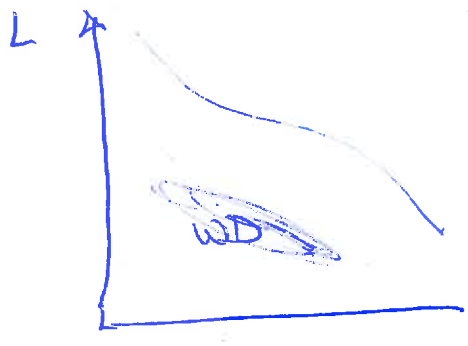
WHITE DWARFS



$T_e \simeq 5000 \div 80.000$ K

DA : only pressure-broadened H absorption lines (2/3 of all WDs)

DB : no H lines (8%) w/ He lines only (absorption)

DC : no lines, only continuum (14%)

DQ : C features in spectra

DZ : metal lines.

Sirius B : $R \sim 5.5 \times 10^{8}$ cm $\simeq 0.008\, R_\odot < R_E$

$\langle \rho \rangle \sim 3 \times 10^{6}\ g/cm^3$

$$P_c \simeq \frac{2}{3} \pi G \rho^2 R_{WD}^2 \sim 3.8 \times 10^{23} \frac{\text{dine}}{\text{cm}^2} \approx 2 \times 10^{6} P_{c,\odot}$$

$$P(r) = \frac{2}{3} \pi G \rho^2 (R^2 - r^2)$$

$$P_{c,\odot} \approx 2 \times 10^{17} \frac{\text{dine}}{\text{cm}^2}$$

from eq. Hydrostatic eq.

$$\frac{dP}{dr} = -\frac{4}{3} \pi G \rho^2 r$$

From $\displaystyle \frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa} \rho}{T^3} \frac{L_r}{4\pi r^2}$

$$\frac{T_{WD} - T_c}{R_{WD} - 0} = -\frac{3}{4ac} \frac{\bar{\kappa} \rho}{T_c^3} \frac{L_{WD}}{4\pi R_{WD}^2}$$

Using $\bar{\kappa} = 0.2 \frac{\text{cm}^2}{g}$ (using $X=0$ for the e^- scattering opacity)

$$\Rightarrow T_c \simeq \left[\frac{3 \bar{\kappa} \rho}{4ac} \frac{L_{WD}}{4\pi R_{WD}} \right]^{1/4} \simeq 7.6 \times 10^{7}\ K$$

$L_{WD} \simeq 0.03\, L_\odot$

⇒ very hot ⇒ white dwarfs cannot be made of H, otherwise nuclear reactions (pp chain & CNO cycle) would make these objects much more luminous.

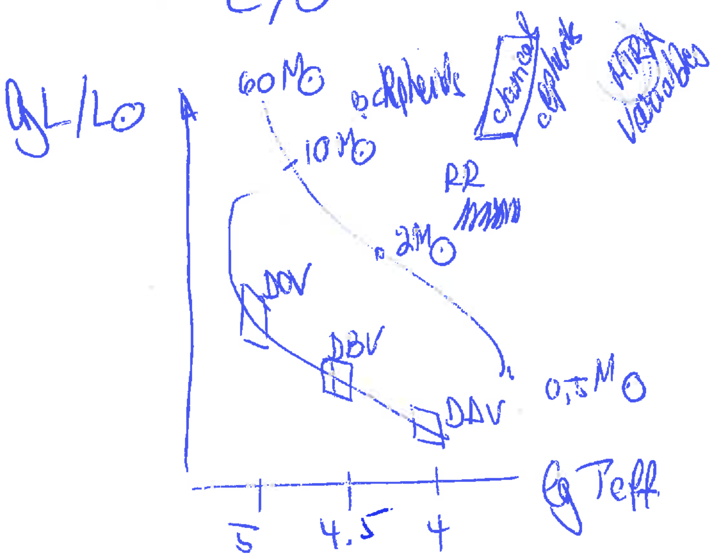
⇒ Thermonuclear reactions are not responsible in producing the energy radiated by WDs.

WDs are made of cores from stars w/ $M < 8\text{-}9 M_\odot$ mostly made of completely ionized C & O.

$M_{DA} \sim 0.56 M_\odot$, w/ 80% in $[0.42, 0.70] M_\odot$

⇒ significant mass loss occurred while on the AGB, through thermal pulses & superwinds.

