THERMONUCLEAR REACTION RATES

, ie: $\epsilon = \epsilon(\rho, T, \{X_z\})$

The fusion of light nuclei into heavier nuclei liberates kinetic energy (at the expense of mass)

$\epsilon :=$ energy liberated per gram of stellar material

It is the slow change of chemical composition that causes the structure of the star to evolve.

KINEMATICS & ENERGETICS:

$a + X \rightarrow Y + b$

$\Updownarrow$ eq.

$X(a, b)\,Y$

ie: particle a strikes nucleus X producing a nucleus Y & a new particle b.

Total energy, momentum, & angular momentum are to be conserved.

NOTE: for stellar interior $\longrightarrow$ low kinetic energies $\longrightarrow$ non-relativistic approach.

The momentum of m_1 relative to the center of mass

$$m_1(\bar{v}_1 - \bar{V}) = \frac{m_1 m_2}{m_1 + m_2}(\bar{v}_1 - \bar{v}_2) = \mu \bar{v}$$

$\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ reduced mass

$\bar{v} = \bar{v}_1 - \bar{v}_2$

velocity of center of mass

$$\bar{V} = \frac{m_1 \bar{v}_1 + m_2 \bar{v}_2}{m_1 + m_2}$$

$$m_2(\bar{v}_2 - \bar{V}) = -\mu \bar{v}$$

ie: in the center of mass, the particles approach each other w/ equal & opposite momenta; the total momentum is zero in the center-of-mass system.

Conservation of momenta $\Longleftrightarrow$

- $\bar{V}$ is not changed by the collision
- total momentum in the center-of-mass system is zero after the collision.

$$(KE)_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \underbrace{\frac{1}{2}(m_1+m_2)V^2}_{} + \underbrace{\frac{1}{2}\mu v^2}_{}$$

(2)

Kinetic energy before collision.

KE associated to (m_1+m_2) moving at the velocity of the center of mass

i.e.: kinetic energy of the center of mass (to be conserved in the collision)

KE of μ moving w/ the relative velocity $\bar{v}$.

energy available for doing work against any force separating the two particles, aka the KE in the center-of-mass system.

NOTE: These non-relativistic formulas are OK if the combined mass of the final particles equals the combined mass of the initial particles.

BUT the source of new kinetic energy comes from a reduction of mass $\Delta KE = -\Delta M c^2$

In low-energy reactions $\frac{\Delta M}{M} \simeq 10^{-3} - 10^{-4}$ to better than 0.1%.

$\Rightarrow$ assumption correct

Conservation of energy $\quad E_{ax} + \underbrace{(M_a + M_x)c^2 = E_{by} + (M_b + M_y)c^2}$

center-of-mass KE of a & X

the sums of the rest masses before & after the reaction are not exactly the same, & KE may be liberated or absorbed.

NOTE: net amount of electric charge is conserved in normal nuclear reactions

$\rightarrow$ nuclear masses $\leftrightarrow$ atomic masses

would be a small error of a few eV.

NOTE: Another conserved quantity is total number of nucleons (n & p)

$\rightarrow$ the atomic weight remains the same on both sides.

Let's define ATOMIC MASS EXCESS IN UNITS OF ENERGY $\quad \Delta M_{AZ} = (M_{AZ} - AM_u)c^2$

$$= [M_{AZ}/amu - A]\, c^2 M_u$$

w/ $M_U = 1$ amu (atomic mass unit) $\equiv \frac{1}{12}\,{}^{12}C = 1.660539 \times 10^{-24}$ g

$$\Delta M_{AZ} = 931.478\,(M_{AZ} - A)\ \text{MeV}$$

M_{AZ}: mass of species (A, Z) in atomic mass unit

$$\Rightarrow\quad E_{aX} + (\Delta M_a + \Delta M_X) = E_{bY} + (\Delta M_b + \Delta M_Y) \qquad (*)$$

by subtracting the atomic weight · the rest-mass energy of 1 amu from both sides. (see Table w/ atomic mass excesses)

Ex: $d + C^{12} \rightarrow C^{13} + p$ w/ d = nucleus of deuterium D

$$E_{d,C^{12}} + 13.1359 + 0 = E_{p,C^{13}} + 7.2890 + 3.1246$$

$$\hookrightarrow E_{p,C^{13}} = E_{d,C^{12}} + 2.7223\ \text{MeV}$$

ie: there is an increase of kinetic energy equal to 2.722 MeV for each such reaction..

CROSS SECTION & REACTION RATE

(*) yields the energy liberated by each nuclear reaction.

$$\underbrace{\frac{\#\,\text{reactions}}{s\ \ cm^3}}_{\substack{\text{it requires the cross section of the reaction}\\ \text{(}\sim\text{ probability per pair of particles of occurrence of a reaction)}}} \cdot \underbrace{[(\Delta M_a + \Delta M_X) - (\Delta M_b + \Delta M_Y)]}_{\text{energy liberated by each reaction}} = \frac{\text{MeV}}{s\ \ cm^3}$$

energy liberated per unit volume per second.

$$\underset{[cm^2]}{\sigma(v)} = \frac{\#\,\text{reactions / nucleus X / unit time}}{\#\,\text{incident particles / cm}^2\text{/ unit time}}$$

$$\Rightarrow\quad \underbrace{v N_a}_{\text{flux of particles } a} \cdot \sigma N_X = \frac{\#\,\text{reactions}}{s\ \ cm^3}$$

N_a: # density of particle a

N_X: # density of particle X

ie: reaction rate per unit volume

More generally, there is some spectrum of relative velocities $\phi(v)$

$\Rightarrow$ reaction rate per unit volume : $r_{ax} = N_a N_x \int_0^\infty v\,\sigma(v)\,\phi(v)\,dv\,(1+\delta_{ax})^{-1}$

$$= N_a N_x \underbrace{\langle \sigma v \rangle}_{} (1+\delta_{ax})^{-1}$$

average value of the product of relative velocity times cross section. $\left(\text{since } \int_0^\infty \phi(v)\,dv \equiv 1\right)$

In the stellar interior, $\langle \sigma v \rangle$ needs to be calculated $\Leftarrow$

takes into account that if $a=X$, then the total # of pairs per unit volume is not N_a^2 but $\frac{1}{2}N_a^2$.

$$(1+\delta_{ax})^{-1} = \begin{cases} 1/2 & \text{if } a \equiv X \\ \text{[redacted]} & \text{if } a \neq X \end{cases}$$

Kronecker delta

$$\Rightarrow \quad r_{ax} = \underbrace{\lambda_{ax}}_{} (1+\delta_{ax})^{-1} N_a N_x$$

$\lambda_{ax} := \langle \sigma v \rangle$ reaction rate per pair of particles.

Let's define the lifetime $\tau_a(X)$ of species X against reactions with species a such that

$$\left(\frac{\partial N_x}{\partial t}\right)_a = - \frac{N_x}{\tau_a(X)}$$

Note: it does not have the usual math meaning

rate of change of the abundance of X due to reactions w/ a

But $\left(\frac{\partial N_x}{\partial t}\right)_a = - r_{ax}$ $\Rightarrow$ $\boxed{\tau_a(X) = (1+\delta_{ax})^{-1}\dfrac{N_x}{r_{ax}} = (\lambda_{ax} N_a)^{-1}}$

The nuclei in stellar interiors are always non-degenerate (except in neutron stars) $\Rightarrow$ v_a, v_x described by Maxwellian-Boltzmann distributions of velocities.

$$\Rightarrow r_{ax} = (1+\delta_{ax})^{-1} N_a N_x \, 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 \sigma(v) e^{-\mu v^2/2kT} dv$$

w/ $\mu = \frac{m_a m_x}{m_a + m_x}$ reduced mass

v relative velocity of m_a & m_x

$\Rightarrow$ The calculation of $\langle \sigma v \rangle$ reduces to performing the integral

$$\lambda = \langle \sigma v \rangle = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 \sigma(v) e^{-\mu v^2/2kT} dv.$$

NON-RESONANT REACTION RATES

NOTE: Coulomb barrier between two particles $V = \frac{Z_1 Z_2 e^2}{R} = \frac{1.44 Z_1 Z_2}{R(fm)}$ MeV

Kinetic energy of interacting particles $kT = 8.62 \times 10^{-8} T$ keV for $T \sim 10 - 100 \times 10^6$ K.

