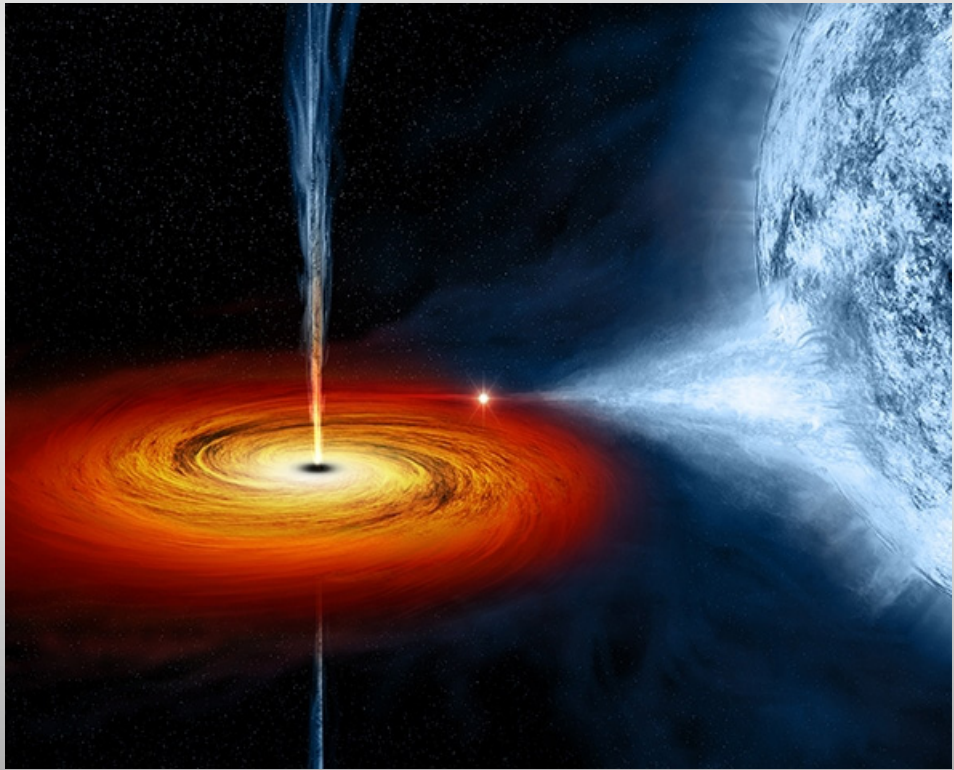
Twisting Strings and Black Holes



Ruth Gregory

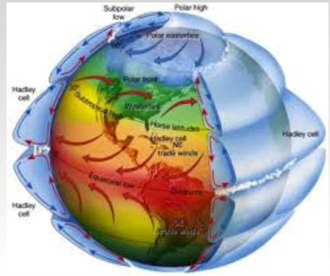
5/2/14

ASU/Tufts

David Kubiznak, and Danielle Wills; +David Gustainis, Rob Mann

Rationale

- Black holes give non-perturbative strong gravity regimes



- Controversy on nature of horizon
- Vortices in field theory : non-perturbative, semi-classical
- Gravity in action – strings and long hair
- Rotation – a new twist!

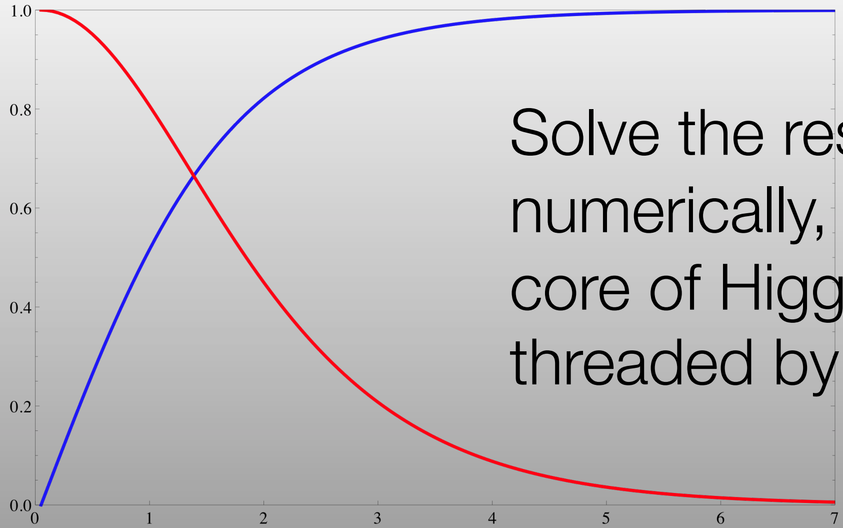
Nielsen-Olesen Vortex

The Higgs phase winds around the string, so rewrite physical degrees of freedom and rescale so that string has unit width.

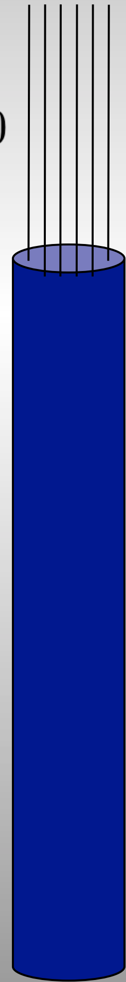
$$\Phi = \eta X(R) e^{i\theta}$$
$$A_\mu = \frac{1}{e}\left(P(R) - 1\right)\partial_\mu \theta$$

$$\longrightarrow$$

$$X'' + \frac{X'}{R} + X P^2 R^2 + \frac{X}{2}(X^2 - 1) = 0$$
$$P'' - \frac{P'}{R} = \frac{X^2 P}{\beta}$$