Thin layer of H, covering a layer of He on top of the C/O core.



ZZ Ceti (DAV stars)

↳ WDs w/ $T_e \sim 12000$ K in the instability strip w/ periods ~ 100 - 1000 sec (variation of T, variations of a few tenths of mag)

DBV → $T_e \sim 27000$ K

DOV → $T_e \sim 10^5$ K

$\rightsquigarrow$ For Sirius B, complete degeneracy in the centre.

$$P_{e,nr} = 1.004 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \frac{\text{dynes}}{\text{cm}^2} \underset{\mu_e \simeq 2}{\approx} 2 \times 10^{23} \frac{\text{dynes}}{\text{cm}^2}$$

└ similar to the previous estimate for hydrostatic eq.

$\Rightarrow$ e^- degeneracy pressure is responsible for maintaining hydrostatic equilibrium in a white dwarf.

By placing $P_{e,nr} = \frac{2}{3}\pi G \rho^2 R_{WD}^2$ w/ $\rho = M_{WD} / \frac{4}{3}\pi R_{WD}^3$

$$\Rightarrow R_{WD} = \frac{7.723 \times 10^{12} \text{ cm}}{(\mu_e)^{5/3} (M_{WD})^{1/3} G} \approx 2.9 \times 10^{8} \text{ cm}$$

$M_{WD} \sim 1 M_\odot$
$\mu_e \sim 2$.

$$\Rightarrow M_{WD} \cdot R_{WD}^3 = \text{const}$$

$$\Updownarrow$$

$$\boxed{M_{WD} V_{WD} = \text{const}}$$

ie: more massive white dwarfs are smaller.

The mass-volume relation comes from the star deriving its support from e^- degeneracy pressure

Chandrasekhar limit $M_{WD} = \frac{5.83}{\mu_e^2} M_\odot \underset{\mu_e \sim 2}{\approx} \boxed{1.45 M_\odot}$

$\mu_e \sim 2$ for C/O WDs.

WDs simply cool off at an eventually constant radius as they slowly deplete their supply of thermal energy (degenerate e^- pressure does not depend on T).

Energy inside the WD is carried by e^- conduction, so efficient that the interior of the WD is almost isothermal. Only at the non-degenerate surface heat is transferred less efficiently (through convection).

From eq. 2 & 5A of stellar structure w/ M the independent variable $\left(\frac{dP}{dM} = -\frac{GM}{4\pi r^4}\ ;\ \frac{dT}{dM} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{L}{16\pi^2 r^4}\right)$

$$\Rightarrow \frac{dP}{dT} = \frac{16\pi a c G M T^3}{3 \kappa L}$$

For a non-degenerate surface $\bar{\kappa}_{bf} = A_{bf}\,\rho / T^{3.5}$

$\bar{\kappa}_{ff} = A_{ff}\,\rho / T^{3.5}$

$$\Rightarrow \frac{dP}{dT} = \frac{16\pi}{3}\frac{GM}{L}\frac{ack}{A\mu m_H}\frac{T^{7.5}}{P} \quad \text{w/ } A = A_{bf} + A_{ff}$$

$$P = \frac{\rho k T}{\mu m_H}$$

integrating w/ respect to T & solving for P

$$\Rightarrow P = \sqrt{\frac{4}{17}\frac{16\pi}{3}\frac{GM}{L}\frac{ack}{A\mu m_H}}\ T^{17/4}$$

For a WD, using $\kappa_0 = 4.34 \times 10^{25}\, Z(1+X)\ \frac{cm^2}{g}$ bound-free opacity

$$\Rightarrow P = \left(\frac{4}{17}\frac{16\pi}{3}\frac{GM_{WD}}{L_{WD}}\frac{ack}{\kappa_0\,\mu m_H}\right)^{1/2} T^{17/4}$$

Using $P = \rho kT/\mu m_H$

$$\Rightarrow \quad \rho = \left(\frac{4}{17} \frac{16\pi a c}{3} \frac{G M_{WD}}{L_{WD}} \frac{\mu m_H}{\kappa_0 k} \right)^{1/2} T^{13/4}$$

