

Baryon production from embedded metastable strings

Johanna Karouby

Massachusetts Institute of Technology

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- 2 What are skyrmions?
- 3 Our model : the embedded pion string
- 4 Baryon production

REFERENCES

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- 2 Johanna Karouby
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“Effects of a Thermal Bath of Photons on Embedded String
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Full Skyrmion in 3 spatial dimension

Skyrmions are topological defects such that $\Pi_3(\mathcal{M}) \neq 1$

Textures : delocalized topological defects, unstable to collapse

Skyrmions=global texture with VEV everywhere, pure gradient energy

They can also describe baryons \Rightarrow skyrme model

- \Rightarrow symmetry of the vacuum manifold = $O(4)$
- \Rightarrow vacuum manifold is a 3-sphere : $\mathcal{M} = S^3$
- \Rightarrow topologically unstable strings since $\Pi_1(S^3) = 1$.
- But $\Pi_3(S^3) \neq 1 \Rightarrow$ textures/skyrmions .

Embedded String Stability

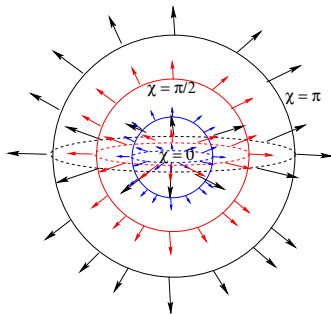
We study the effective potential of the low-energy effective sigma model of QCD in the presence of a thermal bath of photons.

Goal : Stabilize cosmic string by a **thermal bath of photons**

Motivation : Stabilized embedded defects \implies **applications in cosmology**.

- Explanation for the **origin** and **coherence of cosmological magnetic fields on galaxy scale**.
- **CMB** : temperature fluctuations, non-gaussianity.
- very early universe \implies consider **plasma**.
- can contribute to **structure formation**.
- may play a role in **baryogenesis**

Full Skyrmion in 3 spatial dimension

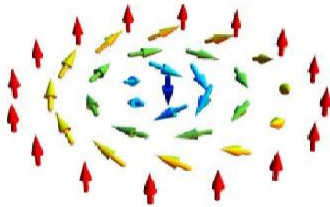


Variation of the chiral field is shown on a ball B^3 .

- 3 ANGLES: $\chi : 0 \Rightarrow \pi$ azimuthal $\phi : 0 \Rightarrow 2\pi$ polar $\theta : 0 \Rightarrow \pi$
- χ varies from 0 (at the center) to $\chi = \pi$ for the boundary of the ball B^3
- \rightarrow *Compactification* of B^3 in the physical space to S^3 .

Full Skyrmion in 2 spatial dimensions

Winding number one Skyrmion configuration in 2d.



South pole mapped to the center of the disk

North pole mapped to the boundary of the disk compactifying it to a two-sphere S^2

The three angles θ, ϕ, χ are related to the 4 scalar fields :

$$\begin{aligned} \pi^0 &= \cos\chi & \text{and} & & \sigma &= \sin\chi \cos\theta \\ \pi^1 &= \sin\chi \sin\theta \cos\phi & \text{and} & & \pi^2 &= \sin\chi \sin\theta \sin\phi \end{aligned} \quad (1)$$

The Pion String

The linear sigma model : simple toy model.

- **Symmetry breaking** occurs when the sigma field takes on its **vacuum expectation value**
- Gives rise to a triplet of **massless pions** $\vec{\pi} = (\pi^0, \pi^+, \pi^-)$.

Lagrangian :

$$\mathcal{L}_0 = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - \eta^2)^2,$$

- \implies symmetry of the vacuum manifold = $O(4)$
- \implies vacuum manifold is a 3-sphere : $\mathcal{M} = S^3$
- \implies topologically unstable strings since $\Pi_1(S^3) = 1$.
- Effectively reducing the vacuum manifold to $S^1 \implies$ strings.

The skyrme model

Skyrme model : Solitons in the linear sigma model can be interpreted as the baryons of QCD

⇒ Baryons productions from a pion string by computing the instanton production.

⇒ Baryon number is a topological quantum number $\Pi_3(S^3) = \mathbb{Z} \Rightarrow$ baryon number can only take on integer values

Effective Lagrangian

Electric charge \implies charged pions fields are coupled to electromagnetism
 \implies Lagrangian with covariant derivatives.