$\Rightarrow kT \ll V$

Probability for two particles of charge Z_1 & Z_2 moving w/ relative velocity v to penetrate their electrostatic repulsion is proportional to

$$\text{penetration} \propto e^{-2\pi Z_1 Z_2 e^2/\hbar v}$$

$\Rightarrow$ cross sections of nuclear reactions are also proportional to this factor

$$\Rightarrow \sigma(E) :\equiv \frac{S(E)}{E} e^{-2\pi Z_1 Z_2 e^2/\hbar v}$$

1 barn $= 10^{-24}$ cm^2

w/ $S(E)$ only slowly dependent on E.

$$\sigma(E) = \frac{S(E)}{E} e^{-bE^{-1/2}} \quad w/ \quad b = 31.28\, Z_1 Z_2 A^{1/2} \ \mathrm{KeV}^{1/2}$$ (6)

$$A = \frac{A_1 A_2}{A_1 + A_2} = \mu / M_u$$

E = energy in the center-of-mass system.

$$\psi(E)\, dE = \phi(v)\, dv = -\frac{2}{\sqrt{\pi}} \frac{E}{KT} e^{-E/KT} \frac{dE}{(KTE)^{1/2}}$$

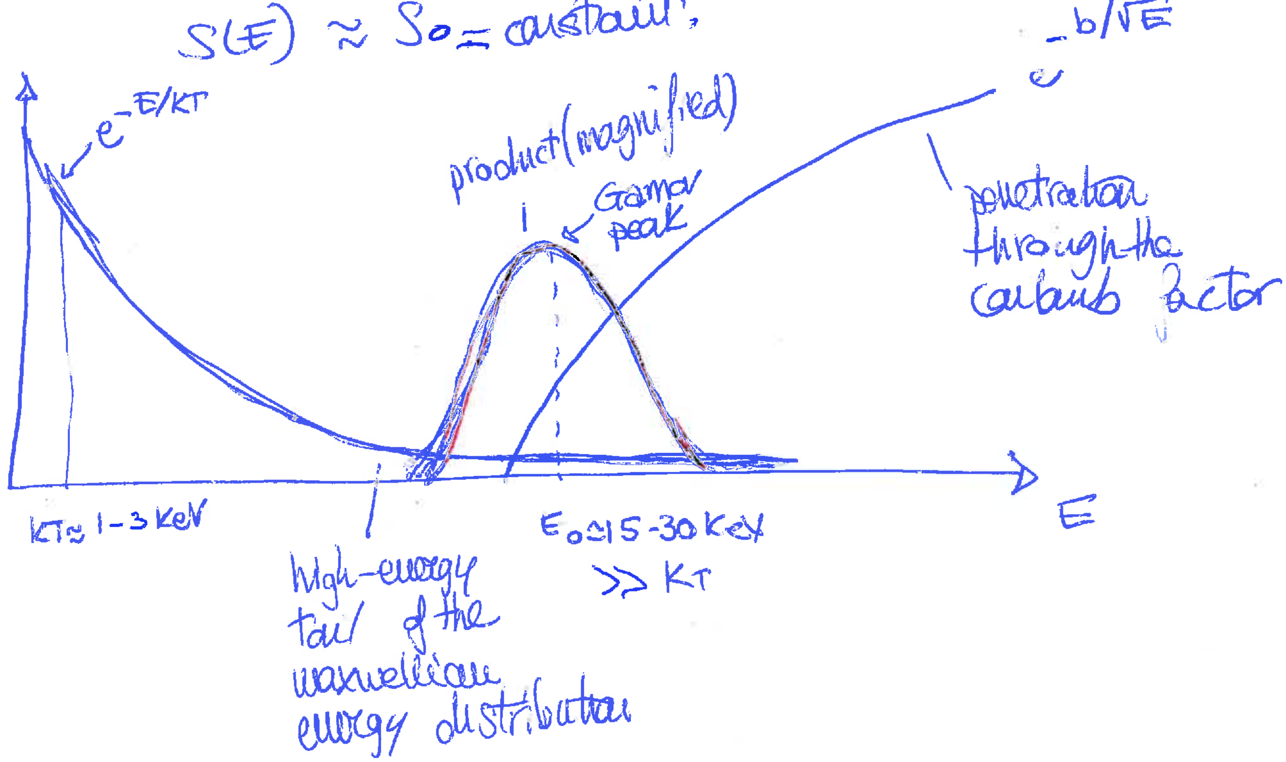
$$\Rightarrow \lambda = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(KT)^{3/2}} \int_0^\infty S(E)\, \underbrace{e^{-\frac{E}{KT} - bE^{-1/2}}}\, dE$$

$e^{-E/KT} \rightarrow 0$ for large E

$e^{-bE^{-1/2}} \rightarrow 0$ for small E

$$\Rightarrow \lambda = \langle \sigma v \rangle = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{S_0}{(KT)^{3/2}} \int_0^\infty e^{-\frac{E}{KT} - bE^{-1/2}}\, dE$$

ie: the major contributions to the integral come from values of E in a very narrow range, so narrow that $S(E) \approx S_0 =$ constant.



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$$\frac{d}{dE}\left(\frac{E}{kT}+\frac{b}{\sqrt{E}}\right)_{E=E_0} = \frac{1}{kT} - \frac{1}{2} b E_0^{-3/2} = 0$$

$$\boxed{E_0 = \left(\frac{bkT}{2}\right)^{2/3} = 1.22\left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3} \text{ keV} \quad \text{w/ } T_6 = T/10^6\,\text{K}}$$

most effective energy for thermonuclear reactions

$E_0 \simeq 10 \div 30$ keV $\gg 0.086\, T_6$ keV

NOTE: $e^{-E/kT - b/\sqrt{E}} \approx C e^{-\left(\frac{E-E_0}{\Delta/2}\right)^2}$, $C = e^{-\frac{E_0}{kT} - b/\sqrt{E_0}} = e^{-3E_0/kT}$

gaussian function

$\Delta = \frac{4}{\sqrt{3}}(E_0 kT)^{1/2}$
$= 0.75\left(Z_1^2 Z_2^2 A T_6^5\right)^{1/6}$ keV

⇒ The most effective particles have energies ranging only ~ 10 keV from E_0.

$$\Rightarrow \lambda = \left(\frac{8}{\mu\pi}\right)^{1/2}\left(\frac{1}{kT}\right)^{3/2} e^{-\tau}\int_0^\infty S(E)\, e^{-\left(\frac{E-E_0}{\Delta/2}\right)^2} dE \quad , \tau = 3E_0/kT$$

& $S(E) \approx S(E_0) = S_0$

$$\Rightarrow \lambda = \frac{4.5\times10^{14}}{AZ_1Z_2} S_0 \tau^2 e^{-\tau} \ \frac{\text{cm}^3}{\text{s}}$$

reaction rate per pair

$$= \frac{7.2\times10^{-19}}{AZ_1Z_2} S_0(\text{keV barns})\, \tau^2 e^{-\tau} \ \text{cm}^3/\text{s}$$

Reaction rate $r_{12} = (1+\delta_{12})^{-1} N_1 N_2 \lambda_{12}$

$$\tau = 42.48 \left(\frac{Z_1^2 Z_2^2 A}{T_6}\right)^{1/3} \propto T^{-1/3}$$

$\Rightarrow$ $\tau = B\, T_6^{-1/3}$, $B = 42.48\left(Z_1^2 Z_2^2 A\right)^{1/3}$

showing the temperature dependence of τ.

