STELLAR EVOLUTION

JEANS CRITERION FOR CONTRACTION

Virial theorem $2K + U = 0$

If $2K < |U| \longrightarrow$ contraction, otherwise expansion.

Let's consider a spherical gas cloud of constant density ρ_0

$$U = -4\pi G \int_0^R M_r \rho r dr \simeq -\frac{3}{5}\frac{G M_c^2}{R_c}$$

U: gravitational potential energy; $M_r = \frac{4}{3}\pi r^3 \rho_0$

w/ M_c & R_c mass & radius of the cloud

$$K = \frac{3}{2} N k T = \frac{3}{2}\frac{M_c}{\mu m_H} kT$$

N: # of particles

$$\Rightarrow \quad 2K < |U| \iff \frac{3 M_c}{\mu m_H} kT < \frac{3}{5}\frac{G M_c^2}{R_c}$$

Replacing $R_c = \left(\frac{3 M_c}{4\pi \rho_0}\right)^{1/3}$ & solving for M_c

$$M_c > \left(\frac{5kT}{\mu G m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} =: M_J$$

↳ Jeans mass

Jeans criterion to initiate the spontaneous collapse of the cloud

Equivalently $R_c > R_J := \sqrt{\frac{15 kT}{4\pi G \mu m_H \rho_0}}$ Jeans length.

Accounting for an external pressure on the cloud due to the surrounding ISM

$\Rightarrow M_c > M_{BE} = \dfrac{C_{BE}\, v_T^4}{P_0^{1/2}\, G^{3/2}}$ w/ $v_T := \sqrt{kT/\mu m_H}$ isothermal sound speed

Bonnor–Ebert mass

$C_{BE} \simeq 1.18$

(NOTE: if $C_J = 5.46$ → Jeans criterion
$C_{BE} < C_J$ due to the external pressure P_0)

EXAMPLE: For a dense core of a giant molecular cloud

$T \sim 10\,K$ $n_{H_2} = 10\ cm^{-3}$

$\rho_0 = m_{H_2} \cdot n_{H_2} = 2 m_H\, n_{H_2} = 3.35 \times 10^{-20}\ g/cm^3$

$\mu \simeq 2$ (molecular hydrogen)

$\Rightarrow M_J = 7.3\ M_\odot$

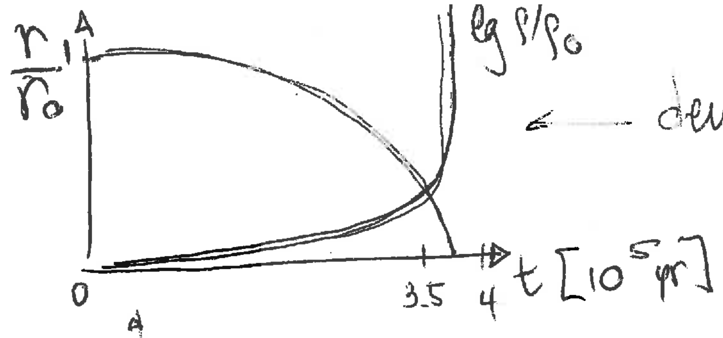
Since the characteristic masses of dense cores are $\sim 10\ M_\odot$ ⇒ the dense cores of GMCs are unstable to gravitational collapse, & are sites of star formation.

$M_{BE} \sim 1.6\ M_\odot$ ⇒ definitely unstable

At the beginning of the contraction, the cloud is essentially in free fall & the temperature can be assumed constant (isothermal contraction), true if the cloud is optically thin & the gravitational potential energy released by the collapse can be efficiently radiated away.

⇒ Free-fall timescale $t_{ff} = \sqrt{\dfrac{3\pi}{32}\dfrac{1}{G\rho_0}}$ for homologous collapse (timescale independent on initial radius)

$\simeq 3.6 \times 10^5\ yr$ for the above GMC



← density increases very rapidly during the final stage of collapse.

slow contraction at the beginning

Stars tend to form in groups, ranging from binary star systems to clusters containing many 10^5 stars.

⇒ FRAGMENTATION of the collapsing cloud: as the cloud collapse, ρ increases while T remains constant ⇒ initial inhomogeneities in density will cause individual parts of the cloud to satisfy the Jeans criterion independently, collapsing locally ⇒ fragmentation.

However the collapse is not isothermal. The opposite is when the energy release cannot be transported out of the cloud → adiabatic collapse

$$\Rightarrow M_J \propto \rho^{(3\gamma-4)/2} = \rho^{1/2}$$

$\gamma = 5/3$ HI

, ie: the collapse results in a minimum mass of the produced fragments, the exact value depending on when the collapse goes from isothermal to adiabatic

$2K = U$

↳ Energy released by the collapse of a spherical cloud just satisfying the Jeans criterion

$$K \simeq \Delta E_g \simeq \frac{3}{10}\frac{G M_J^2}{R_J}$$

$$L_{ff} \simeq \frac{\Delta E_g}{t_{ff}} \sim G^{3/2}\left(\frac{M_J}{R_J}\right)^{5/2} \cdots$$

If the cloud is optically thick & in thermodynamic equilibrium
→ black body emission $L_{rad} = 4\pi R^2 \sigma T^4 e$

$0 < e < 1$ as not in perfect equilibrium.