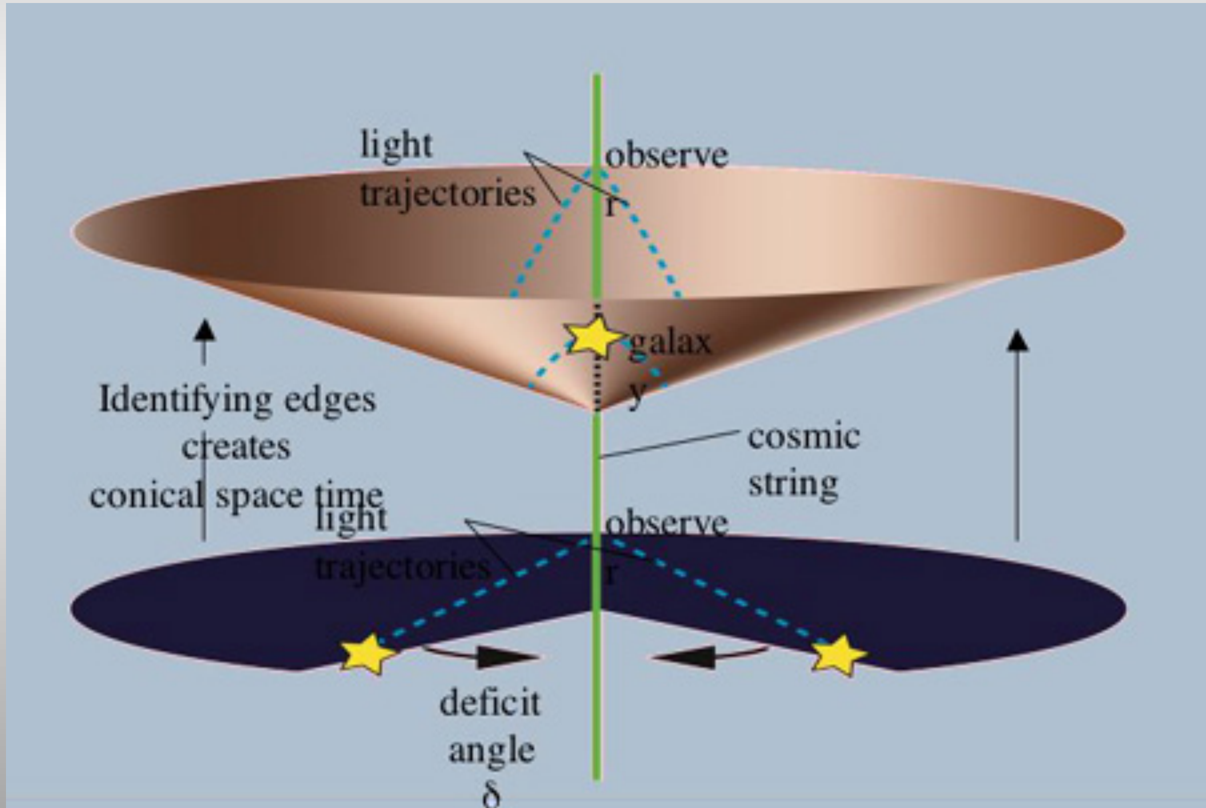


Solve the resulting eom's numerically, to find well resolved core of Higgs condensate threaded by "magnetic" flux.



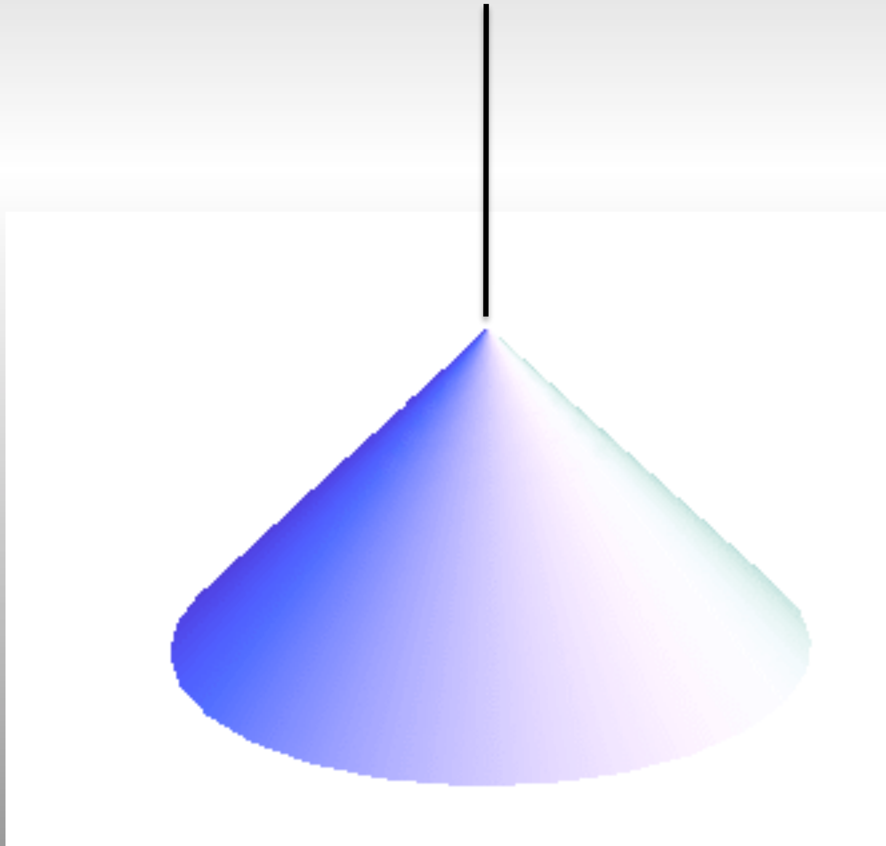
VACUUM GRAVITY

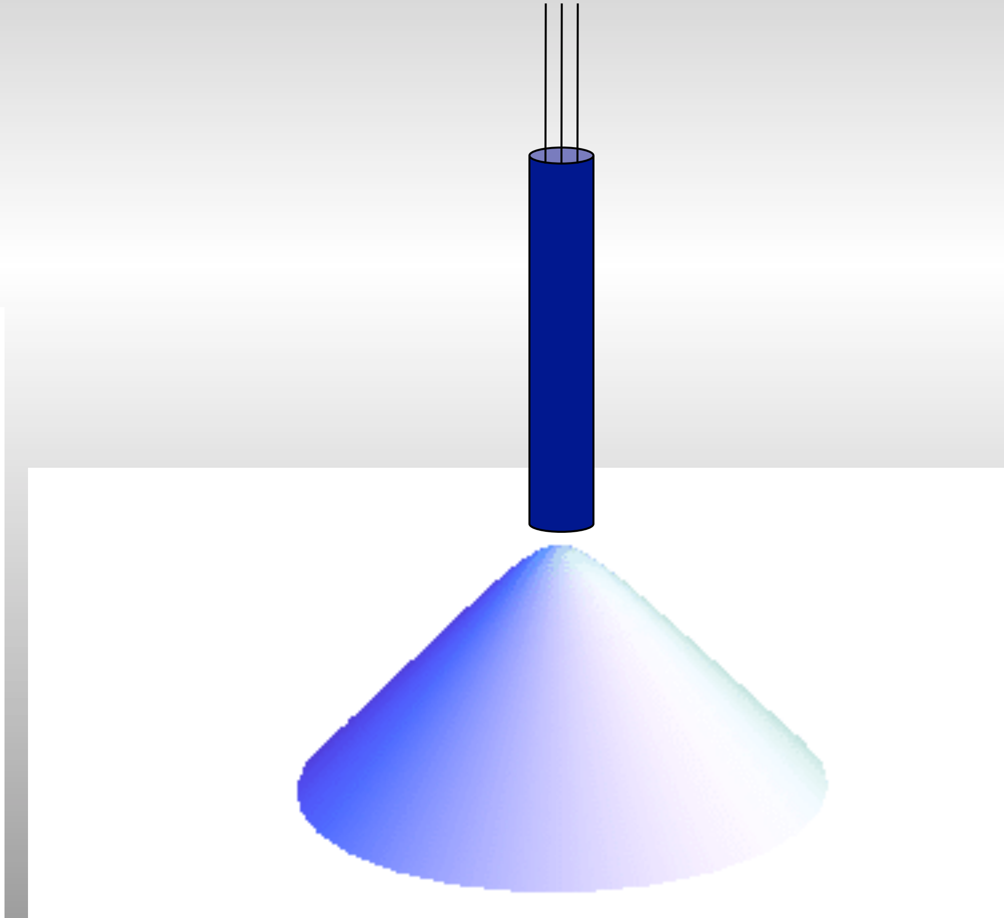
The conical deficit is a vacuum solution in gravity – it gives lensing, but no curvature (away from the string!)