$\Rightarrow$ it easy to determine the manner in which a chosen reaction rate depends upon T.

Since $N_i = \rho \frac{1}{M_u} \frac{X_i}{A_i}$ w/ X_i fraction by mass of species i

$$\boxed{r_{12} = \frac{2.62\times10^{29}}{(1+\delta_{12})} \frac{1}{A Z_1 Z_2} \rho^2 \frac{X_1 X_2}{A_1 A_2} S_0(\text{kev barns})\, \tau^2 e^{-\tau}}$$

$[cm^{-3}\, s^{-1}]$ BASIC NON-RESONANT STELLAR REACTION RATE

NOTE: Three corrections to the above approximation

1. a better approximation $S(E) = S(E_0) + \left(\frac{dS}{dE}\right)_{E_0}(E-E_0)$
2. some error is introduced by substituting a gaussian for the sharply peaked exponential
3. high density of free e^-s near the nuclei increases the reaction rate by reducing the coulomb repulsion.

2. $\longrightarrow$ correction factor $F(\tau) = 1 + \frac{5}{12\tau} + O\left(\frac{1}{\tau^2}\right)$

≈ 1 since τ is a large number

1. correction factor $G(\tau) = 1 + \frac{5}{2}\frac{kT}{S(E_0)}\left(\frac{\partial S}{\partial E}\right)_{E_0}$

One should define $S_0 := S(E_0)\,G(\tau) = S(E_0) + \frac{5}{6}\left(\frac{\partial S}{\partial E}\right)_{E_0} kT$

$\tau = 3E_0/kT$

i.e.: the constant cross-section factor S_0 should be the value of $S(E)$ not at $E = E_0$, but at $E = E_0 + \frac{5}{6}kT$ (9)

$\Rightarrow S_0 \equiv S(E_0 + \frac{5}{6}kT)$ from now on, so we can ignore the correction factor $G(\tau)$.

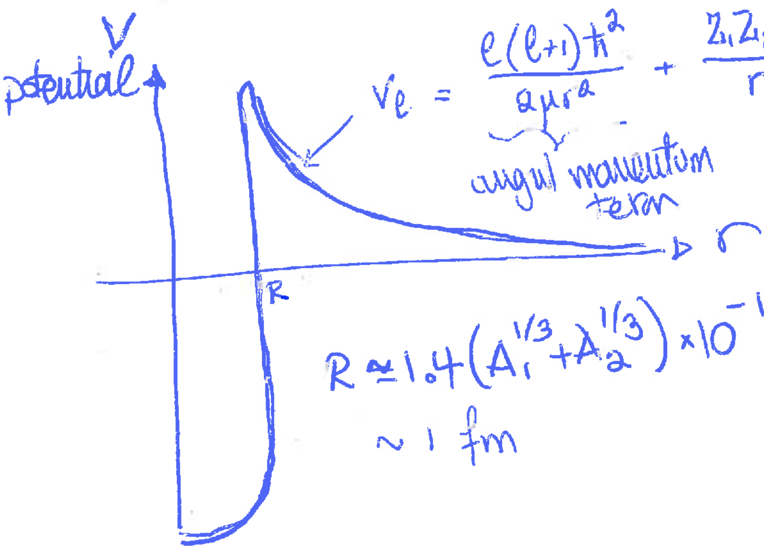
3. $\longrightarrow$ screening factor "f" in r_{12} formula.

COMPOUND NUCLEAR STATES

$W \to a + X$ elastic scattering
$\quad\to a + X^*$ target left in excited state
$\quad\to C + \gamma$ radiative capture (de-excitation through emission)
$\quad\to Y + b$ particle emission

Let's consider $a + X \longrightarrow W$

w/ W compound nucleus & the reaction $a + X$ proceeds through a resonant state of the compound nucleus W.



$$V_\ell = \underbrace{\frac{\ell(\ell+1)\hbar^2}{2\mu r^2}}_{\text{angul momentum term}} + \frac{Z_1 Z_2 e^2}{r}$$

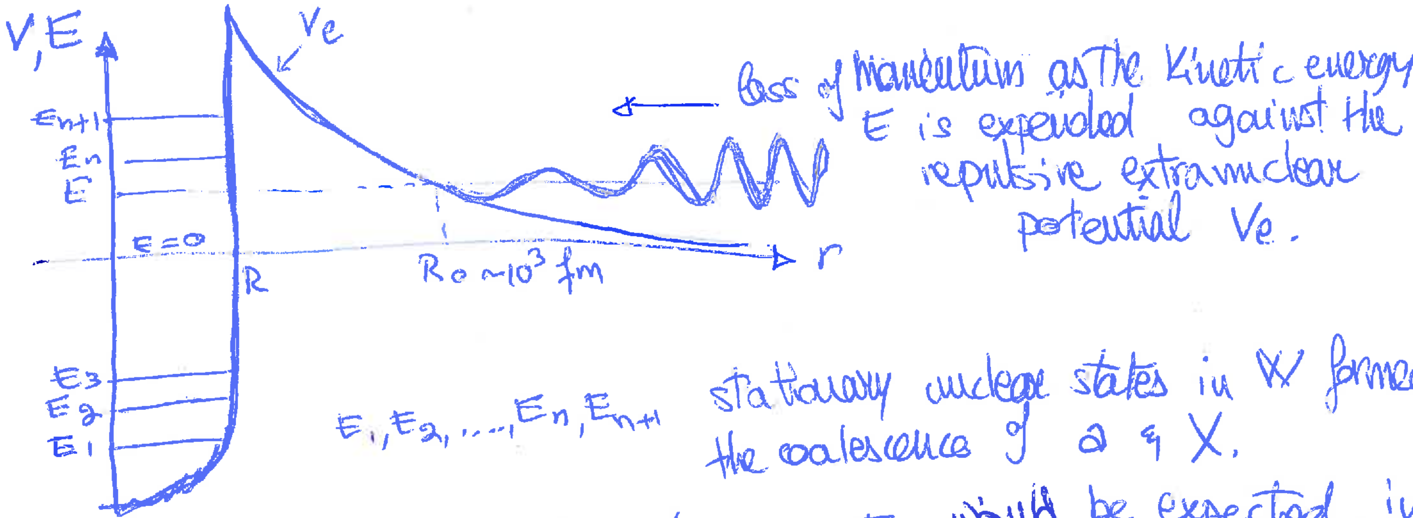
$$R \simeq 1.4 (A_1^{1/3} + A_2^{1/3}) \times 10^{-13}\ \text{cm}$$
~ 1 fm

potential governing the motion of a nucleus relative to another.

For $r < R$, the nuclei are essentially in contact & the nuclear strong force results in a deep negative potential.

$r > R$, the nuclear force is no longer felt, & the Coulomb potential dominates. The angular momentum adds an effective centrifugal potential.

Resonances can occur in nuclear reactions if the kinetic energy E of the particles at infinity is just such that the total energy coincides with one of the quasi-stationary states of the compound nucleus W.



$E_1, E_2, \ldots, E_n, E_{n+1}$ stationary nuclear states in W formed by the coalescence of a & X.

The incoming particle w/ energy E would be expected, in the classical mechanics, to rebound at R_o where $V_e(R_o) = E$, but in the quantum treatment there is a probability that the particle can penetrate the potential wall & reach the nuclear force at $r = R$.

If $E \neq E_i$ → E doesn't coincide w/ a quasi-stationary state & the reaction is non-resonant.