$L_{ff} = L_{rad}$

$\hookrightarrow M_J^{5/2} = \frac{4\pi}{G^{3/2}} R_J^{9/2} \sigma T^4 e$

Eliminating the radius & replacing ρ in terms of M_J (from definition of M_J)

$\left(R_J = \left(\frac{3 M_J}{4\pi\rho} \right)^{1/3} \right)$

$$\Rightarrow M_{J\,eim} \simeq 0.05 \left(\frac{T^{1/4}}{e^{1/2} \mu^{9/4}} \right) M_\odot$$

For $\mu \sim 1$, $e \sim 0.1$, $T = 10^3\,K$ → $M_J \sim 0.9\, M_\odot$

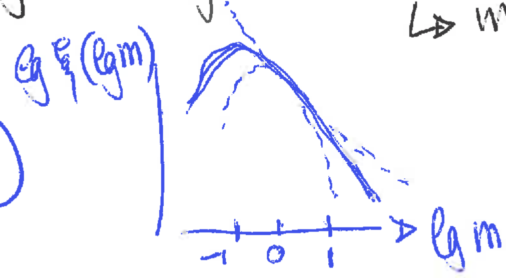
ie: fragmentation ceases when the segments reach the range of solar mass objects.

IMPORTANT PHYSICS:
- cloud is not static, the cloud's outer layers have an initial velocity
- radiation transport through the cloud
- vaporization of dust grains, dissociation of molecules, & ionization of atoms

$M_{J\,eim} \simeq 0{,}01\, M_\odot$ ←

- rotation, nonspherical symmetry, turbulent motions in the gas, magnetic fields ($B \sim 1 \div 100$ nT) ↳ magnetic pressure

⇒ INITIAL MASS FUNCTION (IMF)



PROTOSTELLAR EVOLUTION (before PRE-MAIN SEQUENCE EVOLUTION)

- initially, the free-fall collapse is isothermal (photons can travel long distance). The free-fall timescale is shorter in the center & ρ increases faster there - when $\rho \sim 10^{-13} \frac{g}{cm^3}$, it becomes optically thick & the collapse more adiabatic (opacity due to presence of dust)
- as the collapse becomes adiabatic → pressure increases, slowing the rate of collapse near the core. The central region is nearly in hydrostatic equilibrium w/ $R \sim 5AU$.
- → evolutionary tracks on the HR diagram: in the early stages, as the collapse continues to accelerate both L & Te increase

When $T \sim 1000K$, the dust begins to vaporize & the opacity drops; when $T \sim 2000K$, molecular hydrogen is dissociated into neutral H atoms. Te increases rapidly, due to the opacity drop (flat part, w/ constant L & increase Te). Temperature in the interior gets high enough for D-burning, producing ~60% of the luminosity of a 1M☉ protostar.

- When D is exhausted, track bends sharply downward, almost reaching the "pre-main-sequence phase". Collapse has reached a quasi-static phase.

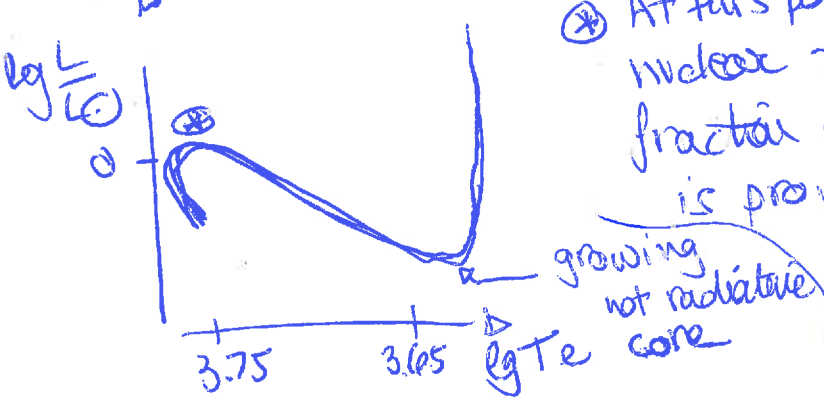
- With the formation of a quasi-static protostar, the rate of evolution becomes controlled by the rate at which the star can thermally adjust to the collapse. The quasi-static contraction to the main sequence takes much longer (eg. 40 Myr for 1 M☉) than the 0.1-1 Myr preceding phase

PRE-MAIN-SEQUENCE CONTRACTION

When the star first achieves hydrostatic equilibrium, it's L is much greater than the MS luminosity for the same mass. At this point, the star is convective throughout, due to the high opacity & very large L. The star keeps contracting, maintaining the largest radius consistent w/ hydrostatic equilibrium ⇒ it approaches the MS by descending the boundary of the forbidden zone (the Hayashi track). This track is actually a boundary between "allowed" stellar models in hydrostatic equilibrium & forbidden. To the right (at lower T_e) there is no mechanism that can adequately transport L out of the star at those low T_e ⇒ no stable star can exist there.

As the star descends this track, T central increases & the opacity in the center drops, $\left|\frac{dT}{dr}\right|_{rad} < \left|\frac{dT}{dr}\right|_{adiab}$ ⇒ a central core in radiative equilibrium develops.

As the contraction slows, L reaches a minimum, but then it begins to rise again. In fact, the hot radiative core w/ low opacity grows outward, & the star becomes less opaque. ($\kappa \propto \rho / T^{3.5}$) ⇒ more energy can flow out radiatively. Since L rises & the star keeps shrinking, T_e increases ⇒ the star moves left & up in the HR diagram.



⊛ At this point, pp chain & CNO bi-cycle nuclear reactions start. When a large fraction of a stellar luminosity is provided by nuclear reactions w/ a strong temperature dependence, the zone is convective.

When the CNO cycle starts by burning C^{12}, reaction that is strongly T-dependent, the core change from a radiative state to a convective state, & it halts the general contraction. The energy to halt contraction is subtracted by the power produced → the luminosity drops & Te as well. Only ~80% of the energy liberated by the nuclear reactions during this phase is available to be radiated.