VILENKIN (1981)

The actual solution is smoothed out in the core, and forms a snub-nosed cone.



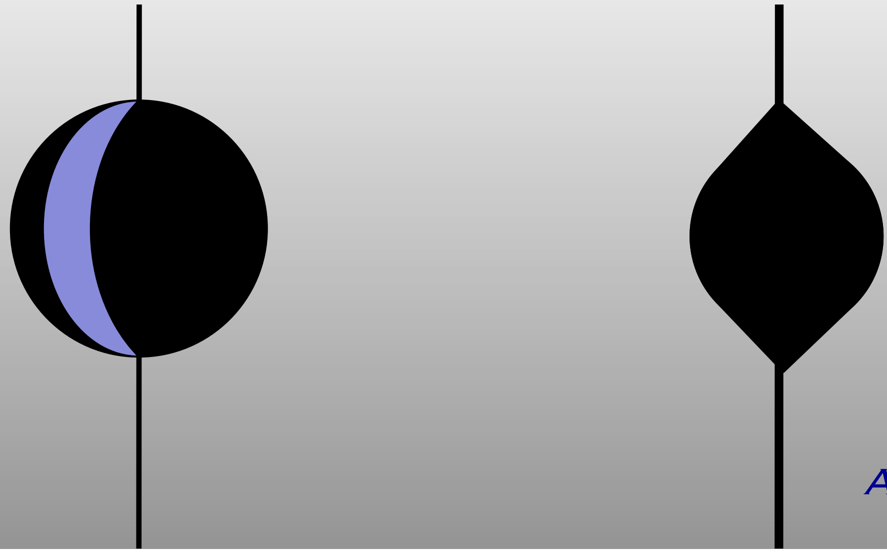


Garfinkle/Gregory (1985/7)

Black Holes & Strings

Schwarzschild is a vacuum nonperturbative solution. Normally we cannot superpose solutions in gravity, but in this case…… we can get a black hole with a conical deficit – what is this physically? Is it possible?

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2\left[d\theta^2 + (1-\delta)^2 \sin^2\theta \, d\phi^2\right]$$



Deficit Angle

Aryal, Ford, Vilenkin (1986)

Black Holes & Vortices

- Prejudice from “no hair” theorems – fields on horizon?
- ODE’s → PDE’s
- Event horizon a singular place
- Has to be done numerically, ...SO......

o First approach on the Schwarzschild background,
o Try guessing a rough “analytic” approximation
o Find numerical solution
o Allow the vortex to back-react

A Achucarro, RG + K Kuijken, PRD52 (1995)

First, Guess:

$$\left(1-\frac{2GM}{r}\right)X_{rr}+\frac{r-GM}{r^2}X_r+\frac{X_{\theta\theta}}{r^2}+\frac{\cot\theta}{r^2}X_\theta+\frac{1}{2}X\left(1-X^2\right)=\frac{XP^2}{r^2}$$

$$\left(1-\frac{2GM}{r}\right)P_{rr}+\frac{2GM}{r^2}P_r+\frac{P_{\theta\theta}}{r^2}-\frac{\cot\theta}{r^2}P_\theta=\frac{X^2P}{\beta}$$

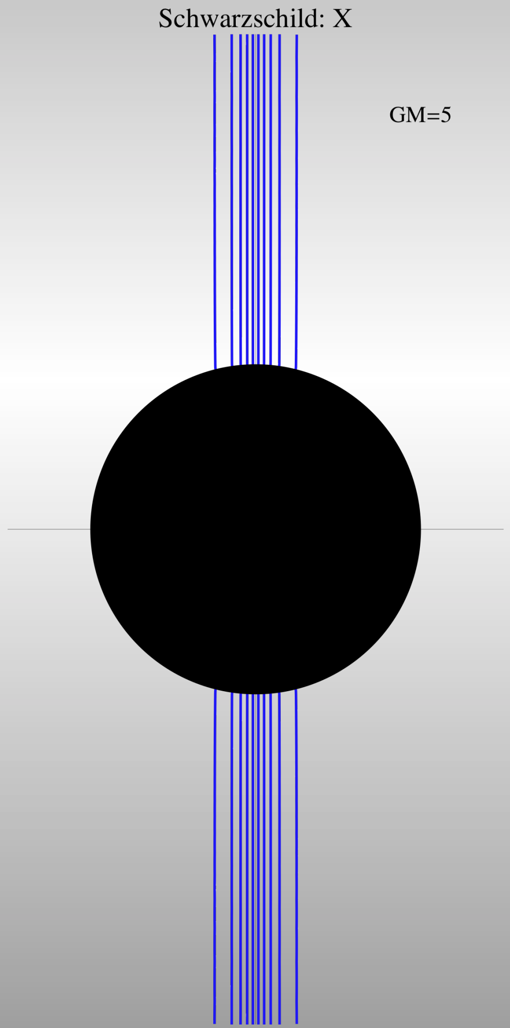
Roughly solved by:

$$X=X_{NO}\left(r\sin\theta\right)$$
$$P=P_{NO}\left(r\sin\theta\right)$$

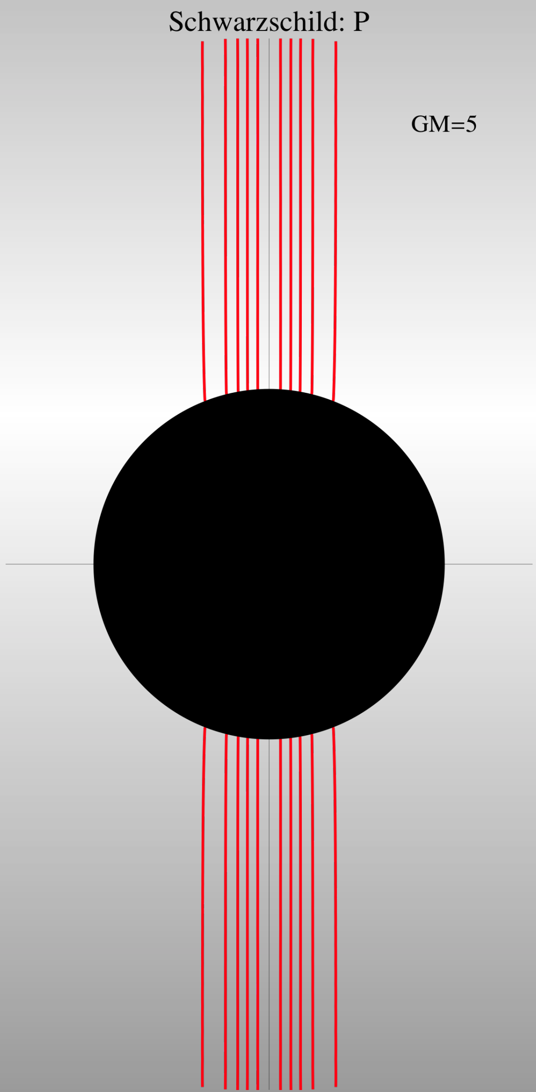
- The Nielsen-Olesen functions

$$X''+\frac{X'}{R}+\frac{1}{2}X\left(1-X^2\right)=\frac{XP^2}{R^2} \quad ; \quad P''-\frac{P'}{R}=\frac{X^2P}{\beta}$$