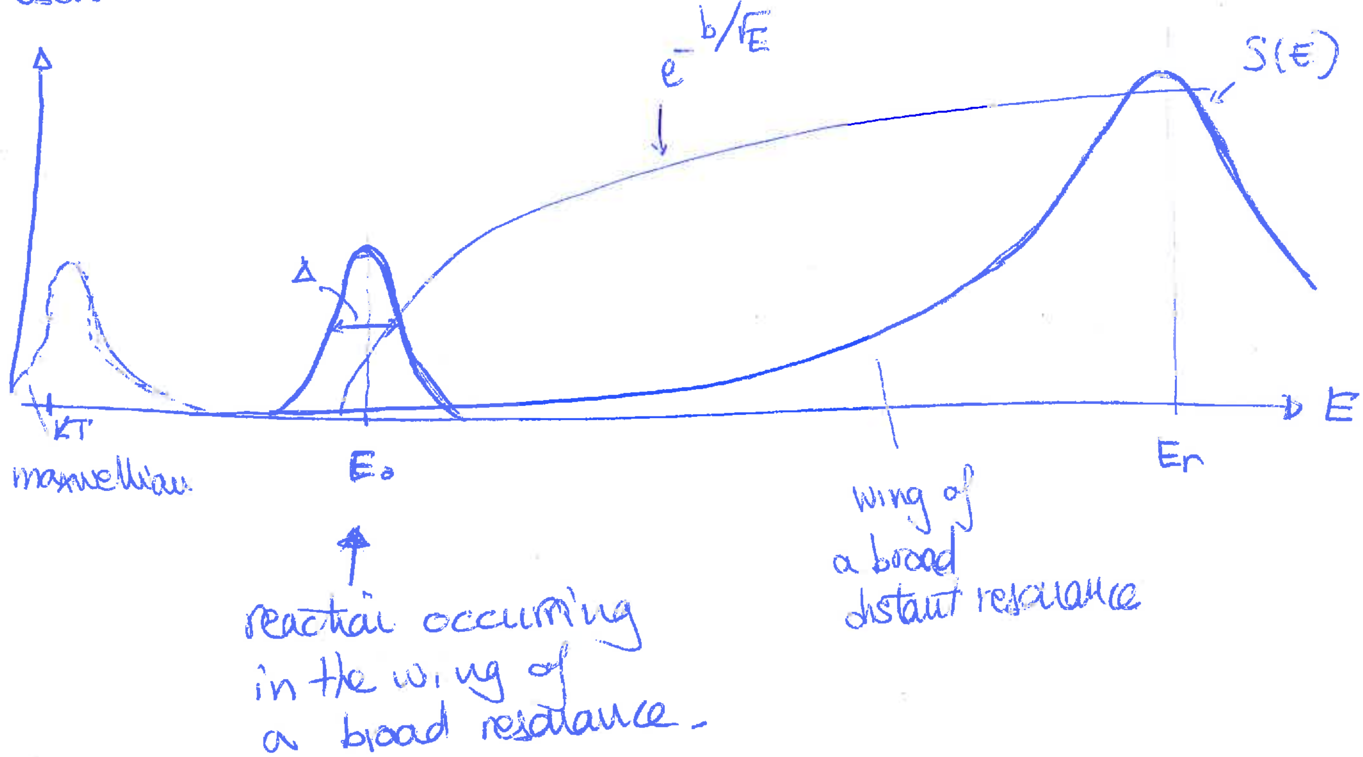
BUT in practice, one of the states E_n will often lie close enough to E so that a resonant reaction rate will have to be employed.

All resonant states are unbound nuclear states, as W can decay back to a & X w/ positive kinetic energy.
The quasi stationary resonant states can live for relatively long times, because the potential barrier $V_e(r)$ inhibits the breakup into a & X in the same way that it inhibits the formations of the states by those particles.
Higher energy states break up increasingly rapidly & into more & more possible combinations of final particles, i.e.: other than a & X.

RESONANT REACTION RATES IN STARS.

When resonances (quasi-stationary states in the compound nucleus) occur in the range of effective stellar energies, the stellar reaction rate is usually dominated by them.

Resonant cross sections are many orders of magnitude greater than non-resonant cross sections at energies near the resonance.



In some cases, the compound nucleus provides a resonance in the range of effective stellar energies.
If the resonance is very far from E_0, then the reaction must proceed through the wings of the resonance & the non-resonant reaction rate formalism must be used.
For resonances in the vicinity of E_0, the full height of the resonant cross section must be used.

ELECTRON SHIELDING

Thermonuclear reaction rates in stars are increased over their lab analogs because of the presence of the dense electron gas. The net negative charge surrounding each nucleus reduces the coulomb repulsion to a value smaller than $\frac{Z_1 Z_2 e^2}{r}$

the

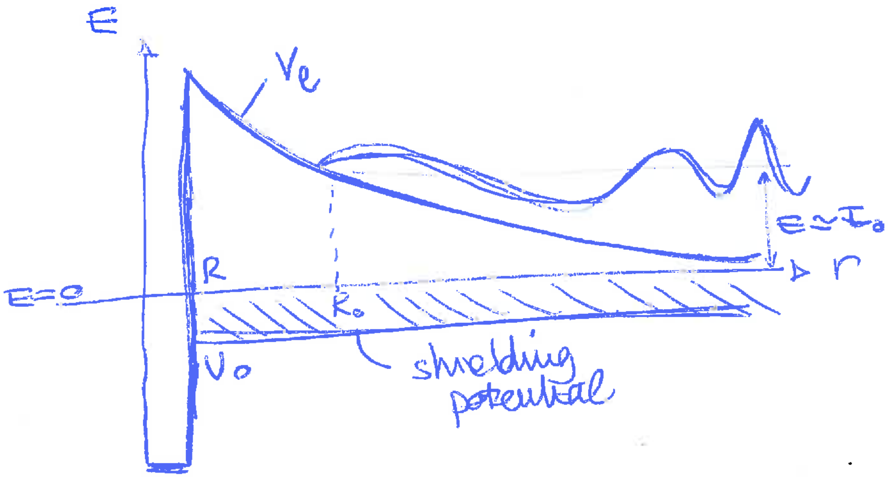
$\longrightarrow$ penetration of coulomb barrier easier

$\longrightarrow$ increased cross section

Each nucleus, even though completely ionized, attracts neighboring es $\Rightarrow$

$$\underbrace{U_{tot}(r_{12})}_{\text{total coulomb interaction energy}} = \frac{Z_1 Z_2 e^2}{r_{12}} + \underbrace{U(r_{12})}_{\text{added interaction due to shielding}}$$

w/ $U(r_{12}) \simeq U_0$, shielding potential at the origin.



$\rightsquigarrow$ e^- shielding ... increases the rate by the factor $f = e^{-U_0/KT} \underset{U_0 \ll KT}{\approx} 1 - \frac{U_0}{KT} + \ldots$

weak screening limit, applicable to the majority of the thermonuclear reactions in astrophysics.

NUCLEAR BURNING STAGES IN STELLAR EVOLUTION

• stars form from interstellar gas by a gravitational instability of that gas (Jean's instability).

When a sufficiently large mass of gas is compressed to a small enough volume, it's force of self-gravitation will cause gravitational collapse. Initially, the cloud is nearly in a condition of free fall, & then pressure forces begin to restrict the collapse → directed motion of free fall converted into random thermal energy in the gas, w/ T rising.

From virial theorem, $\frac{1}{2}\Omega$ is converted into internal thermal energy $(2K+\Omega=0)$

potential energy

Increased T & ρ → pressure rises, & the collapse is decelerated to a slow quasi-static collapse.

From this point on, the virial theorem is the dominant principle governing the subsequent evolution of a star.