In low-mass stars, the radiative core reappears after ^{12}C is exhausted, as the PP chain has a weak enough T dependence to be smoothly distributed over the stellar core.
In high-mass stars, the core remains convective since the CNO cycle dominate the PP chains.

For a 1 M☉ star, gravitational contraction contributes to <1% of the luminosity after $t \sim 50 \times 10^6$ yr.

More massive stars do not come so far down the Hayashi track before achieving radiative equilibrium in their interiors. They also arrive at the main sequence much faster than low-mass stars $t_{contraction} \approx 8 \times 10^7 \frac{M}{M_\odot} \frac{L_\odot}{L}$ yr.
(ex: ~70×10^6 yr, 40×10^6 yr, 1×10^6, 0.1×10^6 for 0.8, 1, 5, 15 M☉) L final MS luminosity

NOTE: Significant mass loss (L_{wind}) will result in a faster contraction to the main sequence. T-tauri, in their final approach to MS, lose mass at a significant rate.

NOTE: The final states of the stars at the end of their contraction tracks reproduce the low-Te envelope of the observe MS quite well → zero age MS.

(8)

NOTE: For a star w/ $M < 0{,}5\,M_\odot$, the track misses the upward branch, as T_{core} is never large enough to fuse C^{12} efficiently in the CNO cycle (T_{core} low enough that opacity remains high → no radiative core, fully convective).

NOTE: At $M < 0{,}072\,M_\odot$, the core never gets hot enough to generate sufficient energy to stabilize the star against gravitational collapse ⇒ not stable H-burning MS.

For $0{,}072 < M/M_\odot < 0{,}06$ → Lithium burning

$0{,}072 < M/M_\odot < 0{,}013$ → D burning

➡ stars w/ $M \in [0{,}013,\ 0{,}072]\,M_\odot$ are BROWN DWARFS (spectral type L & T).

MAIN SEQUENCE

For a given chemical composition, Zero-age MS := locus of points in HR diagram characterizing static stars of homogeneous composition burning H in the core.

The position of a star in HR diagram depends on the chemical composition, but the MS does not. In fact $L \propto \kappa_0^{-1} \mu^{7.5} M^{5.5}$ (from pag. 16, luminosity formula), correct only for the standard model with $\kappa = \kappa_0 \rho T^{-3.5}$. If $\kappa \propto T^{-2.5} \rightarrow L \propto M^{4.5}$. For $M > 7 M_\odot$ stars, $L \propto M^3$.

At $T \sim 10^6 - 10^7$ K, $\kappa \propto Z$ (in pop I stars)
↳ mass fraction of elements heavier than He

> $L \propto T_e^4 \rightarrow \lg T_e = 1.8 \lg \mu$
> $L \propto \mu^{7.5} \rightarrow \lg L = 7.5 \lg \mu$
> ⇒ a change in μ slide the star along a line ~ parallel to MS.

Because of relative insensitivity of R to composition change, $T_e^4 \propto L \propto \mu^{7.5}$ if radius is nearly constant ⇒ change in composition μ of the star MS stars tend to slide the star along a line roughly parallel to the MS itself.
⇒ Zero-age MS shows only weak dependence on μ.
↳ spread of MS.

MS lifetime $t \propto \frac{M X_H}{L}$ — observed, function of M & μ ⟶ $M(\text{given } L) \propto \frac{\kappa_0^{1/5}}{\mu^{1.4}}$

⇒ $t \propto \frac{\kappa_0^{1/5} X_H}{\mu^{1.4}}$ ⇒ MS lifetime of a star w/ given L depends on μ (chemical composition)

Increasing initial He reduces X_H & increases μ ⇒ reduces t.

For very massive stars, the lifetime of the star becomes constant, i.e. $M > 10^2 M_\odot \leftrightarrow t \sim 3 \times 10^6$ yr, & the lifetime is not set by $\sim E/L$ but by $M/\dot{M}$ because of large mass loss ($10^{-3} M_\odot$/yr ↳ 10^5 yr)

In the upper MS, stars have a convective surface & a convective core; in the lower MS, stars have a convective surface & a radiative core.

Convective surface due to very high κ in the subsurface layers, cool enough for H to be partly ionized. Its depth depends on composition.

Convective core occurs when T_c is high enough for energy generation from the CNO cycle to dominate that from the pp chains. It's ~~extent~~ depends on the abundances of CNO nuclei.

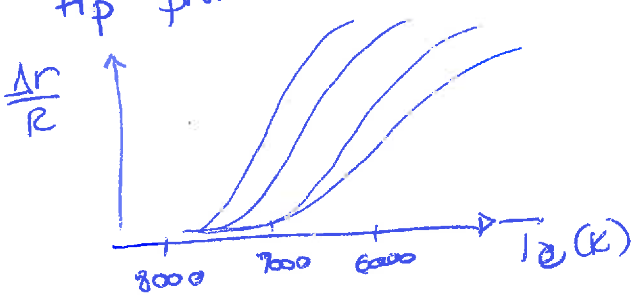
All upper MS stars are pop I.

Lower MS stars are both pop I & pop II.

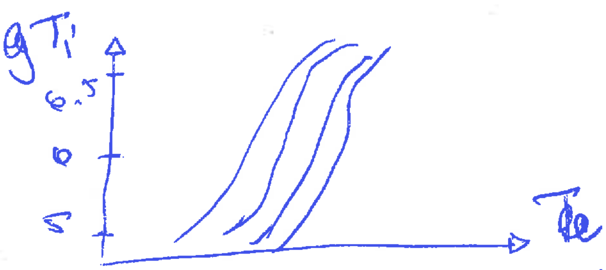
DEPTH OF OUTER CONVECTION ZONE in the stars of the lower MS. Since a theory of convection is not yet fully available, this is still an open problem.