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At the transition between the non-degenerate surface layer & its isothermal degenerate interior:

$$\frac{\rho}{\mu_e} = 2.4 \times 10^{-8} \, T_c^{3/2}$$

interior temperature

$$\Rightarrow \quad L_{WD} = \text{constant} \cdot T_c^{7/2}$$

$$\text{w/ constant} = \frac{4}{17} \frac{16\pi a c}{3} G \frac{\mu m_H}{\kappa_0 k} M_{WD} \frac{1}{(2.4 \times 10^{-8} \mu_e)^2}$$

$$\approx 1.44 \times 10^{4} \left(\frac{M_{WD}}{M_\odot} \right) \frac{\mu}{Z(1+X)}$$

(for L_{WD} in erg/s)

For $M_{WD} = 1 \, M_\odot$

$X = 0, Y = 0.9, Z = 0.1 \rightarrow \mu \simeq 1.4$ $\quad \Rightarrow T_c \simeq 4.4 \times 10^{7}$ K

$L_{WD} = 0.03 \, L_\odot$

COOLING: $\quad \underbrace{-\frac{d}{dt} U}_{\text{depletion of the internal energy providing the luminosity}} = L_{WD}$ $\quad$ w/ $\quad U = \underbrace{\frac{M_{WD}}{A m_H}}_{\text{\# of nuclei in the WD}} \cdot \underbrace{\frac{3}{2} k T_c}_{\text{average thermal energy per nuclei}}$

$$L_{WD} = C \, T_c^{7/2}$$

$$\Rightarrow \quad T_c(t) = \underbrace{T_0}_{\text{initial temperature}} \left(1 + \frac{5}{2} \frac{t}{\tau_0} \right)^{-2.5}, \quad \tau_0 = \frac{3 M_{WD} k}{2 A m_H C \, T_0^{5/2}}$$

NEUTRON STARS

A neutron star (NS) is supported by neutron degeneracy pressure (that you can derive using the mass of n instead of me)

$\Rightarrow$ $R_{NS} \approx \dfrac{5 \times 10^{5}\ \text{cm}}{(M_{NS}/M_{\odot})^{1/3}} \underset{M_{NS} \simeq 1.4 M_{\odot}}{\approx} 4\ \text{km}$ (underestimated by ~ 3 *)

$M_{NS}/m_{n} = \dfrac{1.4\,M_{\odot}}{m_{n}} \approx 10^{57}$ neutrons! extremely compact & dense.

$\langle \rho \rangle_{NS} \simeq 6.65 \times 10^{14}\ \text{g/cm}^3$

Effects of relativity must be included for an accurate description of NS.

At densities of $\sim 10^{7}\ \text{g/cm}^3$: $p^{+} + e^{-} \longrightarrow n + \nu_e$

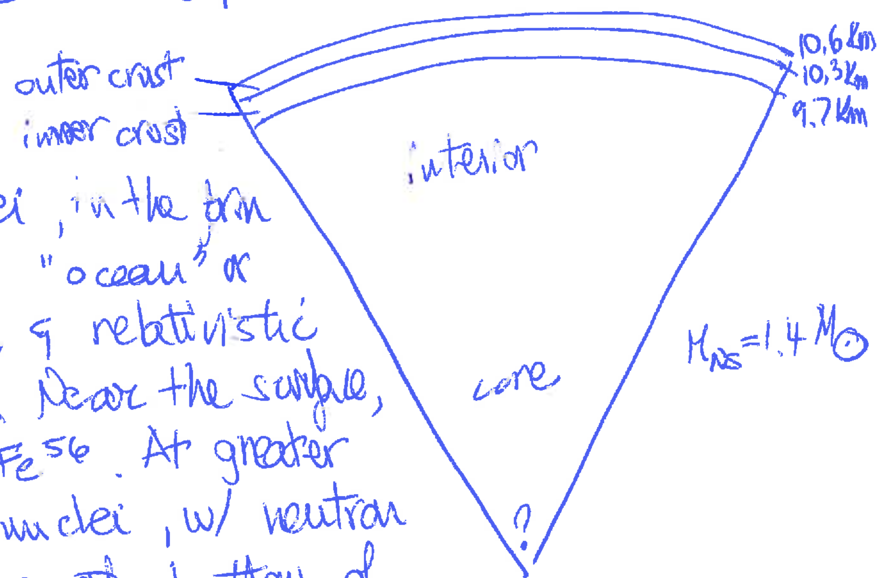
ie: the protons in the Fe^{56} core capture e^{-}s when $\rho > 10^{9}\ \text{g/cm}^3$ (including all the relevant physics)

When the density reaches $\sim 4 \times 10^{11}\ \text{g/cm}^3$ $\rightarrow$ the minimum-energy arrangements is w/ some of the ns outside the nuclei

$\Rightarrow$ neutron drip, w/ appearance of free ns, with a mixture of a lattice of n-rich nuclei, non-relativistic degenerate ns & relativistic degenerate e^{-}s.