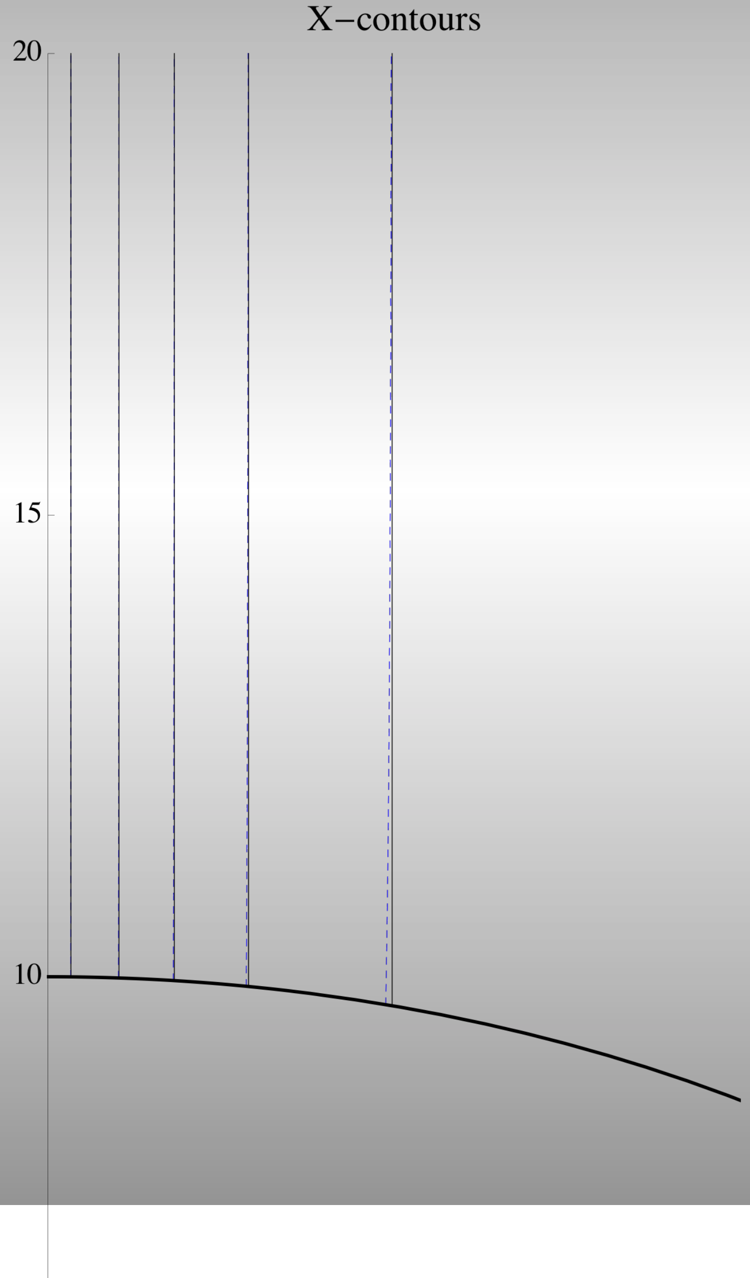
Black holes have long hair!

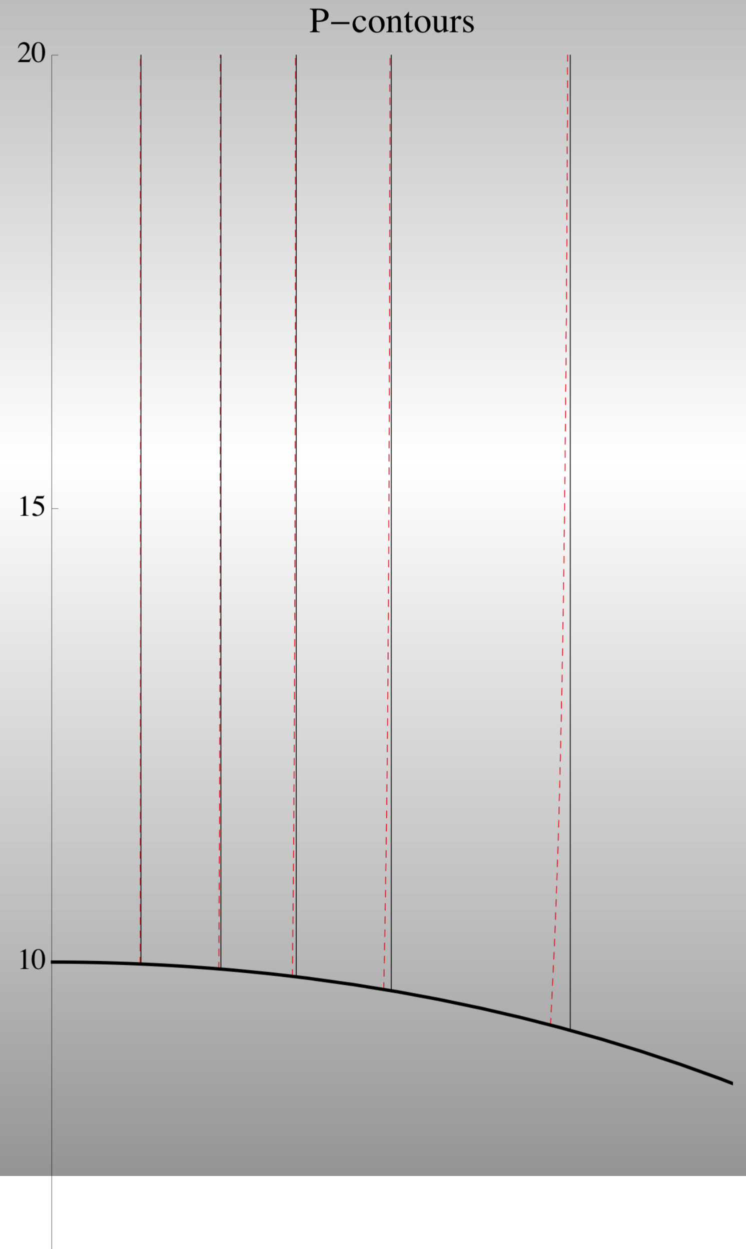


Numerically integrate to check. Elliptic PDE, so use gradient flow.
Wrinkle is horizon – which must be updated as well. This is parabolic system, so still well posed.