Let's define α, so that $l = \alpha H_p$, w/ l effective mixing length & H_p pressure scale height, i.e.: $H_p^{-1} = \frac{1}{P}\left|\frac{dP}{dr}\right| = \frac{GM\rho}{Pr^2}$

models ⟹



→ The convection zone is quite thin at the highest temperatures. As T_e decreases, a transition temperature is reached, where $\Delta r/R$ changes from a small value to an almost linear increase w/ decreasing T_e.



Many stars have Li in their photospheres. At high enough T, Li is destroyed by interactions w/ Ps → the surface Li will slowly be destroyed in stars w/ convection zones extending downward to $T > 10^6$ K.

NOTE: The lowest-mass ZAMS stars have convection zones because their outer convection zone go deep into the interior, making the entire star convective.

There is the possibility of convection zones growing with time ⇒ possibility of bringing nuclear products synthesized in the interior to the surface.

For $1 < \alpha < 2.5$, the transition temperature below which deep convection zones occur is $T_e = 7300 \pm 500K$, which corresponds to $M \approx 1.5 M_\odot$ & spectral type F0.

⇒ class F & cooler as lower-MS stars
A & hotter as upper-MS stars.

CENTRAL STRUCTURE OF MS STARS

In stars w/ $M < 1.2 M_\odot$ → T_c is low enough for only the PP chains to contribute significantly. Since PP chains do not strongly depend on T, the energy source is distributed over a relatively large region of the stellar core ⇒ radiative core. For hotter stars, CNO bicycle becomes more important. But these reactions strongly depend on T, the energy is liberated very near the center of the star → very large fluxes near the center, that cannot be carried away radiatively ⇒ convective core. The size of the convective core increases w/ M & it is half of the total mass at $M = 15 M_\odot$, & ~ 80% for $M \approx 100 M_\odot$

As the upper MS stars consume their central H, several effects occur simultaneously:

1) the convective core shrinks, leaving behind a continual gradation of H concentration
2) R expands
3) the core contracts gravitationally to larger ρ_c & T_c
4) L increases but T_e doesn't drop much due to expanding R.
↳ evolved MS stars are shifted upward to higher L

(12)

For lower MS stars, different values of composition have to be considered, from $Z = 0.04$ (Sun's metallicity) in pop I to $Z \leq 0.001$ in pop II.

$0.6 < M/M_\odot < 1.3 \rightarrow$ stars still alive in globular clusters & w/ pop II characteristics.

A star w/ low heavy-element content has (it's interior opacity is less)
- greater L at fixed mass
- bluer (hotter T_e)
- longer lifetimes as they have greater fuel supply for unit mass.

For a typical low-mass MS star (eg. the Sun), L, R & T have all increased since it first reached the ZAMS, 4.57 Gyr ago.

As the pp chain converts H into He, μ increases $\rightarrow$ P decreases $\Rightarrow$ the core contracts, w/ half of the gravitational potential being radiated & half increasing kinetic energy, ie: T.

$\Rightarrow$ rate of reactions increase $\propto \rho X^2 T^6$. Increase T & ρ offset the decrease in X, w/ L slowly increasing, along with radius & T_e. The envelope is convective $\Rightarrow$ does not expand much $\rightarrow T_e$ remains roughly constant (#1 $\rightarrow$ #3)

$\Rightarrow$ Main-sequence & post-main-sequence evolutionary tracks in the HR diagram w/ $X = 0.68$ $Y = 0.3$ $Z = 0.02$.

Also show Figure

L
R
T_e

t (Age)

(Fig 11.1 from book 11.2, 11.3)

With the depletion of H in the core, the generation of energy via pp chain stops. T has increased enough that H fusion proceeds in a shell around a small, predominantly by He^4 core. T in the core of He^4 (innermost 3% of M) is constant & L is zero. (isothermal core)

The L produced in the shell exceeds that produced by the core during the H-burning phase ⇒ L continue to rise in the HR diagram (point #3); some of this energy goes into a slow expansion of the envelope ⇒ T_e decreases & the track bends to the right. The He^4 produced by H-burning in the shell are added to the isothermal He^4 core, which grows in mass. The isothermal He^4 core is able to support the material above as long as $M_{iso\,core} < f_{SC} M$, w/ $f_{SC} \cdot M$ is the Schönberg - Chandrasekhar limit. When $\frac{M_{iso\,core}}{M} \simeq 0.37 \left(\frac{\mu_{env}}{\mu_{iso\,core}}\right)^2$

⇒ the core collapses on a Kelvin-Helmholtz timescale ($t \sim U_{grav}/L$) & the star evolves rapidly (#4)

⇒ END OF MAIN-SEQUENCE PHASE.

NOTE $\mu_{env} \simeq 0.63$ (complete ionized assumption at core-envelope boundary)

$\mu_{iso\,core} \simeq 1.34$

→ $M_{iso\,core}/M \simeq 0.08$, ie 8%

NOTE: As ρ increases → e^- degenerate pressure

⇒ M_{core} can reach ~13% of the entire mass before collapse

WHEN we derived the VIRIAL THEOREM, we wrote:

$$\int d(PV) - \int P dV = -\frac{1}{3}\int \frac{GM_r}{r} dM_r$$

$\rho dV = dM_r$

Substituting $V = \frac{4\pi}{3} r^3$ & integrating from the center to $M_{iso\,core}$

$$\Rightarrow \int_0^{M_{ic}} \frac{d(4\pi r^3 P)}{dM_r}\, dM_r - \int_0^{M_{ic}} \frac{3P}{\rho}\, dM_r = -\int_0^{M_{ic}} \frac{GM_r}{r}\, dM_r$$ (14)