A fluid w/ free ns has no viscosity $\rightarrow$ superfluid, flowing w/o resistance.

As the ρ increases further, # of free ns increases as the # of e^- decreases. The n degeneracy pressure exceeds the e^- degeneracy pressure when $\rho \sim 4\times10^{12}$ g/cm^3. As $\rho \to \rho_{nuc}$, the nuclei effectively dissolve, resulting in a fluid mixture of free n_s, p_s & e^-_s dominated by n degeneracy pressure, w/ n_s & p_s paired to form superfluids. The fluid of pairs of p_s is superconducting. As ρ further increases $n_s : p_s : e_s \simeq 8:1:1$ limit is reached.



- outer crust : heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, & relativistic degenerate e^-_s. Near the surface, nuclei likely Fe^{56}. At greater depth, n-rich nuclei, w/ neutron drip beginning at bottom of outer crust ($\rho \sim 10^{11}$ g/cm^3)

- inner crust : lattice of nuclei (such as Kr^{118}), a superfluid of free n_s & relativistic degenerate e^-_s. Bottom of inner crust, $\rho \sim \rho_{nuc} \to$ nuclei dissolve.

- interior : superfluid of n_s, w/ a small number of superfluid, superconducting p_s & relativistic degenerate e^-_s.

- core ? : made of pions or other sub-nuclear particles. $\rho_c \sim 10^{15}$ g/cm^3.

NS also obey $M_{NS} V_{NS} = \text{const.}$

↳ they become smaller & more dense w/ increasing mass.

$$M_{NS}^{max} \simeq \begin{array}{l} 2.2\, M_\odot \text{ if static} \\ 2.9\, M_\odot \text{ if rapidly rotating} \end{array}$$

⟶ <u>Black hole</u>

Treating a star like a sphere w/ moment of inertia

$I = CMR^2 \longrightarrow I_i \omega_i = I_f \omega_f$

↳ $\omega_f = \omega_i \, (R_i / R_f)^2$ angular velocity

$P_f = P_i \, (R_f / R_i)^2$ period.

$$R_{core} / R_{NS} \simeq \frac{m_N}{m_e} \left(\frac{Z}{A} \right)^{5/3} \sim 512$$

$\Rightarrow P_{NS} \approx 3.8 \times 10^{-6} \, P_{core} \approx 5 \times 10^{-3}\ \text{s}$

$P_{core} \sim 1350$ s (rotation period for a WD)

⇒ NS rotates very rapidly when they are formed, w/ period of a few milliseconds

The magnetic flux through the surface of a WD is conserved as it collapses to form a NS. (9)

$B_i \, 4\pi R_i^2 = B_f \, 4\pi R_f^2$ (since $\phi = \int_S \vec{B} \cdot d\vec{A}$ — magnetic field flux through a surface)

$\Rightarrow B_{NS} \simeq B_{WD} \left(\frac{R_{WD}}{R_{NS}} \right)^2 \approx 1.3 \times 10^{10}$ T

$B_{WD} \sim 5 \times 10^4$ T (extreme case)

$\Rightarrow$ Huge magnetic field ($B_{sun} \sim 2 \times 10^{-4}$ T)

T of NS when formed out of SN explosion is extremely high, $T \sim 10^{11}$ K. During the first day, the NS cools by emitting neutrinos via the URCA processes

$n \longrightarrow p^+ + e^- + \bar{\nu}_e$

$p^+ + e^- \longrightarrow n + \nu_e$

$\nu_e / \bar{\nu}_e$ carry away energy, cooling the NS.

When $T \sim 10^9$ K, ps & ns become degenerate (after 1 day) & those processes stop. Other ν-dominated processes dominate the cooling in the following $\sim 10^3$ yrs, & then photons from the surface take over. $T_{interior} \sim 10^8$ K when $t \sim$ a few 10^2 yrs, w/ $T_{surface} \sim$ several 10^6 K.