⇒ Ω $\xrightarrow{\Omega/2}$ kinetic energy of the particles

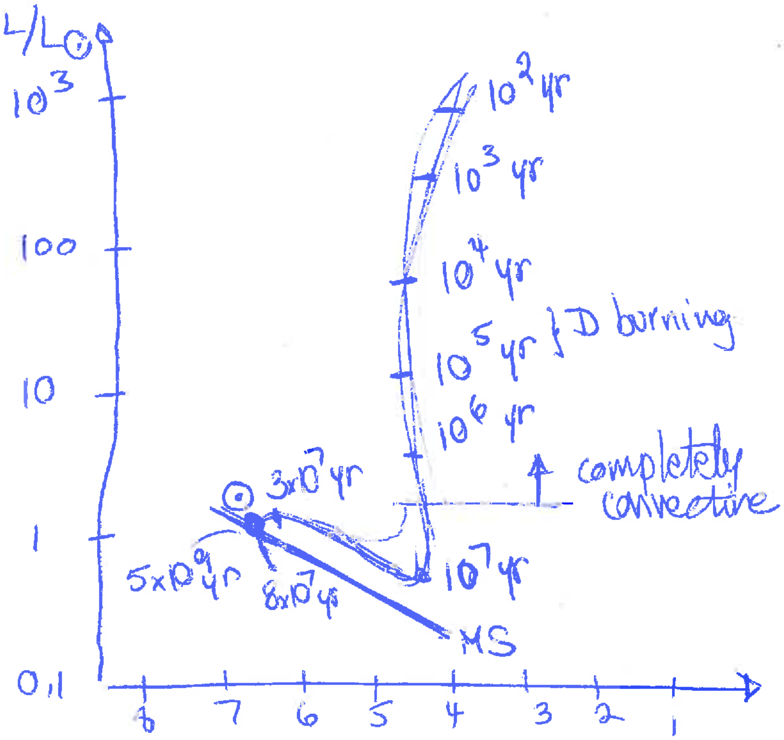
 $\xrightarrow{\Omega/2}$ production of radiation

The radiation escapes at first, but then stellar matter becomes opaque as the collapse is slowed down by the pressure buildup ⇒ thermodynamic equilibrium is established

Much of the radiation still escapes, mostly consumed in ionizing the matter (initially neutral).

The interior temperature cannot rise above 10^4 K until H has been ionized, since ~90% atoms are H. Hydrostatic equilibrium is established when most H & He is ionized.

(14)



Stars contracting toward the main-sequence have very large L due to the required high T_e & R.

Since L & κ (opacity) are large during the contraction phase the star is convective to get the energy out fast enough.

A star contracts along a nearly vertical path in the HR diagram until near the MS. The early contraction is very rapid & luminous, & the energy source is entirely due to gravitational work of the contraction. As it approaches the MS, the evolution becomes slow. The star is fully convective for a few million years, after which a central core in radiative equilibrium begins to grow, slowly moving outward until it reaches its final MS size. The subsurface convection zone shrinks to its final MS size. During this process, the internal temperature has been increasing. T rises until it's high enough to cause thermonuclear reactions to occur at a rate adequate to supply the power radiated from the surface - When this balance is achieved → static stellar configuration.

Most energy-generating reactions involving light particles are non-resonant ⇒ $r_{12} \propto N_1 N_2\, e^{-42.48\,[Z_1^2 Z_2^2\; A/T_6]^{1/3}}$

as T increases, r_{12} increases rapidly

Because exponent is large, the major reactions are those for which $Z_1 Z_2$ is as small as possible.

However $p + p \rightarrow He^2$ (unstable) $\rightarrow p + p$
$p + He^4 \rightarrow Li^5$ (unstable) $\rightarrow p + He^4$
$He^4 + He^4 \rightarrow Be^8$ (unstable) $\rightarrow He^4 + He^4$

major 2-particle combinations (involving p & He^4) have unstable ground states

- The first thermonuclear reaction to proceed is $H^1 + D^2 \rightarrow He^3 + \gamma$
This converts D (deuterium) into He^3 during the pre-MS contraction
⇒ D is quickly exhausted, & its effect is to slow the contraction somewhat during the "Deuterium burning" phase.
- Similarly, Li, Be, & B are destroyed effectively by reactions w/ ps @ T ~ a few million K (therefore alter the cosmological abundances of D, Li, Be, B)
- Reactions converting H into He: → proton-proton chains
→ CNO cycle

PROTON - PROTON REACTION

- $H^1 + H^1 \rightarrow He^2 \rightarrow D^2 + e^+ + \nu_e$ (plus 0.42 MeV of kinetic energy of the e^+ & ν)
$\hookrightarrow e^+ + e^- \rightarrow 2\gamma$ with 1.02 MeV
For a total of 1.442 MeV

$p \rightarrow n + e^+ + \nu$
decay of p through weak interaction decay.
(β^+ decay)

The cross-section is very small, $10^{-47}\, cm^2$ @ 1 MeV.

NOTE: Because of the low coulomb barrier in the p-p reaction, a star would consume its H quickly if it weren't slowed down by the weakness of the beta decay.

$$S(0) = (3.78 \pm 0.15) \times 10^{-22}\ \text{keV barn}$$

$$\frac{dS}{dE} = 4.2 \times 10^{-24}\ \text{barn}$$

⇒ $$r_{pp} = 11.05 \times 10^{10} \rho^2 X_H^2 T_6^{-2/3} e^{-33.81 T_6^{-1/3}} (1 + 0.0123 T_6^{1/3} + 0.0109 T_6^{2/3} + 0.00095 T_6)\ cm^{-3} s^{-1}$$

The lifetime of Ps against p-p reactions at $T = 15\times10^6$ K, $\rho = 100$ g/cm^3, & $X_{He} = X_H = 0.5$ ⇒ $\tau_p(H) \simeq 10^{10}$ yr

The 1.442 MeV is converted to local heat, of which $\bar{E}_\nu = 0.262$ MeV are carried away by the neutrinos.
⇒ average heat input from each reaction is $1.442 - 0.262 = \underline{1.18 \text{ MeV}}$

● PP I CHAIN (~69% of times in the Sun)

NOTE: many reactions proceed at a negligible rate, either because the cross sections are too small or the product of the abundances are too small.
Eg: $D + D \rightarrow He^4 + \gamma$ BUT small cross section & D is negligible (as destroyed into He^3).

(A) $H^1 + H^1 \rightarrow D^2 + e^+ + \nu_e$ $r_{pp} = \lambda_{pp} H^2/2$

(B) $D^2 + H^1 \rightarrow He^3 + \gamma$ $r_{pd} = \lambda_{pd} HD$

(C) $He^3 + He^3 \rightarrow He^4 + 2H^1$ $r_{33} = \lambda_{33} (He^3)^2/2$

number densities

Since $\frac{dD}{dt} = \lambda_{pp}\frac{H^2}{2} - \lambda_{pd} HD$

↳ this is self-regulating, & D reaches an equilibrium value

$$\left(\frac{D}{H}\right)_{equil} = \frac{\lambda_{pp}}{2\lambda_{pd}} = \frac{\tau_p(D)}{2\tau_p(H)} \simeq 2.8\times10^{-18} \text{ @ } T = 15\times10^6 \text{ K}$$

D^2 burning reaction is so fast that the lifetime of D inside a star may be on the order of seconds!