$4\pi R_{ic}^3 P_{ic}$ $(r=0 \text{ @ } M_r=0)$

$- \frac{3 M_{ic} k T_{ic}}{\mu_{ic} m_H} = -3 N_{ic} k T_{ic} = -2K_{ic}$

U_{ic} gravitational potential energy

$N_{ic} := \frac{M_{ic}}{\mu_{ic} m_H}$ # of gas particles in the core

$K_{ic} = \frac{3}{2} N_{ic} k T_{ic}$

NOTE: core supported also by degenerate e^-s pressure...

$$\boxed{4\pi R_{ic}^3 P_{ic} - 2K_{ic} = U_{ic}}$$

generalized form of virial theorem for stellar interiors in Hydrostatic eq.

Using $U_{ic} \sim -\frac{3}{5} \frac{G M_{ic}^2}{R_{ic}}$

$K_{ic} = \frac{3 M_{ic} k T_{ic}}{2 \mu_{ic} m_H}$ & solving for P_{ic}

$$\Rightarrow P_{ic} = \frac{3}{4\pi R_{ic}^3} \left(\frac{M_{ic} k T_{ic}}{\mu_{ic} m_H} - \frac{1}{5} G \frac{M_{ic}^2}{R_{ic}} \right)$$

$$\left.\frac{\partial P_{ic}}{\partial M_{ic}}\right|_{P_{ic}\ max} = 0 \longleftrightarrow R_{ic} = \frac{2}{5} \frac{G M_{ic} \mu_{ic} m_H}{k T_{ic}}$$

$$\boxed{P_{ic,max} = \frac{375}{64\pi} \frac{1}{G^3 M_{ic}^2} \left(\frac{k T_{ic}}{\mu_{ic} m_H} \right)^4}$$

maximum value of the surface pressure that can be produced by an isothermal core.

As M_{ic} increases, $P_{ic,max}$ decreases, & at some point the core can no longer support the overlying layers of the star's envelope. ←

We need to estimate the envelope pressure.

From (2) $\frac{dP}{dr} = -\frac{GM_r \rho}{r^2} \rightarrow \int dP = -G \int \frac{\rho \, dr \, M_r}{r^2}$

$\Rightarrow \int dP = -G \int \frac{M_r \, dM_r}{4\pi r^4}$

$\rho = \frac{dM_r}{4\pi r^2 dr}$

$$\Rightarrow P_{ic,env} = -\int_M^{M_{ic}} \frac{GM_r}{4\pi r^4} dM_r \simeq -\frac{G(M_{ic}^2 - M^2)}{8\pi \langle r^4 \rangle}$$

w/ $\langle r^4 \rangle$ average of r^4 between $r = R$ & $r = R_{ic}$

$M_{ic}^2 \ll M^2$

$\langle r^4 \rangle \simeq R^4/2 \Rightarrow P_{ic,env} \simeq \frac{G}{4\pi} \frac{M^2}{R^4}$

But from ideal gas law, $T_{ic} = \frac{P_{ic,env} \mu_{env} m_H}{\rho_{ic,env} k}$

w/ $\rho_{ic,env} \sim \frac{M}{4\pi R^3/3}$

$\rightarrow R^4 = \frac{GM^2}{4\pi P_{ic,env}} = \frac{GM^2 \mu_{env} m_H}{4\pi T_{ic} \rho_{ic,env} k} = \frac{GM^2 \mu_{env} m_H}{4\pi T_{ic} k} \frac{4\pi R^3}{M \, 3}$

$\Rightarrow R \simeq \frac{GM}{3T_{ic}} \frac{\mu_{env} m_H}{k}$

$\Rightarrow P_{ic,env} \simeq \frac{G}{4\pi} \frac{M^2}{R^4} \simeq \frac{81}{4\pi} \frac{1}{G^3 M^2} \left(\frac{k T_{ic}}{\mu_{env} m_H} \right)^4$

$\Rightarrow P_{ic} = P_{ic,env} \Longleftrightarrow \frac{M_{ic}}{M} \sim 0.54 \left(\frac{\mu_{env}}{\mu_{ic}} \right)^2$

approximate solution

For more massive stars, the evolution on the MS is similar, but the core is convective, which mixes material, keeping core composition nearly homogeneous. As the H is consumed, the convection zone in the core shrinks, more for more massive stars.

For $M = 5 M_\odot$, when $X = 0{,}05$ → core contracts, L increases, T_e increases as the radius decreases (#2)

➡ end of life on MS. ($M = 5 M_\odot$ ~ intermediate)

NOTE: #1 → 2: the envelope is radiative → it has to expand to radiate the larger amount of L produced ⇒ cooler)

EVOLUTION OFF THE MS

- The end of the MS phase occurs when H burning ends in the core (#3 for $1 M_\odot$ & #2 for $5 M_\odot$)
- For $1 M_\odot$ star: the core contracts, while thick H-burning shell appears, T & ρ rises (due to contraction) & the large T in the shell makes it produce more energy than the core did on the MS → L increases, the envelope expands slightly (being convective), & T_e decreases.
- For $5 M_\odot$ star: after #2, the entire star contracts, releasing gravitational energy w/ L increasing slightly, R decreasing → T_e has to increase: evolution #2→#3. When T outside the He core is large enough → H burning shell (#3)

SUB GIANT BRANCH:

As the continues to consume H, the core steadily increases it's mass, becoming more isothermal. At #4, the Schonberg-Chandrasekhar limit is reached, & the core contracts rapidly. The gravitational energy released causes the envelope to expand → T_e drops

➡ redward evolution on HR, i.e. SGB phase.

