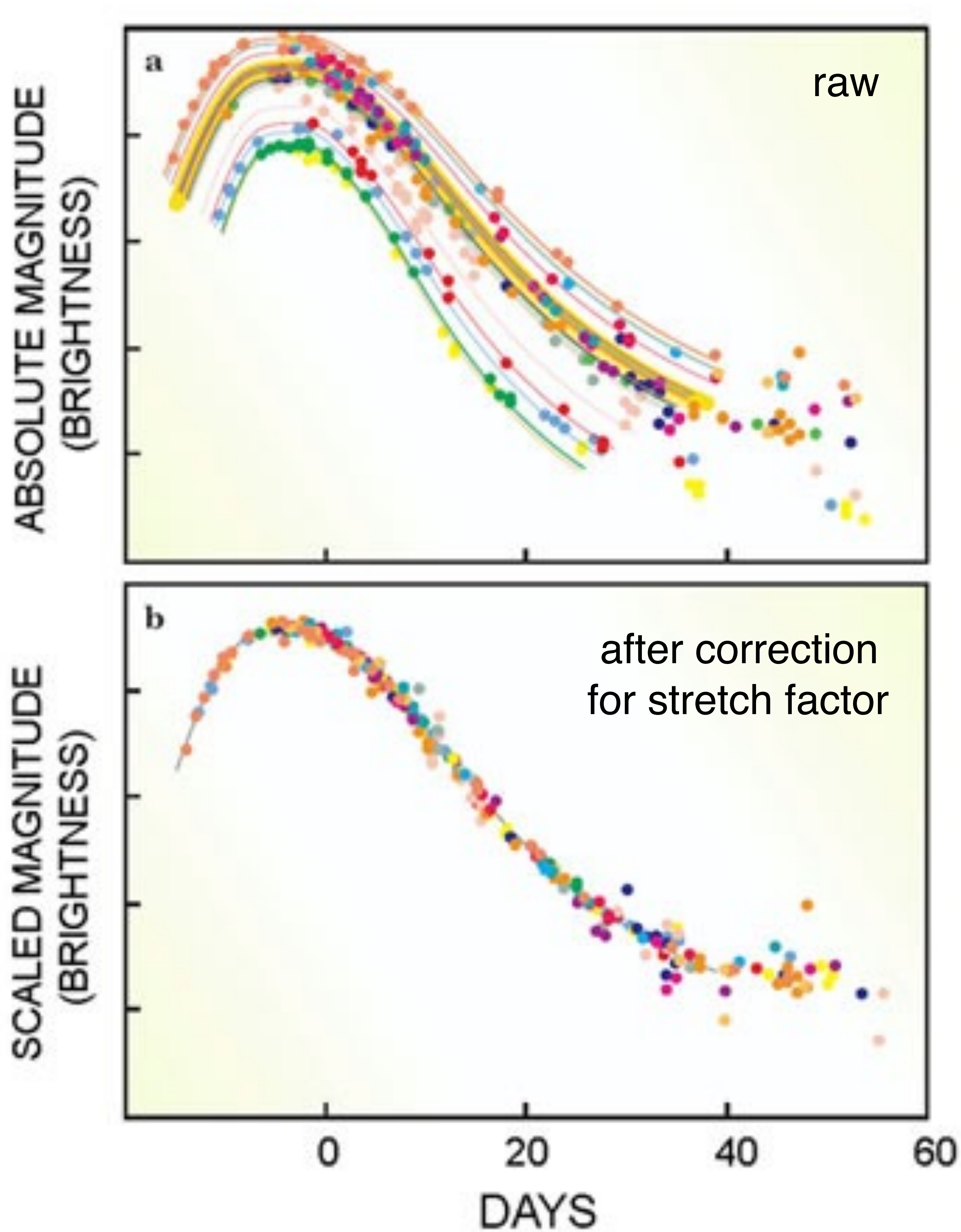
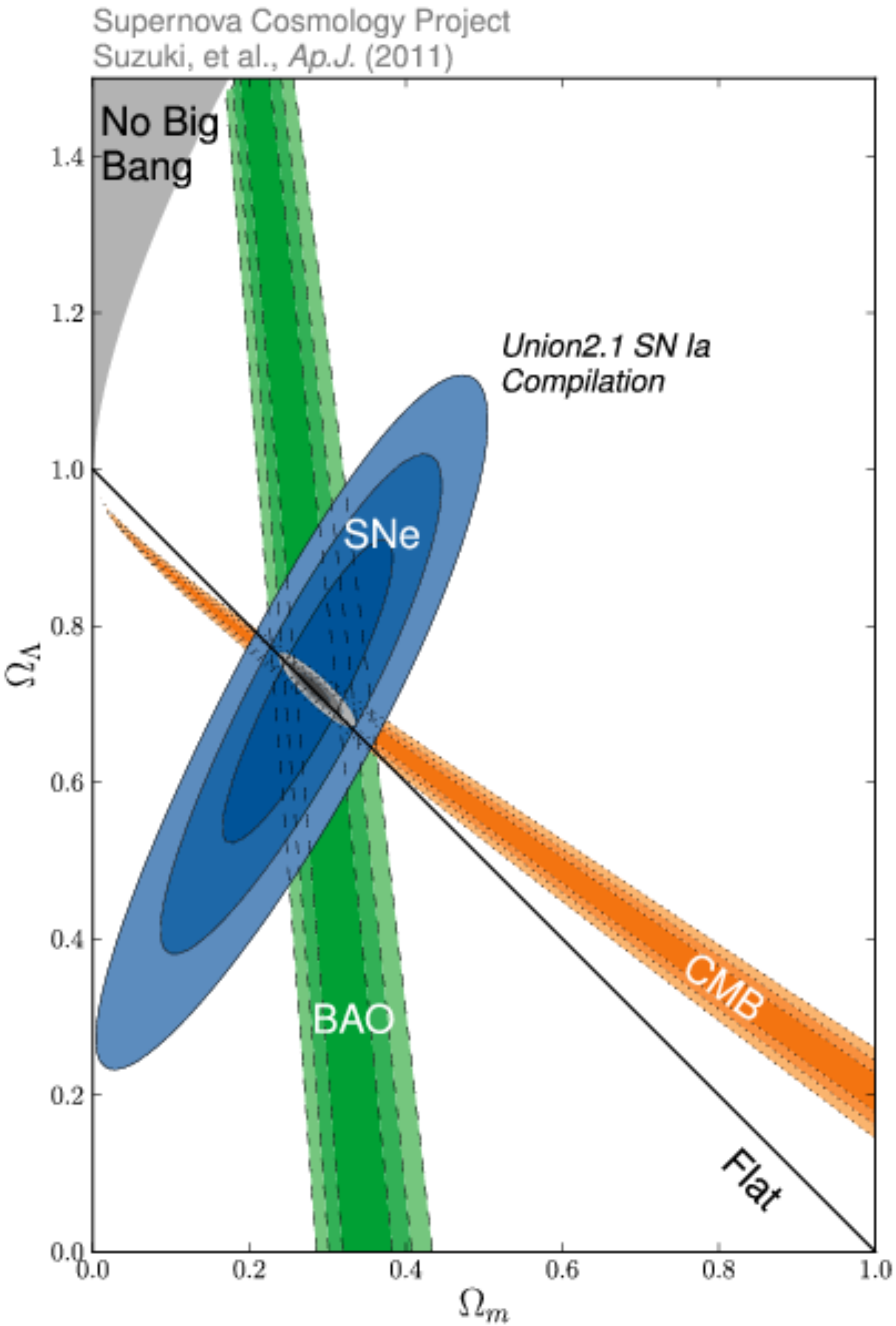


Empirical adjustment can be made so all have the same absolute magnitude, i.e. luminosity.

The same luminosity means that if you identify that something is a Type Ia supernova, it can be used as a **standard candle to get distance**.



This is used to constrain cosmological models specifically the fact that we are dominated by dark energy (here Ω_Λ)

(more on this in Astro 32!)

White Dwarfs

Reading assignment
THURSDAY 12/3: 16.6, 16.7

Homework Assignment #5 due by:
TUESDAY 12/15 before 9AM.