Check against guess





Add Gravity

Can show the solution is indeed a combination of the black hole and the smooth snub-conical gravitating vortex.

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2\left[d\theta^2 + (1-\delta)^2 \sin^2\theta \, d\phi^2\right]$$

$$M_{ADM} = (1-\delta)M$$

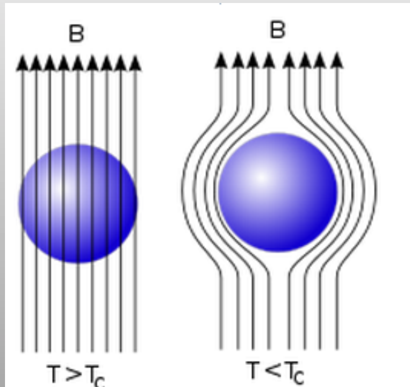
$$Area = 16\pi(1-\delta)M^2$$

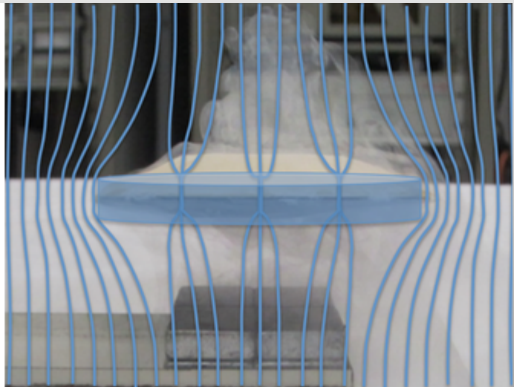
$$\delta(R) = G\mu\left(-\int_0^R R\mathrm{E}\,dR + \frac{1}{R}\int_0^R R^2\mathrm{E}\,dR\right)$$

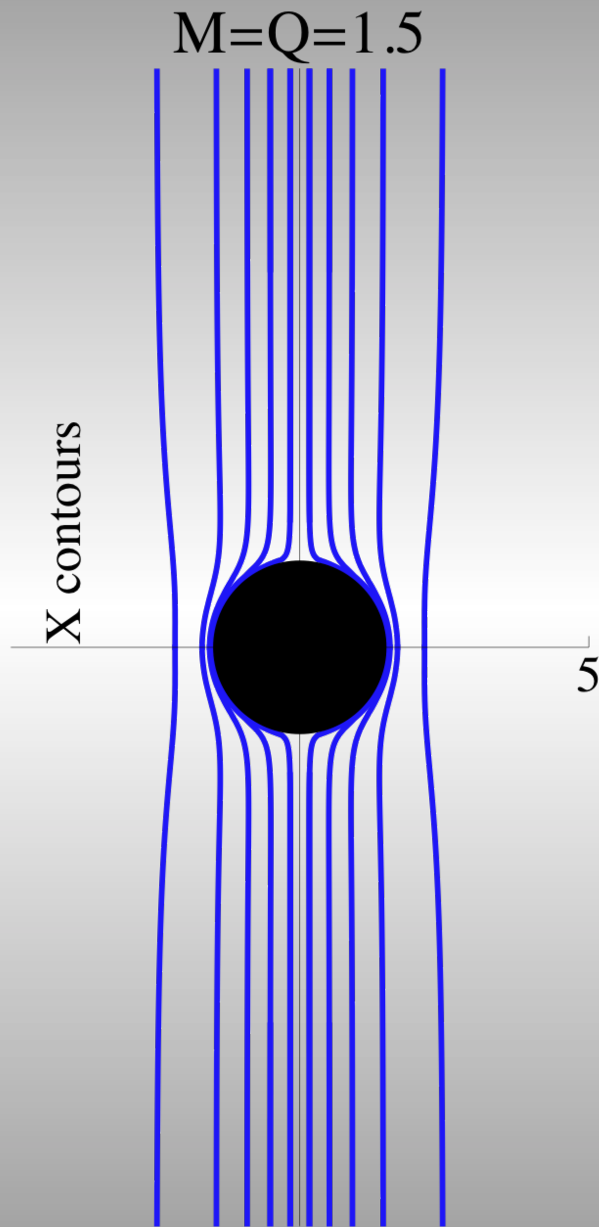
RG, Hindmarsh (1995)

Add Charge

Adding charge does not change the picture much, except for extremal (m=q) black holes, then a Meissner effect is observed for small black holes.

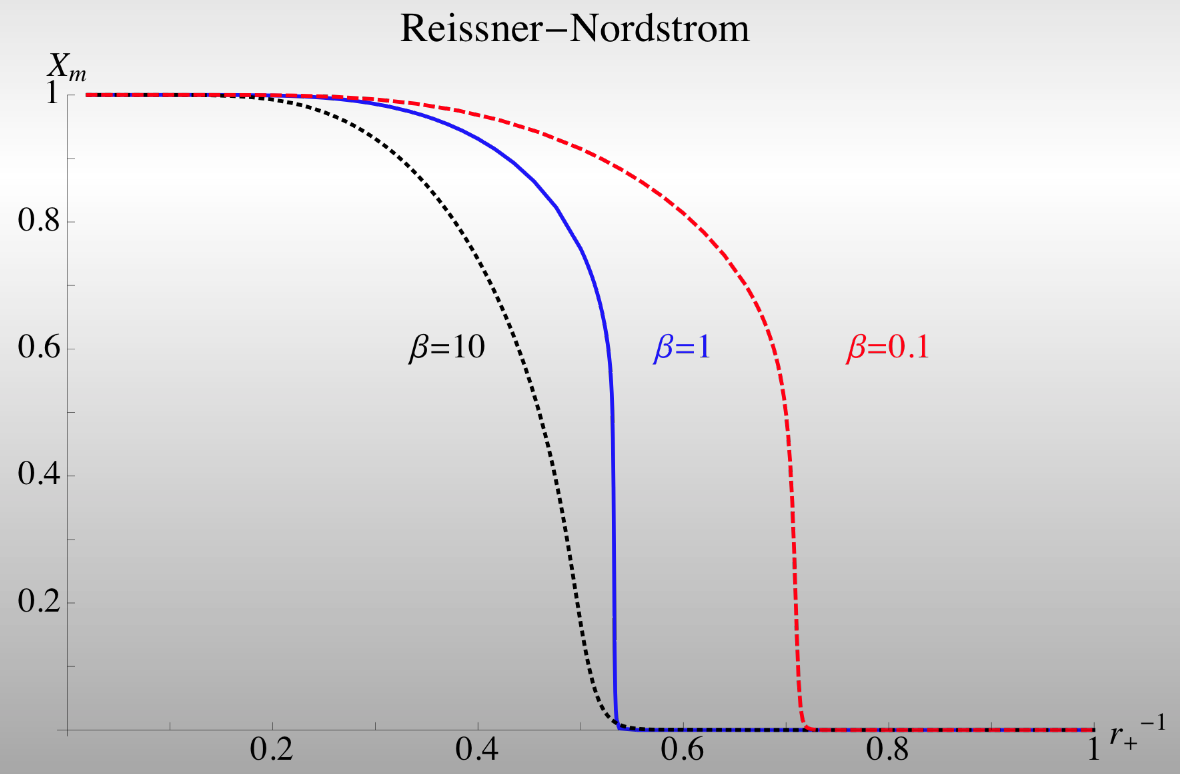






Bonjour, Emparan, RG (1998/9)