#4 → #5 : as the core contracts, T & ρ in H burning shell increase, making energy production rate to increase rapidly ⇒ envelope expands, absorbing energy produced by shell → lower T_e.
For $M = 5 M_\odot$, the energy absorbed is large enough for the L to decrease slightly.

RED GIANT PHASE

The expansion of envelope & decrease of T_e ⇒ photosphere opacity increases due to H^- ion contribution ⇒ convection zone near the surface, extending deep into the interior as the star evolves to #5.
Due to the star being dominated by convection, energy is transported very efficiently to the surface ⇒ star moves rapidly upward along the RGB (red giant branch). This is the same path followed during the pre MS phase.
As the star moves upward, the convection zone deepens into the regions where nuclear fusion modified the chemical composition ⇒ 1) Li burns ($T > 2.7 \times 10^6$ K) ⇒ Li nearly depleted over most of the star.
2) the processed material becomes mixed : Li abundance at the surface will decrease & He^3 will increase; C^{12} is transported inward & N^{14} outward
⇒ C^{12}/N^{14} at the surface (ie, observable) decreases

1ST DREDGE UP phase : transport of material from deep interior to the surface.

RED GIANT TIP: #6

At the tip of the RGB, T (1.3×10^8 K for $5 M_\odot$) & ρ (7.7×10^3 g/cm^3) are high enough for 3α processes to produce C^{12}, some of which is further processed into ^{16}O (strongly T-dependent nuclear reaction)
The core expands
↳ core convective

& the this pushes the H-burning shell outward, cooling it & the rate of energy output to decrease ⇒ abrupt decrease in L. At the same time, the envelope contracts & T_e increases.

He FLASH

$M < 1.8 M_\odot$: as the core collapse during the evolution to the top of the RGB, it becomes strongly e^--degenerate when T & ρ are large enough to start 3α processes ($T \sim 10^8$ K, $\rho \sim 10^4$ g/cm³) ⇒ energy released explosively.

Pe is not dependent on T ←

L generated during He flash ~ $10^{11} L_\odot$!! but lasting for only a few seconds, with most of this energy not reaching the surface, as absorbed by the overlying layers & causing some mass loss from surface. The energy generated removes the degeneracy, & then it goes into thermal (kinetic) energy needed to expand the core → ρ decreases, T decreases, slowing reaction rate.

still very poorly known transition

core convective ⇒ quiescent He core burning & H shell burning (HORIZONTAL BRANCH).

HORIZONTAL BRANCH: 1/4 of time spent on HR diagram (2/3 on main sequence).

As the envelope contracts following the RGT, the H burning shell compresses → increase energy output from the shell → L increases → T_e increases → deep convection zone in envelope shrinks, with a convective core

⇒ BLUE WARD HB evolution.

At HB, the μ in core has increased that the core begins contracting, along w/ expansion & cooling of the envelope ⇒ begin of redward of HB. Shortly after, He in the core is

exhausted, converted to C & O. The inert CO core contracts
→ fast evolution on the redward HB.

NOTE: during the evolution on the HB ⇒ instability strip
→ periodic pulsation w/ variations in L, T_e, R, v_{surf}.
↳ test of stellar structure theory.

with increasing T_{core} associated w/ contraction ⇒ thick He burning shell outside CO core; as the core continues to contract → the shell gets narrows & strengthens
→ material above the shell expands & cools ⇒ temporary turn off of H-burning shell.

As ρ increases & T in the CO core decreases due to ν_s energy losses → e^- degeneracy becomes important.

EARLY ASYMPTOTIC GIANT BRANCH (#9→10)

When the redward evolution on the HB reaches the Hayashi track
→ the evolutionary track bends upward, along the ASYMPTOTIC GIANT BRANCH (AGB). This is analog to the H-burning shell, but w/ He-burning shell.

For $M = 5 M_\odot$ → $T_c \sim 2\times10^8$ K, $\rho \sim 10^6$ g/cm³

In the EARLY-AGB, the star has He- & H-burning shells, with the He-burning shell dominating the energy output, w/ the H-burning shell nearly inactive.

The expanding envelope initially absorbs much of the energy produced by the He burning shell → as T_e decreases, the convective envelope deepens, extending downward to the chemical discontinuity between the H-rich outer layer & the He-rich region above the He-burning shell.

→ mixing, ie. 2ND DREDGE-UP phase, increasing He & N content of the envelope.

THERMAL-PULSE AGB

Near the upper portion of the AGB (TP-AGB), the dormant H-burning shell reignites, dominating the energy output of the star. The narrowing He-burning shell begins to turn on & OFF quasi-periodically → intermittent He-SHELL FLASHES due to H-burning adding He onto the He layer below: as He increases in the shell, the base of the shell becomes degenerate; as the T at the base slightly increases, the He flash occurs, driving the H-shell outward, causing it to cool & shutting off temporarily. The process repeats itself.

The period between pulses, function of mass of star, ranges from thousands of years for $M \sim 5 M_\odot$, to hundreds of 10^3 yrs for $M \sim 0.6 M_\odot$, with pulse amplitude growing with each event → abrupt changes of luminosity at the surface. When the He flash happens, the L drops (due to shutting off of H-burning shell), the radius decreases, & T_e increases. The He-shell energy output decreases when degeneracy is lifted → H-burning shell moves inward & starts again.

Long-period variables (LPVs) are AGB stars. ←

P ~ 100-700 day, including MIRA variable stars.

THIRD DREDGE-UP & CARBON STARS

He-burning shell → convection zone between He-shell & the H-burning shell (here, C is more abundant than O by ≈5-10 while in atmospheres is O more abund.)