For $D^2 + H^1 \rightarrow He^3 + \gamma$; $S(0) = 2.5\times10^{-4}$ keV barn
$dS/dE = 7.9\times10^{-6}$ barn

For (C) $\frac{dHe^3}{dt} = \lambda_{pd} HD - 2\lambda_{33}\frac{(He^3)^2}{2} \simeq \lambda_{pp}\frac{H^2}{2} - 2\lambda_{33}\frac{(He^3)^2}{2}$

$\lambda_{pp}\frac{H^2}{2} \simeq \lambda_{pd} HD$
(ie: D in equilibrium)

He^3 also builds toward an equilibrium abundance given by

$$(He^3/H)_{equil} = (\lambda_{pp}/2\lambda_{33})^{1/2} \Rightarrow \frac{dHe^3}{dt} = \lambda_{33}H^2\left[\left(\frac{He^3}{H}\right)^2_{equil} - \left(\frac{He^3}{H}\right)^2\right]$$

The destruction of He^3 is predominantly by interactions with itself after it has built up to only 1% of its equilibrium value. Only for very low He^3 abundances, He^3 is destroyed w/ D_J although it is created much faster by (A+B) in the PPI chain.

$$(A+B) \overset{eq.}{\Longleftrightarrow} 3H \to He^3 + \gamma$$

w/ energy liberated
6.936 MeV − 0.263 MeV
carried off by νs.

$$\Rightarrow \rho\epsilon(3H \to He^3) = 1.069\times10^{-5}\, r_{pp}\ \frac{erg}{cm^3 s}$$

(C) liberated 12.858 MeV of kinetic energy.

$$\Rightarrow \rho\epsilon_{PPI} = 1.069\times10^{-5}\, r_{pp} + 2.06\times10^{-5}\, r_{33}\ erg/cm^3/s$$

when He^3 hasn't reached its equilibrium value

$$= 2.099\times10^{-5}\, r_{pp}\ erg/cm^3/s$$ at equilibrium.

At equilibrium $\epsilon_{PPI} = 2.32\times10^6 \rho X_H^2 T_6^{-2/3} e^{-33.81T_6^{-1/3}}(1+0.0123T_6^{1/3}+0.0109T_6^{2/3}+0.00095T_6)$ erg/g/sec

(30.9%) (0.1% in the Sun)

PP II & PP III chains.

(A) $H^1 + H^1 \to D^2 + e^+ + \nu_e$

(B) $D^2 + H^1 \to He^3 + \gamma$

He^4 serves as a catalyst

(C II/III) $He^3 + He^4 \to Be^7 + \gamma$

(D II) $Be^7 + e^- \to Li^7 + \nu$ (0.8 MeV)

(E II) $Li^7 + H^1 \to He^4 + He^4$

Li^7 quickly destroyed by this reaction

PP III

(D III) $Be^7 + H^1 \to B^8 + \gamma$

(E III) $B^8 \to Be^8 + e^+ + \nu_e$ (β^+ decay)

ν_e (7.2 MeV) average energy

(F III) $Be^8 \to 2He^4$

Be^8 unstable

NOTE: in stars there is no free ns ⇒ $p+n \to {}^2D$ not allowed (only in cosmo nucleosynthesis)

⇒ Same total energy released, but different amount carried away by the νs.

NOTE: For ppII & ppIII, He^4 is produced with only one p-p reaction (rather than two for the ppI) 18

⇒ rate of production of He^4 is twice as much for ppI (r_{pp} rather than $\frac{1}{2} r_{pp}$).

⇒ increase energy generation (up to a factor of 2 minus neutrino loss)

NOTE: ppI, II, III will occur simultaneously & details depend on T, ρ, & composition.

NOTE: Neutrino losses differ markedly

ppI : $\dfrac{2 \times 0.263}{26.73} = 2\%$

ppII : $\dfrac{0.263 + 0.80}{26.73} = 4\%$

ppIII : $\dfrac{0.263 + 7.2}{26.73} = 27.9\%$

⇒ Total rate of energy liberation

$0.98\, F_{ppI} + 0.96\, F_{ppII} + 0.721\, F_{ppIII}$

fraction of He^4 produced by the ppI ...

$\phi(\alpha) = f(T, X/Y)$

$$\epsilon_{pp} = \frac{\epsilon_{ppI}}{0.98}\, \phi(\alpha) \left(0.98\, F_{ppI} + 0.96\, F_{ppII} + 0.721\, F_{ppIII}\right)$$

- at $T_6 \lesssim 14$, He^4 is produced predominantly by ppI
- at $T_6 \sim 14$, ppII takes over from ppI
- at $T_6 \sim 23$, ppIII takes over from ppII.

$(T_{\odot c} \sim 16 \times 10^6\, K)$ ⟶ ppII

NOTE: $B^8 \rightarrow Be^8 + e^+ + \nu_e$ (19)

is the major source of solar neutrinos of sufficiently high energy to be absorbed efficiently

$Be^7 + e^- \rightarrow Li^7 + \nu_e$

$\phi_\nu(Be^7) = (1.2 \pm 0.5)\, 10^{10}\ \#\ cm^{-2}\, s^{-1}$ neutrino flux

$\phi_\nu(B^8) = (2.25 \pm 1)\, 10^7\ \#\ cm^{-2}\, s^{-1}$

$\phi_\nu(Be^7) \simeq 500\, \phi_\nu(B^8)$ but those from B^8 more capable of producing detection on Earth experiments

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CNO cycle

With the exception of pop III stars (only H & He) & pop II objects (very little else other than H & He), most stars have formed from gas w/ mixture of heavier elements ⇒ CNO cycle

CN cycle: $\boxed{C^{12}} + H^1 \rightarrow N^{13} + \gamma$

$N^{13} \rightarrow C^{13} + e^+ + \nu_e$ (0.71 MeV) ↔ $\tau = 870$ sec

$C^{13} + H^1 \rightarrow N^{14} + \gamma$

2 weak interactions decays

$N^{14} + H^1 \rightarrow O^{15} + \gamma$

$O^{15} \rightarrow N^{15} + e^+ + \nu_e$ (1 MeV) ↔ $\tau = 178$ sec

$N^{15} + H^1 \rightarrow \boxed{C^{12}} + He^4$

⇒ eg. $C^{12} + 4H \rightarrow C^{12} + He^4 + 2e^+ + 2\nu$

w/ C^{12} acting as catalyst, & the cycle occurs w/ any of the four nuclei $C^{12}, C^{13}, N^{14}, N^{15}$ as catalysts

For every 10^4 cycles, I have 4 cycles in which instead of the last reaction I have

$N^{15} + H^1 \rightarrow O^{16} + \gamma$

$O^{16} + H^1 \rightarrow F^{17} + \gamma$

$F^{17} \rightarrow O^{17} + e^+ + \nu_e$ ↔ $\tau = 95$ sec (0.94 MeV)

$O^{17} + H^1 \rightarrow N^{14} + He^4$ only resonant reaction

⇒ There are really two cycles, the CNO bi-cycle.

The lifetime in years of a non-resonant reaction against protons is

$$\frac{1}{\tau_p(2)} = 2.45 \times 10^{16}\, \rho X_H f S_0 \left[\frac{(A_2+1) Z_2}{A_2}\right]^{1/3} T_6^{-2/3}\left(1+\frac{5}{12 B T_6^{1/3}}\right) e^{-BT_6^{-1/3}}\ \mathrm{yr}^{-1}$$

f: electron-screening factor

⇒ lifetimes $\propto \frac{1}{X_H}$ — hydrogen abundance

⇒ $\tau \rho X_H$ = function of only temperature

Since $\rho X_H \simeq 100\ \mathrm{g/cm^3}$ at the centers of MS stars

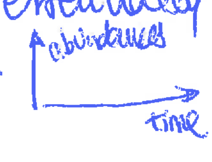
⇒ $\lg \frac{\rho X_H}{100} \tau_p$ is the typically calculated quantity (see Table of $\lg \frac{\rho X_H}{100} \tau_p$ vs T_6)

At $T_6 = 25$, characteristic of most CNO burning temperatures in the upper main sequence, the sequence of lifetimes in increasing order is
$\tau_p(N^{15})$, $\tau_p(C^{13})$, $\tau_p(C^{12})$, $\tau_p(O^{17})$, $\tau_p(N^{14})$, $\tau_p(O^{16})$
ie: the fastest-burning species is N^{15}, & the slowest O^{16}.