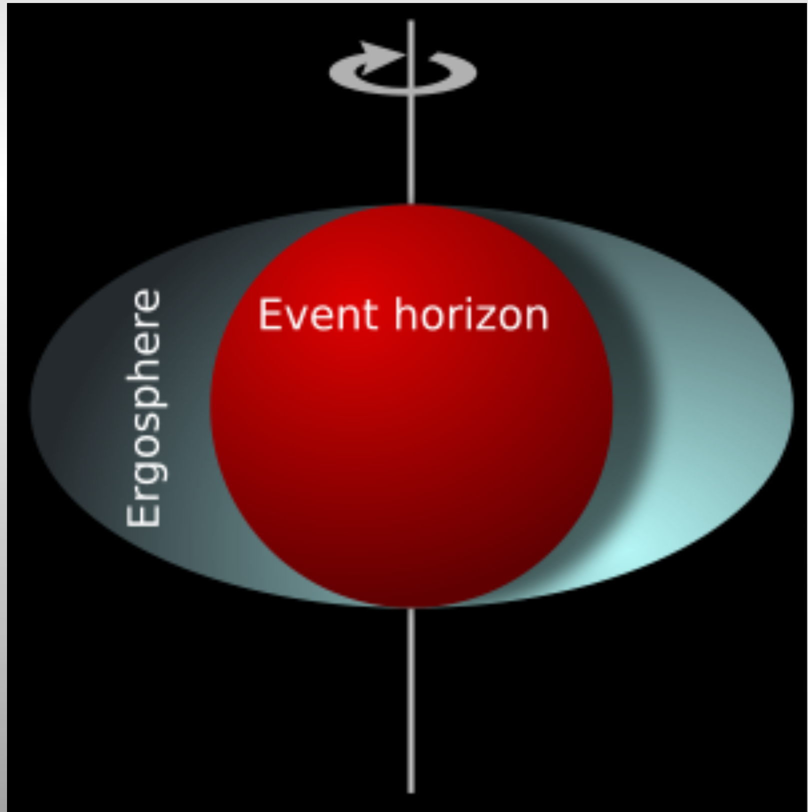
Tracking the order parameter at the equator of the horizon shows a standard phase transition between piercing and expelling.



Kerr Black Holes

David Kubiznak, and Danielle Wills

Expect all black holes to rotate – the Kerr metric. This has far richer structure, and vastly complicates the vortex fields.



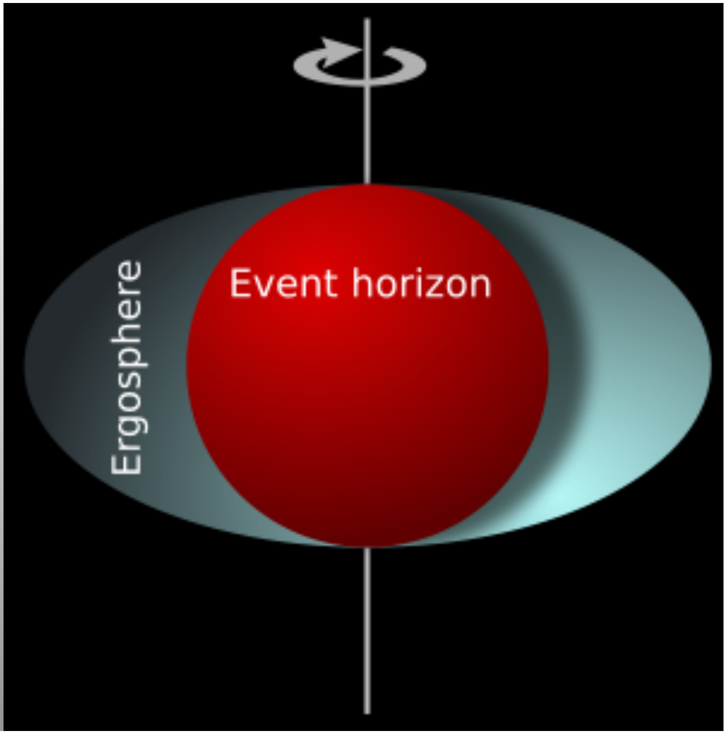
Kerr Rotation

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{4GMar\sin^2\theta}{\Sigma} dt d\varphi - \Sigma d\theta^2 - \frac{\Gamma}{\Sigma} \sin^2\theta \, d\varphi^2 - \frac{\Sigma}{\Delta} dr^2 \,,$$

where $a = J/M$ and

$$\Sigma = r^2 + a^2 \cos^2\theta \,, \quad \Delta = r^2 - 2GMr + a^2 \qquad \Gamma = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta \,.$$

The t-φ cross-term means that locally space is dragged around the black hole: a local “rest” frame rotates around the black hole relative to infinity. This means we cannot simply have an angular gauge field – which angle? Gauge fields must have time component as well – i.e. we generate ELECTRIC fields.



Guess:

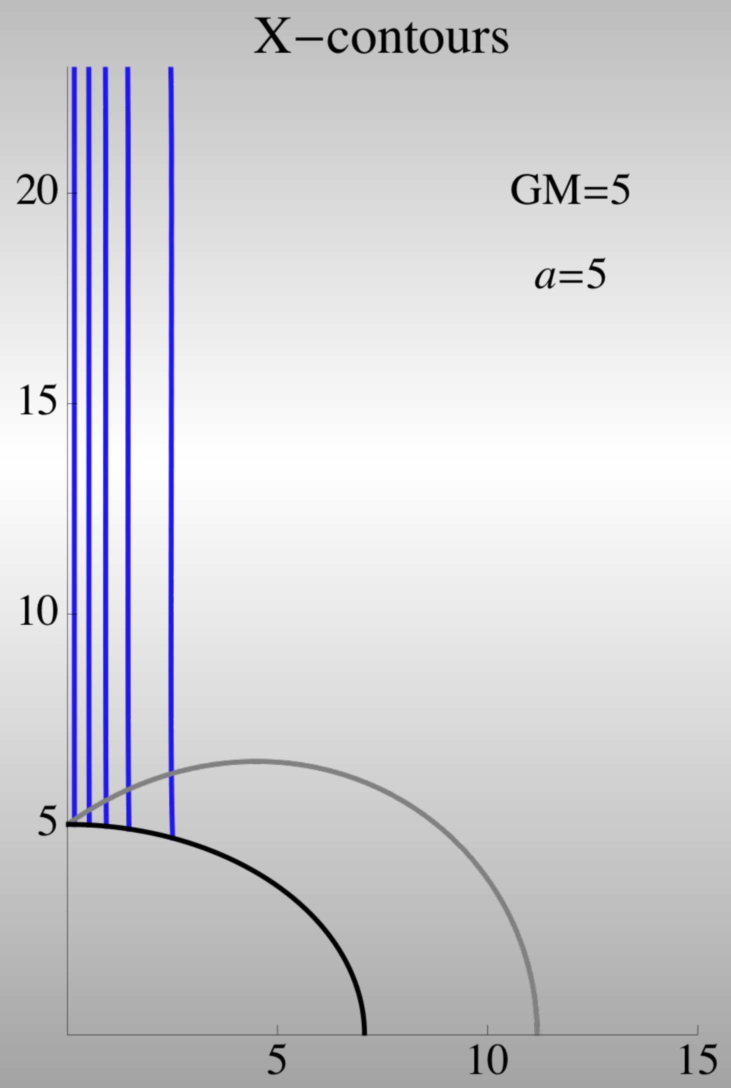
$$X = X_{NO}\left(\sqrt{r^2 + a^2}\,\sin\theta\right)$$

