ASTRONOMY 191 - Fall 2024
Homework Assignment #3
DUE by Sunday, December 1, 2024
250 points

1. Problem 1 [20 points]

Estimate the value of the internal magnetic field B responsible for the spin-orbit interaction for the level 3d in Hydrogen. Remember that the total energy of the level including the spin-orbit interaction is $E_{n,l,j}=E_n+a_{n,l}\vec{l}\cdot\vec{s}$, with $a_{n,l} = (|E_n|\alpha^2 Z^2)/[\hbar^2 nl(l+1)(l+1/2)]$ for H-like atoms, and that $\Delta E_{LS}=(\mu_B/\hbar)\vec{s}\cdot\vec{B}$.

2. Problem 2 [30 points]

Draw the energy levels of the multiplets ^{3}F and ^{3}D, showing the allowed optical transitions. Since the spin-orbit interaction energy shift can be written as $E_{LS} = a_{n,l}\vec{L}\cdot\vec{S}$, evaluate the relative separation of the terms in the same multiplet. Notes: (i) first, determine S, L, and J using $^{2S+1}L_J$; (ii) remember that $\vec{L}\cdot\vec{S}=(\hbar^2/2)[J(J+1)-L(L+1)-S(S+1)]$; (iii) use the selection rules $\Delta L=\pm1$, $\Delta J=0,\pm1$.

3. Problem 3 [50 points]

The green line in the spectrum of Hg (Mercury) is emitted in the transition between two energy levels, n and n', characterized by the quantum numbers $j'_n=1$ and $j'_{n'}=2$, and by the Landé factors $g_n=2$ and $g_{n'}=3/2$. (1) Determine and draw the number of magnetic sub-levels of the two levels and their separations under the presence of a weak external magnetic field B_{ex}; and (2) determine the number of lines that will be observable for the Zeeman effect. Notes: (i) Assume that the energies of the n and n' levels without B_{ex} are E_1 and E_2, respectively; after adding the external magnetic field, the energy level will split into m'_j levels with energies $E_n + \mu_B B_{ex} g_n m'_j$; (ii) use the following selection rules for the allowed transitions: $\Delta m'_j=0,\pm1$, excluding $m'_j=0 \rightarrow m'_j=0$ if $\Delta j'=0$.

4. Problem 4 [50 points]

Assume neutral Hydrogen gas. Using Boltzmann's equation, write N_2/N_1, the ratio between the number of atoms in the ground state N_1 and the number of atoms in the first excited state N_2, as a function of temperature T. Calculate the temperature at which $N_2 = N_1$, i.e., half of atoms are in the ground state, and half are in the first excited state. Make a plot of $N_2/(N_1 + N_2) = (N_2/N_1)/[1 + (N_2/N_1)]$ as a function of temperature in the range 5,000-25,000 K. What is the range of the fraction (in percent) of H atoms in the first excited state compared to total?

5. **Problem 5** [50 points]

Assume a gas of pure Hydrogen, with a pressure of free electrons $P_e = 20$ N m^{-2}. Using Saha's equation, derive the fraction of ionized H atoms, i.e., $N_{II}/N_{tot} = N_{II}/(N_I + N_{II}) = (N_{II}/N_I)/[1+(N_{II}/N_I)]$, as a function of temperature T, and plot it over the temperature range 5,000-25,000 K. Use $G_{II} = 1$; $G_I \approx g_1$ (since approximately all neutral H is in the ground state at these temperatures - from problem 4); 13.6 eV as the ionization energy of HI. At what temperature $N_{II}/N_{tot} = 0.5$, i.e., half of the H is ionized? What is the range in temperature corresponding to $N_{II}/N_{tot} = 0.1$ and $N_{II}/N_{tot} = 0.9$, i.e, to go from 10% to 90% of the H ionized?

6. **Problem 6** [50 points]

Assuming a gas of pure Hydrogen, with a pressure of free electrons $P_e = 20$ N m^{-2}, and using the findings in problems 4 and 5, plot the strength of the Balmer line (in absorption) as a function of temperature in the range 5,000-25,000 K. The Balmer line in absorption is caused by the transition of the electron in the H atom from the first excited level to the second excited state. Therefore, the strength of the Balmer line is proportional to N_2/N_{tot}, which can be re-written using the Boltzmann's and Saha's equations as:

$$\frac{N_2}{N_{tot}} = \frac{\frac{N_2}{N_1}}{1 + \frac{N_2}{N_1}} \frac{1}{1 + \frac{N_{II}}{N_I}} \tag{1}$$

Discuss why the strength of the Balmer line is negligible at temperatures $T \lesssim 8,000$ K and it is also suppressed at temperatures $T \gtrsim 15,000$ K.