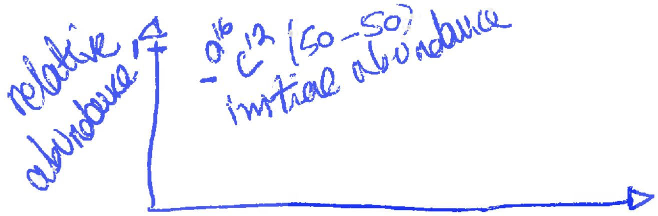
We then write the differential equations for the abundances of the CNO nuclei...

NOTE: The solar abundance ratio is $C^{12} : N^{14} : O^{16} \simeq 3.2 : 1 : 4.7$ by #
& much smaller amounts of the other nuclei.
These ratios are fairly representative of the entire pop I stars. C^{13}, N^{15}, & O^{17} are very little abundant in the interstellar gas ⇒ they are not significant seed nuclei for the CNO cycle. The major seed nuclei are C^{12} & O^{16}, w/ N^{14} a distant third.

The CNO bicycle, if it has time to achieve equilibrium, essentially converts all the CNO nuclei to N^{14} (see Fig.



$T_6 = 20$ / CNO cycle



↳ O^{16} has not yet achieved equilibrium after even 10^3 proton captures, i.e. 3×10^8 yrs.

➡ For pop I stars, there may well be insufficient H ever to drive oxygen to equilibrium. The H is consumed so rapidly by the CNO cycle that it vanishes before the Oxygen can be depleted.

The rate of energy generation is the sum over all reactions of the difference of the energy release & the reaction rate & the neutrino loss.

$$\epsilon_{CNO} = 8.67 \times 10^{27} \rho X X_{CNO} C_{CNO} T_6^{-2/3} e^{-152.28 T_6^{-1/3}} \quad \frac{erg}{s\,g}$$

w/in 20% uncertainty

total mass fraction of C, N, O

correction term

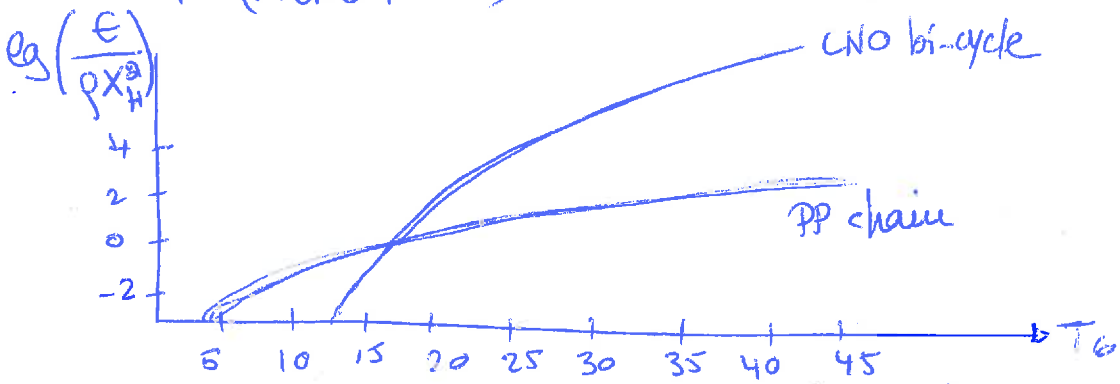
$$\simeq \epsilon'_{0,CNO} \rho X X_{CNO} T_6^{19.9}$$

i.e.; the CNO cycle is much more strongly temperature dependent than is the PP chain (with $\epsilon_{PP} \propto T^4$)

In most hydrogen-burning stars, the PP chain & the CNO bi-cycle operate simultaneously. Which of the two dominates the energy generation depends on the relative abundance of H & CN nuclei & on the temperature.

Since $\epsilon_{pp} \propto \rho X_H^2$ $\epsilon_{CNO} \propto \rho X_H X_{CNO}$ (22)

$\Rightarrow$ $\epsilon / \rho X_H^2$ is a function of only temperature for PPI & $(X_{CNO} / X_H) \cdot f(T)$ for the CNO cycle.



For solar metallicity $X_{CNO}/X_H = 0{,}02$ (pop I stars)

$\hookrightarrow$ The CNO cycle takes over from the PP chains @ $T_6 \sim 18$

$\Rightarrow$ low-mass stars, with smaller central T, are dominated by the PP chains during the H-burning evolution, whereas more massive stars, w/ higher central T, convert H into He by the CNO cycle.

H-burning in stars is found to occur as the central energy source for main-sequence stars, and as a shell source in later stages of stellar evolution.

When H is converted into He, the mean molecular weight μ of the gas increases $\Rightarrow$ if T & ρ remain the same, the gas pressure decrease $\Rightarrow$ the star would no longer be in hydrostatic equilibrium & collapse would begin. The collapse raises both T & ρ, compensating the increase of μ. When T & ρ are high enough, the nuclei can fuse.

HELIUM BURNING

The present ratio $He^4/H \sim 0.1$ (in number) is the result of H-burning in early cosmological stages followed by ~ 13 Gyr of star formation, death, & remixing.

There are no stable nuclei with $A=5$ & $A=8$ (forbidding the fusion of two He^4 into an $A=8$ nucleus)

Since $C^{12} \leftrightarrow p_s + n_s = 12 \longleftrightarrow 3\,He^4$ $(p_s+n_s=4)$

$O^{16} \leftrightarrow p_s+n_s = 16 \longleftrightarrow 4\,He^4$

↳ these nuclei might be the results of more-than-two-body alpha-particle collisions.

3α REACTION

$\alpha \leftrightarrow He^4$

(A) $He^4 + He^4 \rightleftharpoons Be^8$: Be^8 is unstable, breaking up into 2 He^4, but the Be^8 lifetime is 2.6×10^{-16} s, i.e. much longer than the scattering of two He^4 → a small concentration of Be^8 nuclei builds up in the helium gas until the rate of break up of Be^8 is equal to its rate of formation.

NOTE: Both (A) & (B) are resonant reactions

At $T=10^8$ K & $\rho = 10^5$ g/cm^3, there is ~ 1 Be^8 nucleus for 10^9 He^4 nuclei. This is sufficient to allow a third He^4 particle to interact w/ the Be^8

(B) $Be^8 + He^4 \longrightarrow C^{12} + \gamma$.

$$\boxed{\epsilon_{3\alpha} = 5.09\times10^{11}\,\rho^2 \frac{X_\alpha^3}{T_8^3} f_{3\alpha}\, e^{-44.027/T_8}} \ \frac{\text{erg}}{\text{s g}}, \quad \text{w/ } X_\alpha = Y$$

$f_{3\alpha}$: screening factor for triple alpha process.

$$\boxed{\approx \epsilon'_{0\,3\alpha}\,\rho^2 Y f_{3\alpha} T_8^{41}}$$

@ $T=10^8$ K → very strong dependence on temperature

⇒ even a small increase in T will produce a large increase in the amount of energy generated per second.
In a stellar center supported by e^- degeneracy, the onset of the burning is accompanied by an explosive reaction, the HELIUM FLASH.

NUCLEOSYNTHESIS DURING He BURNING.

CARBON BURNING: $C^{12} + He^4 \longrightarrow O^{16} + \gamma$ (non resonant reaction)
OXIGEN BURNING: $O^{16} + He^4 \longrightarrow Ne^{20} + \gamma$ (non resonant @ $T_8 = 1$ but resonant @ $T_8 > 2.1$)

NEON BURNING: $Ne^{20} + He^4 \longrightarrow Mg^{24} + \gamma$

⋮

Continued successive α-particle captures can occur in principle, but the increasing coulomb barrier severely limits the number of α-particle captures at temperatures low enough for some He still to remain.
C^{12} & O^{16} are produced in significant amounts by stars of moderate mass. The final product is almost entirely O^{16} for $M > 10\,M_\odot$ stars.
Ne^{20} is produced appreciably only at high temperatures, ie: for massive stars $M \gg 20\,M_\odot$ (during He burning, in a star with $M = 15\,M_\odot$, the final weight fraction of Ne^{20} at the center is only 1%)

He burning happens in giant stars.

