Homework # 4

Due before Tuesday, November 10th

README: Make sure you write down all steps to solve the problems, so that I can give partial credit if needed. If I do not see the procedure you followed to solve the problem, you will get a zero on that problem. Also, be careful with units: they are extremely important! Only use the constants and values provided to solve the problems. Finally, make sure you write neatly so that I can understand it: if I don't understand what you wrote, you will get a zero. Scan the solution (into a single PDF, preferably) and send it back to me before the deadline. When a comparison between quantities is asked for, you need to calculate the ratio between the two quantities. When asked to make a figure/plot/diagram, make sure you produce this figure electronically, either with python coding (preferably) or with Excel (or similar).

CONSTANTS and OTHER VALUES (only use the provided constants/values to solve the problems):

1 pc = 3.0857×10^{16} m	1 radian = 206264 arcsec	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
1 year = 3.1557×10^{7} s	$c = 3 \times 10^{5}$ km s^{-1}	$R_{Earth} = 6.378136 \times 10^{6}$ m
$M_{Sun} = 1.989 \times 10^{30}$ kg	$R_{Sun} = 6.95508 \times 10^{8}$ m	$L_{Sun} = 3.839 \times 10^{26}$ W

Bolometric absolute magnitude of Sun: $M_{bol,Sun} = 4.74$
Boltzmann constant: $k = 1.38065 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
Planck constant: $h = 6.62607004 \times 10^{-34}$ m^2 kg s^{-1}
Stefan-Boltzmann constant sigma = 5.6704×10^{-8} W m^{-2} K^{-4}
Mass of proton (or neutron): $m_p = 1.6726 \times 10^{-27}$ kg
1 eV = 1.0622×10^{-19} J
Permittivity of free space: $8.854187817 \times 10^{-12}$ F/m
charge of electron $e = 1.602176487 \times 10^{-19}$ C
1 fm = 10^{-15} m

1) Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is 0.37921 arcsec and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semi-major axis of the reduced mass is 7.61 arcsec. The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_A/a_B=0.466$.

   A. Find the mass of each member of the system in units of solar mass.
   B. The absolute bolometric magnitude of Sirius A is 1.36, and Sirius B has an absolute bolometric magnitude of 8.79. Determine their luminosities in units of solar luminosity.
   C. The effective temperature of Sirius B is 24,790K. Estimate its radius, expressed in terms of the radius of the Sun and the radius of Earth.

2) Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, estimate how long the Sun could shine at its current rate through chemical processes alone. For simplicity, assume that the Sun is composed entirely of Hydrogen. Is it possible that the Sun's energy is entirely chemical? Why or why not?

3) Taking into consideration the Maxwell-Boltzmann velocity distribution,
   A. what temperature would be required for two protons to collide if quantum mechanical tunneling is neglected? Assume that nuclei having velocities 10 times the root-mean square (rms) value for the Maxwell-Boltzmann distribution can overcome the Coulomb barrier. Also assume that the two protons are separated by 2 fm at collision. Compare your answer with the estimated central temperature of the Sun;
   B. using the equation of the Maxwell-Boltzmann distribution, calculate the ratio of the number of protons having velocities 10 times the rms to those moving at the rms velocity;
   C. assuming, incorrectly, that the Sun is pure Hydrogen, estimate the number of Hydrogen nuclei in the Sun. Could there be enough protons moving with a speed 10 times the rms value to account for the Sun's luminosity? Hint: from previous calculations, derive the number of reactions, then the total produced energy, and finally the resulting lifetime of the Sun at its current luminosity.

4) Estimate the Eddington luminosity of a 0.072 M_{Sun} star and compare your answer to its main-sequence luminosity ($\log(L/L_{Sun})=-4.3$). Assume an opacity of 0.001 $m^2\ kg^{-1}$. Is radiation pressure likely be significant in the stability of a low-mass main-sequence star?

5) Estimate the Eddington luminosity of a 120 M_{Sun} star with $\log(T/K)=4.727$ and $\log(L/L_{Sun})=6.252$. Assume that the opacity is due to electron scattering (using X=0.7, fraction by mass of Hydrogen). Compare your answer with the actual luminosity of the star.