The depth of the envelope convection zone increases with the pulse strength of the flashes.

For $M > 2M_\odot$ → the two convection zones will merge & extend down to regions where C has been synthesized.

⇒ DREDGE-UP phase: C-rich material is brought to the surface, decreasing ratio O/C

⇒ the O-rich spectrum of a star transforms over time to a C-rich spectrum

O-rich giants vs C-rich giants

↳ CARBON STARS

CARBON STARS: C spectral type (overlapping K & M types)

abundance of C-rich molecules in their atmospheres, such as SiC (rather than SiO in M stars)

MASS LOSS

All luminous stars lose mass at some small continuous rate due to mechanical heating of the most tenuous outer layers by the dissipation of sonic, hydromagnetic, or gravity waves.

⇒ slow rates of mass loss ⟺ stellar wind

Stellar winds can change the evolution of the star in at least 3 major aspects:

1. if mass of surface layer lost over the total evolutionary lifetime is a non-negligible fraction of total mass, the chemical composition of the evolved surface may reveal nuclear products not otherwise expected on the surface.
2. Rotational velocity at surface reduced if mass loss carries away more angular momentum than it had on the photosphere, i.e.: via the magnetic field.
3. The age of the star is less than its age computed with the assumption of constant mass.

As $L \propto M^4$ for main sequence stars, the past power expenditures of a star with non-negligible mass loss will have been greater than that of a corresponding star w/ constant mass.

AGB evolution & mass loss

AGB stars lose mass at a rapid rate, as high as $\dot{M} \sim 10^{-4}$ M⊙/yr. $T_e \sim 3000$ K (cool) (OH/IR sources, stars surrounded by optically thick dust clouds)

⇒ dust grains form in the expelled matter

Silicate grains tend to form in an environment rich in O
Graphite " " " " " " " " " C

⇒ ISM composition is related to the relative number of O- & C- stars.

For $M > 8 M_\odot$ → the core will undergo significant further nuclear burning

For $M < 8 M_\odot$ ⇒ the He-burning shell converts more & more He into C & then O, increasing the mass of the C-O core as the star evolves up the AGB. The density of the core gets large enough that e^- degeneracy pressure dominates.

For $M < 4 M_\odot$ ⇒ the C-O core will never get hot enough to ignite C/O burning ⇝ white dwarf

For $4 < M/M_\odot < 8$ ⇒ mass loss prevents catastrophic core collapse, experiencing additional nucleosynthesis in the core, leading to core compositions of O, Ne, Mg, w/ $M_{core} < 1.4 M_\odot$ (Chandrasekhar limit)

$$Ne^{22} + He^4 \rightarrow Mg^{25} + n$$
$$Ne^{22} + He^4 \rightarrow Mg^{26} + \gamma$$

Post-Asymptotic Giant Branch

As the cloud around the OH/IR source continues to expand, it becomes optically thin, exposing the central star, which exhibits a spectrum of an F or G supergiant.
The star moves horizontally to lower T_e ⟹ post-AGB.
The star's envelope is expelled, the H & He-burning shells are extinguished, & the luminosity drops rapidly.
The hot central object, revealed, cool to become a WHITE DWARF, the degenerate C-O (or ONeMg) core, surrounded by a thin layer of H & He.

Planetary Nebulae

The expanding shell of gas around a WD progenitor is called a planetary nebula. The UV light emitted by the hot, central WD is absorbed by the gas, exciting & ionizing it. Atoms de-excite emitting optical photons.
Typical temperature are 10^4 K, w/ emission from O III] 4958.9, 5006.8
O II], Ne III], & H balmer & N II]. Typical velocity is 10-30 km/s. After ~ 50K yrs, the planetary nebula dissipate into the ISM.

- **LUMINOUS BLUE VARIABLES (LBVs)** have high effective temperatures $T_e \sim 15000 - 30000$ K, w/ $L > 10^6 L_\odot$
  ⇒ upper left portion of HR diagram, all very massive $> 85 M_\odot$
  ➡ evolved, post-MS stars, along w/ strong emission lines,
- **WOLF-RAYET STARS (WR)**
  WR are very hot, $T_e \sim 25000 - 10^5$ K, losing mass at a rate $> 10^{-5} M_\odot/yr$ w/ wind speeds ~ 800 ÷ 3000 km/s.
  Progenitors w/ M as low as $20 M_\odot$.

WN : spectra dominated by emission lines of He, N

WC : " " " " " of He, C, without N & H.

WO : prominent O lines.

WNs have lost all of the H-dominated envelopes; convection has brought up CNO-cycle-processed material to the surface ... WC is when further mass loss ejects CNO-material, exposing He-burning material produced by the 3α process. Finally, mass loss strips away all but O component of 3α → WO.

MASSIVE STARS: $M > 8 M_\odot$

$10 < M/M_\odot < 20$: O → RSG (red supergiant) → BSG → SN

$20 < M/M_\odot < 25$: O → RSG → WN → SN

$25 < M/M_\odot < 40$: O → RSG → WN → WC → SN

$40 < M/M_\odot < 85$: O → supergiant w/ emission lines → WN → WC → SN

$M/M_\odot > 85$: O → " → LBV → WN → WC → SN

For a star w/ $M > 8 M_\odot$, the temperature in the core can get high enough for C & O burning, ending its life as a CORE-COLLAPSE SUPERNOVA (type Ib, Ic, II).

He-burning shell adds ash to the C/O core, the core continues to contract, T rises until C burning ignites, generating O^{16}, Ne^{20}, Na^{23}, Mg^{23}, Mg^{24} (very dependent on mass of the star)

→ onion-like shell structure.