The products of nucleosynthesis during He burning is quite uncertain, & causes a corresponding uncertainty in the subsequent evolution of the star. If C^{12} is a substantial remnant, the next nuclear burning phase will be from interactions of C^{12} with itself. If little C^{12} is produced that burning phase will be omitted, & the star will progress directly from the burning to O burning.

ADVANCED BURNING STAGES

When the He burning ceases to provide sufficient power to the star, gravitational contraction begins again. From the virial theorem, T of the He-exhausted region rises during contraction, until T & ρ are high enough for the next nuclear burning stage, or e⁻ degeneracy halting the contraction.

Stars w/ $M > 0.7 M_\odot$ must contract until T is large enough for carbon to interact w/ itself, while less massive stars settle into degenerate white-dwarf configurations.

After He burning, the most abundant nuclei in the gas are expected to be C^{12} & O^{16}.

$T \sim 6-7 \times 10^8 K$

$C^{12} + C^{12} \longrightarrow$
- $Mg^{24} + \gamma$
- $Na^{23} + p$
- $Ne^{20} + \alpha$
- $Mg^{23} + n$ * (requires too much energy to be important)
- $O^{16} + 2\alpha$ *

resonant reactions

low probability compared to others

→ the direct products of C reaction are Na^{23}, Ne^{20}, ps.

Most of the protons $C^{12} + p \longrightarrow N^{13} \longrightarrow C^{13} + e^+ + \nu_e$ (beta decay)

$C^{13} + He^4 \to O^{16} + n$

→ free ps are converted in free ns, while He^4 & C^{12} are converted into O^{16}.

The ns will be captured → nucleosynthesis of heavy elements by n-capture chains.

By the end of C burning, the initial C^{12} nuclei have been converted into O^{16}, Ne^{20}, Na^{23}, Mg^{24}, & Si^{28} (through α-capture by C^{12}, O^{16}, Ne^{24} & Mg^{24} too)

$$\epsilon_c \simeq 2.6\times10^{40}\, \rho\, X^2_{C^{12}}\, \lambda_{C^{12},C^{12}}\ \frac{\text{erg}}{\text{g s}}$$

Note: T is high enough that the role of the neutrino emission has to be examined with care.

NOTE: Reactions between C^{12} & O^{16} are not important, since the larger coulomb barrier makes the rate too slow to be important at the C-burning temperatures, while C^{12} is completely exhausted when T get's large enough.

$T \sim 10^9$ K

$$O^{16}+O^{16} \rightarrow \begin{array}{l} S^{32}+\gamma \\ P^{31}+p \\ S^{31}+n \\ Si^{28}+\alpha \\ Mg^{24}+2\alpha \ * \end{array}$$

NOTE: Photo disintegration of Ne^{20} happens at the same temperatures.

The major final nucleus synthesized appears to be Si^{28}, with nuclear energy generation $\epsilon_O \approx 2\times10^{10}\, \rho\, X^2_{O^{16}}\, \lambda_{O^{16},O^{16}}\ \frac{\text{erg}}{\text{s g}}$

Neutrino losses will be high during O-burning phase, so much that most of the generated energy is radiated as L_ν. Because of this, O burning happens at $T > 10^9$ K to replace the heavy neutrino losses.

PHOTODISINTEGRATION

At the temperatures during C & O burning, disintegration of nuclei by the thermal photon bath becomes important. This is similar to ionization of atoms w/ $T \sim 10^4$ K, but with the difference of nuclear force as the binding energy.

The first photodisintegration of importance is N^{13}, formed through $C^{12} + p \rightarrow N^{13}$. For $T > 7.5 \times 10^8$ K, N^{13} photo-disintegration is faster than it's beta decay
$\Rightarrow$ C^{13} concentration (from $N^{13} \rightarrow C^{13} + e^+ + \nu_e$) is sharply reduced, as is its effectiveness as a source of free n_s.

During O burning, ie: $T > 10^9$ K, $\gamma + Ne^{20} \rightarrow O^{16} + He^4$; at $T > 1.3 \times 10^9$ K, the rate of Ne^{20} photo-disintegration becomes greater than the rate of Ne^{20} production ($O^{16} + He^4 \rightarrow Ne^{20} + \gamma$)
$\Rightarrow$ Ne^{20} is effectively disintegrated. The liberated He^4 is likely captured by remaining Ne^{20}

$$\Rightarrow \quad 2Ne^{20} \rightarrow O^{16} + Mg^{24} + 4.583 \text{ MeV} \quad \text{(net effect)}$$

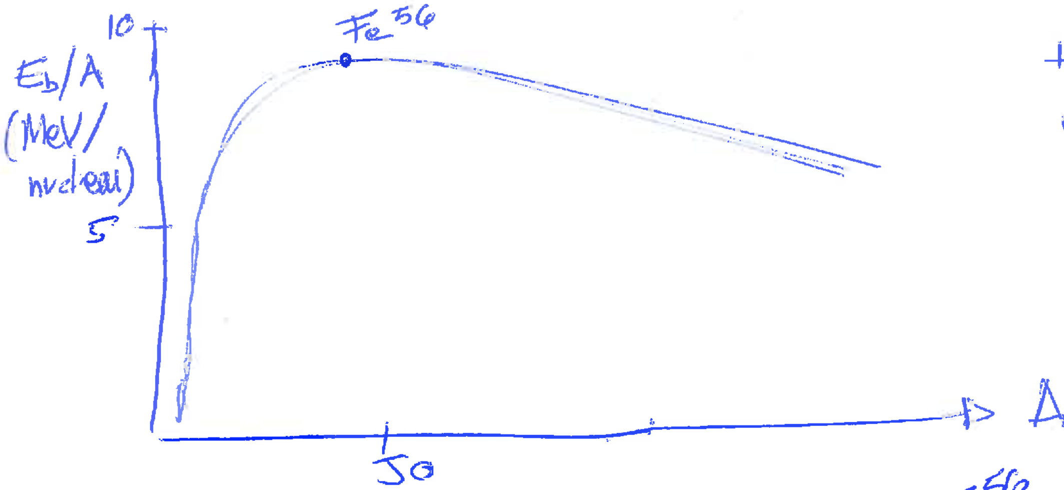
$$\epsilon_{Ne^{20}} = 10^{17.34} X_{Ne^{20}} \lambda_\gamma (Ne^{20}) \ \frac{\text{erg}}{\text{s g}}$$

(what Ne^{20} is observed in nature was produced in C burning in a shell surrounding an even more advanced core)

At the conclusion of O burning, the gas continues to heat up. Subsequent nuclear reactions are primarily of a rearrangement type, ie: a particle is photo ejected from one nucleus & capture by another $\Rightarrow$ converting nuclear particles to their most stable forms

$$E_b = \Delta m c^2 = \left[Z m_p + (A - Z) m_H - m_{\text{nucleus}} \right] c^2$$

binding energy $\longrightarrow$ E_b / A binding energy per nuclear



He^4 & O^{16} are unusually stable nuclei relative to others of similar mass. (most abundant elements in the universe, along w/ H^1)

There is a maximum in E_b/A at Fe^{56}.
⇒ the rearrangement attempts to convert O-burning remnants into nuclei in the vicinity of Fe^{56}.
The timescale of this is controlled primarily by the photodisintegration of Si^{32} ...

The most abundant nuclear species in the cosmos are H^1, He^4, O^{16}, C^{12}, Ne^{20}, N^{14}, M^{24}, Si^{28} & Fe^{26}, the result of the dominant nuclear reaction processes occurring in stars.