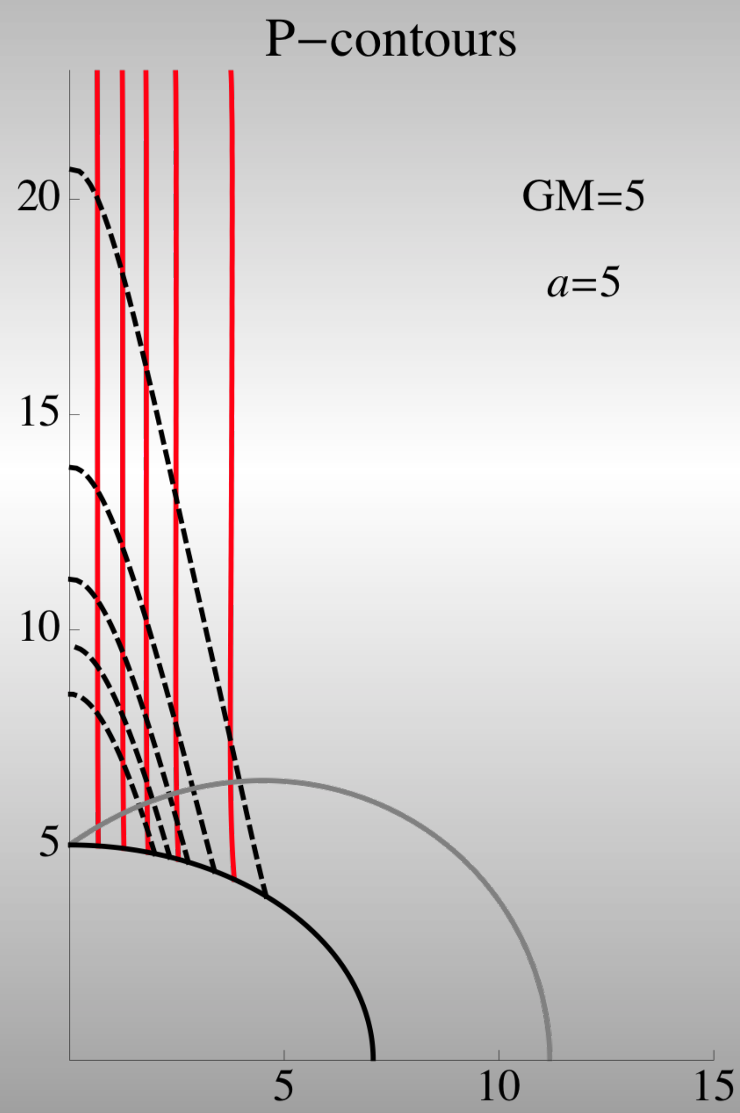
$$P_\varphi = P_{NO}\left(\sqrt{r^2 + a^2}\,\sin\theta\right)$$

$$P_t = -\frac{2GMar}{\left(r^2 + a^2\right)^2} P_{NO}\left(\sqrt{r^2 + a^2}\,\sin\theta\right)$$