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + D_\mu^+ \pi^+ D^{\mu-} \pi^- - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + V_0,$$

where $D_\mu^+ = \partial_\mu + ieA_\mu$, $D_\mu^- = \partial_\mu - ieA_\mu$.

- 2 complex scalar fields : $\pi_c = \pi^1 + i\pi^2$ and $\phi = \sigma + i\pi_0$.
- Partition function of the system, $Z[T]$

$$Z[T] = \int \mathcal{D}\Phi \mathcal{D}\pi_c \mathcal{D}A^\mu e^{-S[A^\mu, \Phi, \pi_c]} = \int \mathcal{D}\Phi \mathcal{D}\pi_c e^{-S[\Phi, \pi_c]} e^{\frac{V_{\text{eff}}(\Phi, \pi_c) V}{T}}$$

At high-temperature, we can truncate the series above

$$V_{\text{eff}}(\Phi, \pi_c, T) = \frac{\lambda}{4} (|\Phi|^2 + |\pi_c|^2 - \eta^2)^2 - \frac{\pi^2 T^4}{45} + \frac{e^2 |\pi_c|^2 T^2}{12} - \frac{e^3 |\pi_c|^3}{6\pi} T$$

The effective vacuum manifold now reduced :

$\mathcal{M} = S^1 \implies$ stable string.

Quantum tunneling

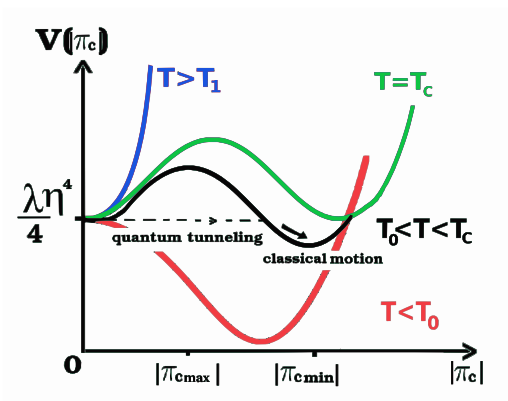


Figure: Finite temperature effective potential in the core of the string.

$$V_{\text{eff}}(\phi, \pi_c, T) \simeq \frac{\lambda}{4} (|\phi|^2 + |\pi_c|^2 - \eta^2)^2 + \frac{e^2 |\pi_c|^2}{12} T^2 - \frac{e^3 |\pi_c|^3}{6\pi} T \quad (2)$$

- Equation of motion for $r \sim R$:

$$\frac{d^2 \pi_c}{dr^2} + \frac{3}{r} \frac{d\pi_c}{dr} = V'(\pi_c)$$

- $$S_{sphere} = \pi^2 \int r^3 dr \left[\frac{1}{2} \left(\frac{\partial \pi_c}{\partial r} \right)^2 + V(\pi_c) \right]$$

$$= -\frac{\pi^2}{2} R^4 \Delta V + 2\pi^2 R^3 S_1$$

- Extremizing S_{sphere} : $\frac{\partial S_E}{\partial R} = 0$

$$R(T) = \frac{3S_1}{\Delta V} = \sqrt{\frac{3}{2e}} \frac{1}{\epsilon \sqrt{T^2 - T_0^2}}$$

- T dependent decay rate per unit volume:

$$\frac{\Gamma_{sphere}}{V} \sim P_4 \exp \left[-\pi^2 \frac{1}{48 \lambda \left(\sqrt{\frac{e^4}{3\pi^2 \lambda} \frac{T^2}{T^2 - T_0^2}} - 1 \right)^3} \right] \quad (3)$$

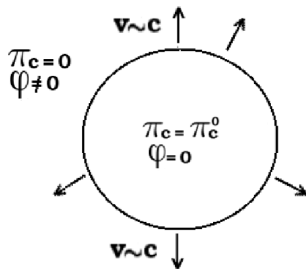
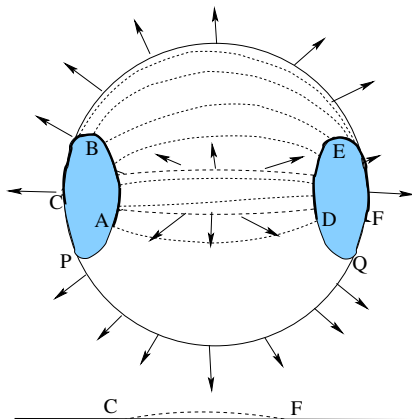


Figure: Bubble Instanton in the core of the string

Consider 2 neighboring tunneling regions on the string \Rightarrow opposite patches on the vacuum manifold.