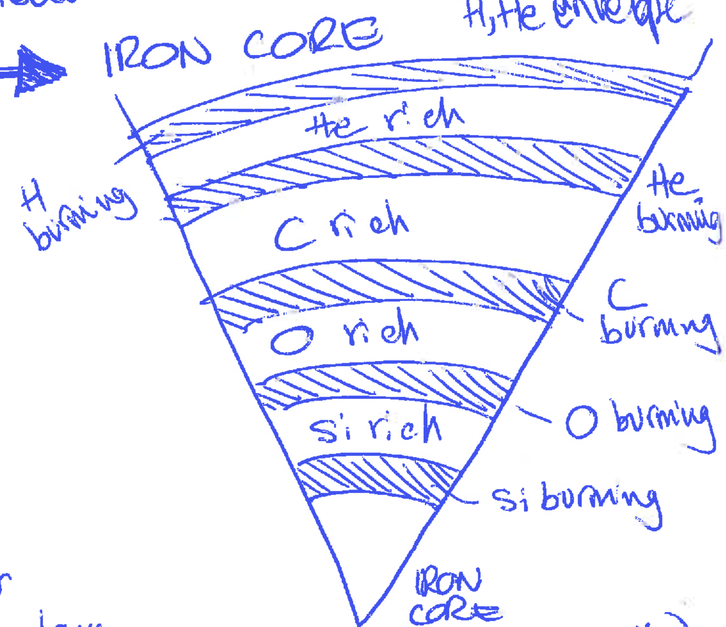
Following C burning, the O in the resulting Ne/O core will ignite, producing a new core dominated by Si^{28}.
At $T \sim 3\times10^{9}$ K, Si-burning begins: $Si^{28} + He^{4} \rightleftharpoons S^{32} + \gamma$

$$S^{32} + He^{4} \rightleftharpoons Ar^{36} + \gamma$$

$$\vdots$$

$$Cr^{52} + He^{4} \rightleftharpoons Ni^{56} + \gamma$$

→ Si burning produces nuclei centered near Fe^{56}, most abundant being Fe^{54}, Fe^{56}, Ni^{56} → IRON CORE



(1.3 Mo for M = 10 Mo)
(2.5 for 50)

At each succeeding reaction, less & less energy is generated per unit mass of fuel → the timescale of each sequence becomes shorter

For $M \sim 20 M_\odot$:

- MS = 10^{7} yr
- He core burning ≑ 10^{6} yr
- C " " ≑ 300 yr
- O " " : 200 days
- Si " " : 2 days

When the mass of the contracting Iron core is large enough & T is sufficiently high

$$Fe^{56} + \gamma \rightarrow 13He^{4} + 4n$$
$$He^{4} + \gamma \rightarrow 2p^{+} + 2n$$

} photo disintegration

processes highly endothermic (energy is needed)

↓

Thermal energy is removed from the gas that would otherwise have provided the pressure to support the core

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$T \sim 8\times10^{9}$ K

$\rho_c \sim 10^{10}$ g/cm^3

(for $M = 15 M_\odot$)

↓

$p^{+} + e^{-} \rightarrow n + \nu_e$ ⇔ e^{-} capture by heavy nuclei

└ which were providing support via e^{-} degeneracy pressure

During Si burning, the photon luminosity of a star with $M = 20\,M_\odot$ is $\sim 10^7$ times smaller than the energy leaving with neutrinos! 27

→ most of the support for the core (e^- degeneracy pressure) is suddenly gone, & the core collapses extremely rapidly (speeds $\sim 7\times10^4$ km/s) → w/in 1 sec the size of Earth is compressed to the size of 50 km.

The collapse of the inner core continues until $\rho \sim 8\times10^{14}$ g/cm^3 $\simeq$ 3 $\rho_{\text{atomic nucleus}}$
At this point, neutron degeneracy pressure dominates, halting the collapse; the core rebounds, sending pressure waves outward into the falling material from the outer core
→ shock wave moving outward. The shock will drive the envelope & the remainder of the nuclear-processed matter in front of it. The total kinetic energy in the expanding material is $\sim 10^{51}$ erg, roughly 1% of the energy liberated in ν_s. At $r \sim 10^{15}$ cm $\simeq$ 100 AU, the material becomes optically thin → 10^{49} erg of energy liberated as photons w/ peak luminosity nearly $10^{43}\,\frac{\text{erg}}{\text{s}} \simeq 3\times10^9\,L_\odot$

→ CORE-COLLAPSE SN.

Type II SN are generally red supergiants in the extreme upper-right corner of the HR diagram.

Type Ib & Ic SNs have lost various amounts of ~~the~~ their envelopes prior to detonation; products of exploded WR stars.

Type Ib ↔ WN
Ic ↔ WC

If $M \lesssim 25 M_\odot \rightarrow$ neutron star, supported by degenerate neutron pressure

$M \gg 25 M_\odot \rightarrow$ black hole

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s- & r- process nucleosynthesis

$${}^{A}_{Z}X + n \rightarrow {}^{A+1}_{Z}X + \gamma$$

$$\quad \rightarrow {}^{A+1}_{Z+1}X + e^- + \bar{\nu}_e + \gamma$$

beta decay

If the beta-decay half-life is short compared to the timescale for n capture ➡ slow-process ("s"). s-process reactions tend to yield stable nuclei.

If half-life of beta-decay is long compared to the timescale for n capture ➡ rapid-process ("r"), resulting in n-rich nuclei. s-processes tend to happen in normal phase of stellar evolution, whereas r-processes occur during SN when large flux of ν_s exists.

These processes account for the abundance ratios of nuclei w/ $A > 60$.