$L = 4\pi R^2 \sigma T_e^4 \approx 7.13 \times 10^{32}$ erg/s $\approx L_\odot$ but in the X-rays (2.9 nm)

Pulsars $\longrightarrow$ rapidly rotating NS.
Crab nebula : SN explosion is AD 1054 + pulsar at it's center.
$\hookrightarrow$ synchrotron radiation

Synchrotron radiation is produced when relativistic e_s spiral along magnetic field lines.

$\vec{F}_m = q(\vec{v} \times \vec{B})$ magnetic force on a moving charge q

$\Rightarrow$ $\vec{v}$ perpendicular to $\vec{B}$ produces a circular motion around the field lines

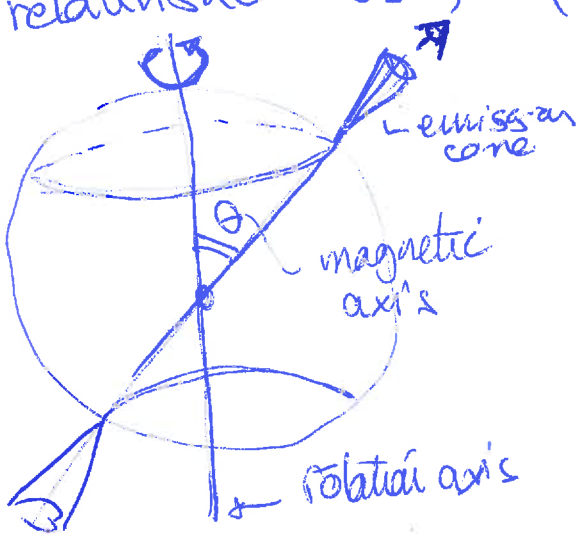
$\vec{v}$ parallel to $\vec{B}$ is not affected.

$\Rightarrow$ the charge follows the curved field lines, accelerating & emitting elm radiation.

$\Rightarrow$ synchrotron radiation if motion is primarily around the field lines

curvature radiation if motion primarily along the field lines.

The rotating NS provides the required energy to accelerate the expansion of the nebula, the relativistic e_s, & the magnetic field.



Pulsar: rapidly rotating NS w/ strong dipole magnetic field, inclined to the rotation axis at an angle Θ

Both B & fast rotation natural in NS

The energy per second emitted by the rotating magnetic dipole is

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}$$

the magnetic field at any point in space changes rapidly due to the pulsar rotation
→ this induces an electric field at that point
⇒ the time-varying electric & magnetic fields form an elm wave that carries energy away (magnetic dipole radiation)

w/ B strength of the field at the magnetic pole of the star w/ radius R ; P rotation period

↳ the energy is taken away at the expense of the rotational energy of the NS, which will slow down.

$$\frac{dE}{dt} = \frac{dK}{dt}$$ — rotational kinetic energy lost

$$\Rightarrow -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

$$\Rightarrow B = \frac{1}{2\pi R^3 \sin\theta}\sqrt{\frac{3\mu_0 c^3 I P\dot{P}}{2\pi}}$$

For crab nebula $P = 0.0333$ s
$\dot{P} = 4.21 \times 10^{-13}$
$\theta = 90°$ (assumed)

$\Rightarrow B = 8 \times 10^8$ T

NOTE: the emission of radiation is the mostly poorly known/understood aspect of pulsars.

When a WD is the primary component of a semi-detached binary system

↳ dwarf nova
classical nova } cataclysmic variables
supernova (type Ia) — single-degenerate

double degenerate models
(2 WDs in a binary system emitting gravitational waves → merging)

long quiescent w/ outbursts w/ increase in luminosity by a factor of 10 (dwarf nova) to 10^6 (classical novae).

$\langle M_{WD} \rangle \sim 0.86 M_\odot$

+ MS star of type G or later

Mass is transferred & goes ~~to~~ form an accretion disk.

↳ Black body emission from the disk

dwarf nova ⟷ sudden increase in the rate at which mass flows onto the accretion disk.

classical nova : higher accretion rates
(10^{-8}–10^{-9} $M_\odot$/yr)

The H-rich gases accumulate on the surface of the WD, where compressed & heated. At the base of the layer, H is mixed w/ C, O & N, & supported by e^- degenerate pressure.

When 10^{-4}–10^{-5} $M_\odot$ of H has accumulated & $T \sim$ few$\cdot 10^6$ K

→ shell of CNO-cycle H-burning develops

FD explosively → ejection of material

Recurrent (10^4–10^5 yr)