#### ASTRONOMY 121 - Fall 2023 Homework Assignment #2 DUE by Monday, October 23, 2023 400 points

# NOTE: Be sure to show all calculations clearly, and box/circle/highlight the final result.

## Problem 1. [50 points]

Using  $n(E) = \frac{g(E)}{e^{E/KT}-1}$  and  $p_{\nu} = h\nu/c$ , derive  $u(\nu)d\nu = E(\nu)n(\nu)d\nu$ , i.e., the energy density of photons of frequency  $\nu$  in the range  $d\nu$  in thermal equilibrium. Calculate the same, but expressed in terms of wavelength rather than frequency. Plot the energy density as a function of both frequency and wavelength (on two different plots).

#### Problem 2. [30 points]

From Problem 1., calculate the total energy density of the photon field in thermodynamic equilibrium, and comment on the Stefan-Boltzmann's law. NOTE:  $\int x^3 dx/(e^x - 1) = \frac{\pi^4}{15}$ 

#### Problem 3. [30 points]

The radius of the Sun is 16 arcmin of arc as seen from the Earth. The radiant flux at the top of the Earth's atmosphere is  $1.338 \times 10^6$  erg s<sup>-1</sup> cm<sup>-2</sup> (a.k.a., the solar constant). Calculate the effective temperature of the Sun using only the data given. NOTE: from the solar constant and the distance of the Sun, the solar luminosity is calculated.

#### Problem 4. [50 points]

Calculate the position of the maximum of the blackbody spectrum in the form  $\lambda_{max}T = constant$ . The wavelength of maximum intensity in the solar spectrum is 5000Å. Assuming the Sun radiates as a blackbody, compute its surface temperature.

#### Problem 5. [50 points]

The mass of the Hydrogen atom is equal to 1.007825 amu, and the mass of the Helium atom is 4.002603 amu. Calculate the energy released by nuclear binding when four Hydrogen atoms combine to form one Helium atom. How much energy is liberated when 1 g of Hydrogen is converted to 1 g of Helium? Assume 1 amu =  $1.66053886 \times 10^{-24}$  g.

#### Problem 6. [30 points]

If an O5 star were at 35,000 K blackbody, where would the peak in the continuous spectrum be?

# Problem 7. [50 points]

Calculate the radii of B0, A5, and M0 main-sequence stars, given their absolute bolometric magnitudes  $M_{bol} = -6.7$ , 1.7, and 7.6, respectively, and effective surface temperatures of  $T_{eff} = 21000$ , 8100, and 3300 K, respectively. Compare to the solar radius, i.e., express the radii in units of solar radius.

# Problem 8. [30 points]

Calculate the radius of a white dwarf having a luminosity  $L=10^{-2} L_{\odot}$  and an effective temperature  $T_{eff}=10000$  K.

#### Problem 9. [30 points]

If the apparent magnitude of the  $M_V=0$  RR Lyrae variables in a given cluster is  $m_V=15$ , what is the distance to the cluster?

# Problem 10. [50 points]

Estimate the approximate main-sequence lifetime of a star with absolute visual magnitude  $M_{Bol}=0$  (A0 star),  $M_{Bol}=-10.6$  (O5 star), and  $M_{Bol}=7.6$  (M0 star). Plot the estimated main-sequence lifetime of a star as a function of its mass (from 0.1  $M_{\odot}$  to 100  $M_{\odot}$ ).