- small patches on S^3 since only such instantons dominate the tunneling process.
- size of patch depends on T
- strong fluctuations \Rightarrow large patches



Skyrmion production

Instanton meeting with opposite patches in the vacuum manifold

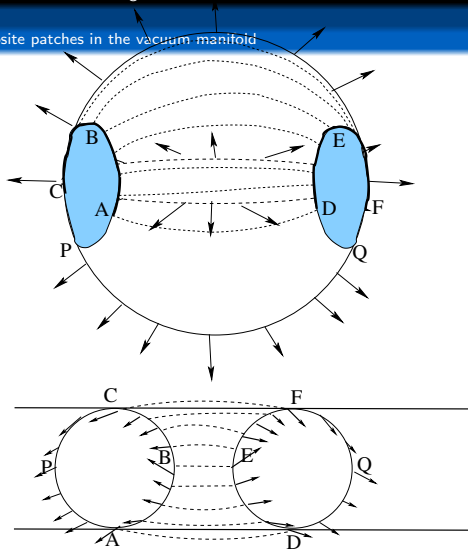


Figure: Top 2 patches ABCP and DEFQ on the vacuum manifold $S^2 \Rightarrow$
 Bottom Region in-between the two patches \Rightarrow sensitive to the parts of the
 boundaries of the two patches
 Intermediate region \Rightarrow dashed lines shown on the S^2

Skyrmion formation

Argued in Phys. Rev. **D 43**, 1047 (1991), A.M. Srivastava:

- Skyrmions never form with full winding number in phase transition \Rightarrow partial winding skyrmions
- If a partial winding configuration covers more than 50% of the vacuum manifold $S_3 \Rightarrow$ evolve to a complete integer winding Skyrmion \Rightarrow **baryons**
- If the configuration covers less than 50% of $S_3 \Rightarrow$ it will shrink down and should decay in perturbative **meson excitations**.
- Seen in simulations of the formation of textures in the context of early Universe

Estimates of baryon production by string decay

The number of instanton formed per unit volume per unit time is equal to the decay rate Γ .

Assume $\Gamma > H$ then within a Hubble time the number of bubbles per unit length is

$$n(t) = \pi R_0^2 \frac{\Gamma}{V} H(t)^{-1} \text{ w here } R_0 = \frac{1}{\sqrt{\lambda\eta}} \sim r_{string}$$

- on S_2 : average size of patch on S_2 : $\frac{1}{8} \times \text{Area}$
- $\Rightarrow P(\text{opposite patches}) = P(\text{full windings skyrmion}) = \frac{1}{8}$
- on S_3 : $P(\text{opposite patches}) = \frac{1}{16}$

\Rightarrow number of baryon-anti-baryon produced per unit length : $n_b = \frac{n}{16}$.

The largest number of bubbles nucleated on the string during Hubble time can be simply estimated

$$n \simeq (2R(T))^{-1} \simeq \sqrt{\frac{e}{6}} \epsilon(T) \sqrt{T^2 - T_0^2} \quad (4)$$

Conclusion : Embedded String Stability

We have studied the effective potential of embedded strings of a thermal bath of photons and studied baryon production.

- Studied of 1st order phase transition for strings : eg superconducting strings
- **The plasma effects** lift the potential in direction of the charged pion fields.
- This lead to an effective vacuum manifold which admits **cosmic string solutions**, the pion strings.
- **Topological defects embedded defects** of the full theory with the property that they are **stabilized in the early Universe** for the right values of the parameters.
- Decay of the string naturally leads to **baryon production** and to **meson excitations**
- Our arguments are general and apply to many theories beyond the Standard Model.
- Possibilities to do same mechanism with **domain walls**