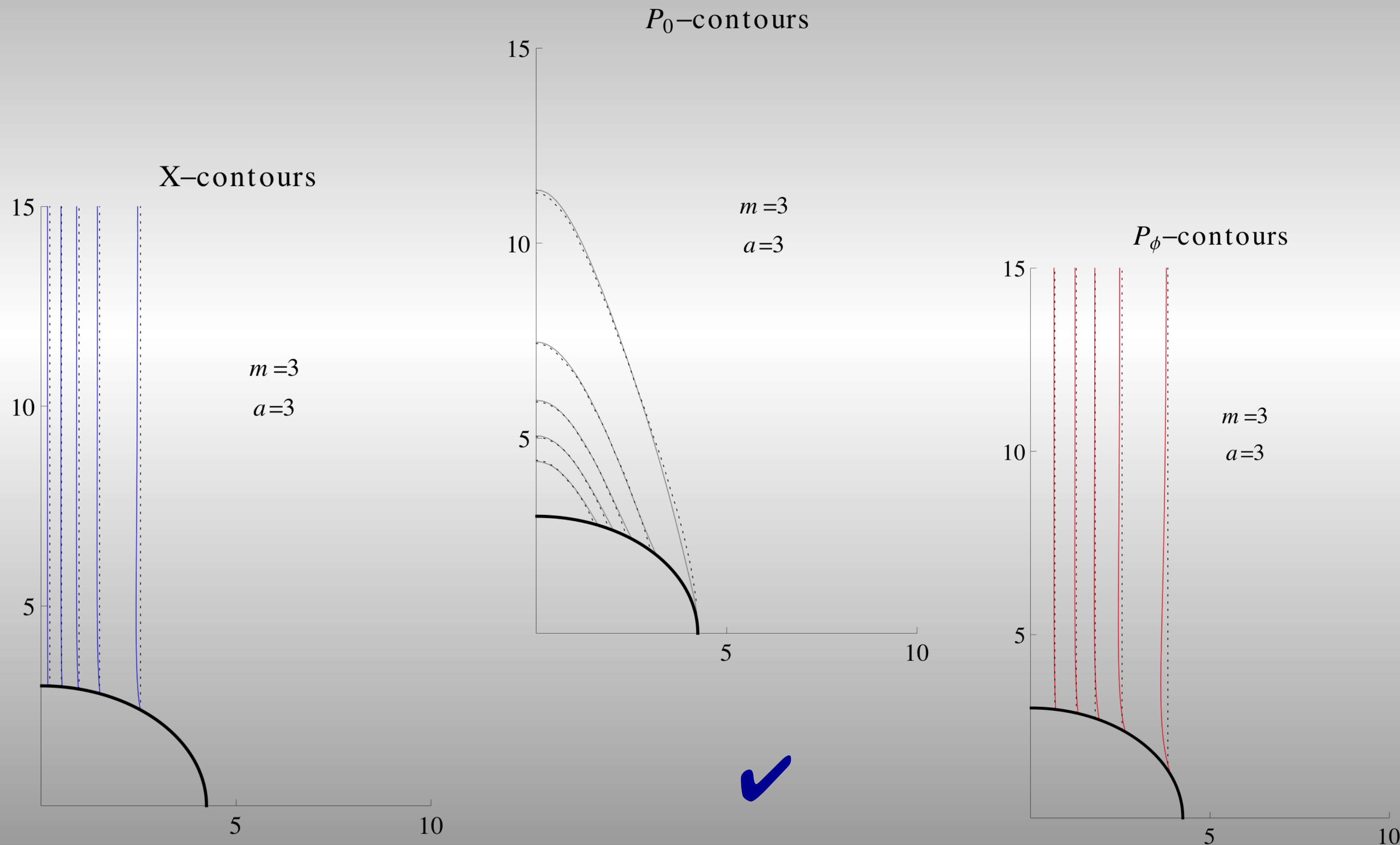
$$\boxed{\begin{aligned}
\frac{X^2}{\beta}P_\phi &= \frac{\Delta}{\Sigma}\partial_r\partial_r P_\phi + \frac{1}{\Sigma}\partial_\theta\partial_\theta P_\phi + \frac{2GM\rho^2}{\Sigma^3}(r^2 - a^2\cos^2\theta)\partial_r P_\phi \\
&\quad -\frac{\cot\theta}{\Sigma^3}(\Sigma^2 + 4GMra^2\sin^2\theta)\partial_\theta P_\phi - \frac{4a^3GMr}{\Sigma^3}\cos\theta\sin^3\theta\partial_\theta P_t \\
&\quad +\frac{2GMa\sin^2\theta}{\Sigma^3}\left[2r^2\Sigma + \rho^2(r^2 - a^2\cos^2\theta)\right]\partial_r P_t\,, \\
\frac{X^2}{\beta}P_t &= \frac{\Delta}{\Sigma}\partial_r\partial_r P_t + \frac{1}{\Sigma}\partial_\theta\partial_\theta P_t + \frac{4GMra}{\Sigma^3}\cot\theta\left(\partial_\theta P_\phi + a\sin^2\theta\partial_\theta P_t\right) + \frac{\cot\theta}{\Sigma}\partial_\theta P_t \\
&\quad +\frac{2GMa}{\Sigma^3}(\Sigma - 2r^2)\partial_r P_\phi - \frac{1}{\Sigma^3}\left[2GM(2r^2\rho^2 - a^2\sin^2\theta\Sigma) - 2r\Sigma^2\right]\partial_r P_t\,, \\
0 &= \frac{\Delta}{\Sigma}X_{,rr} + \frac{2(r-GM)}{\Sigma}X_{,r} + \frac{X_{,\theta\theta}}{\Sigma} + \frac{\cot\theta X_{,\theta}}{\Sigma} + \frac{1}{2}X(1 - X^2) + XP_\mu^2\,,
\end{aligned}}$$

Numerical Results

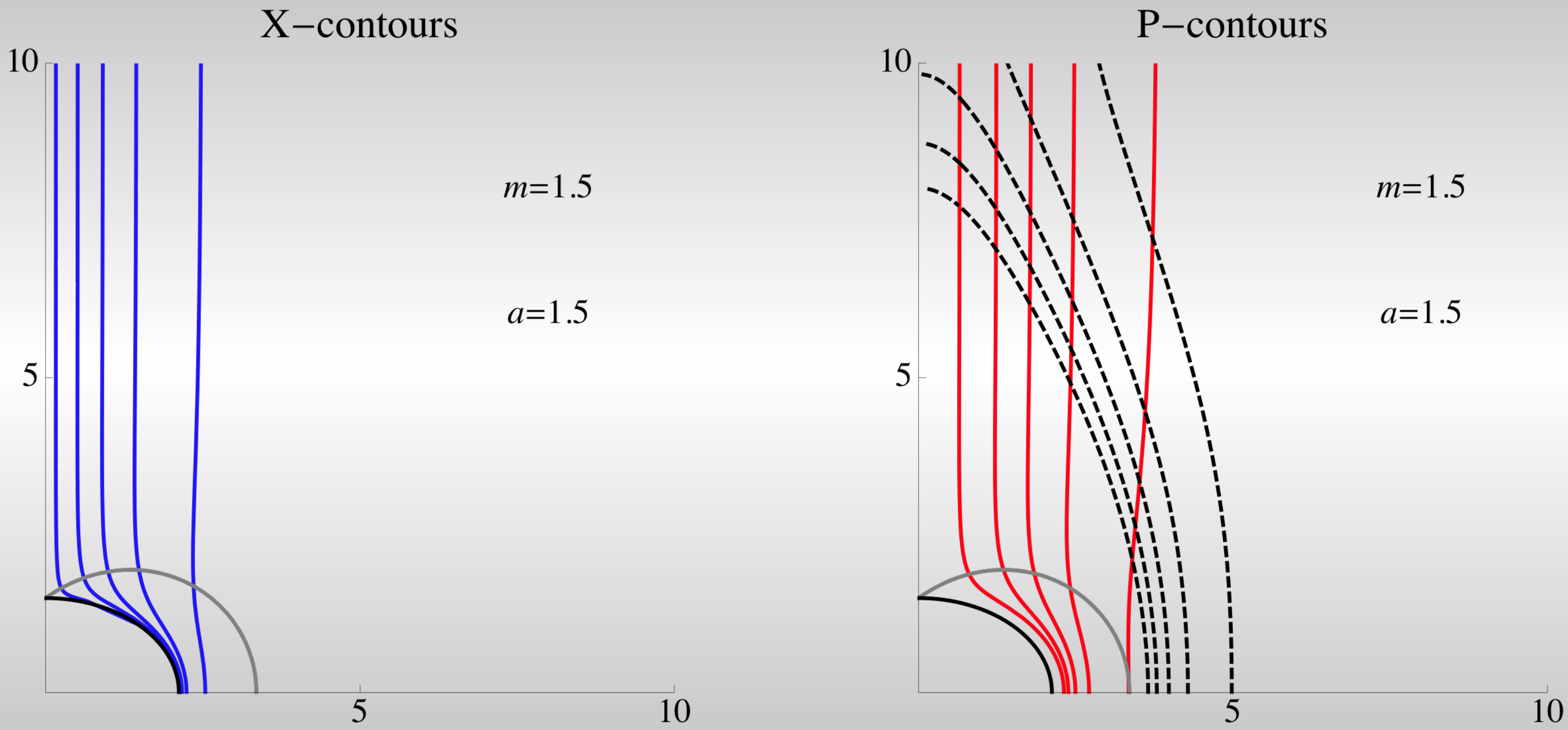




Check against guess

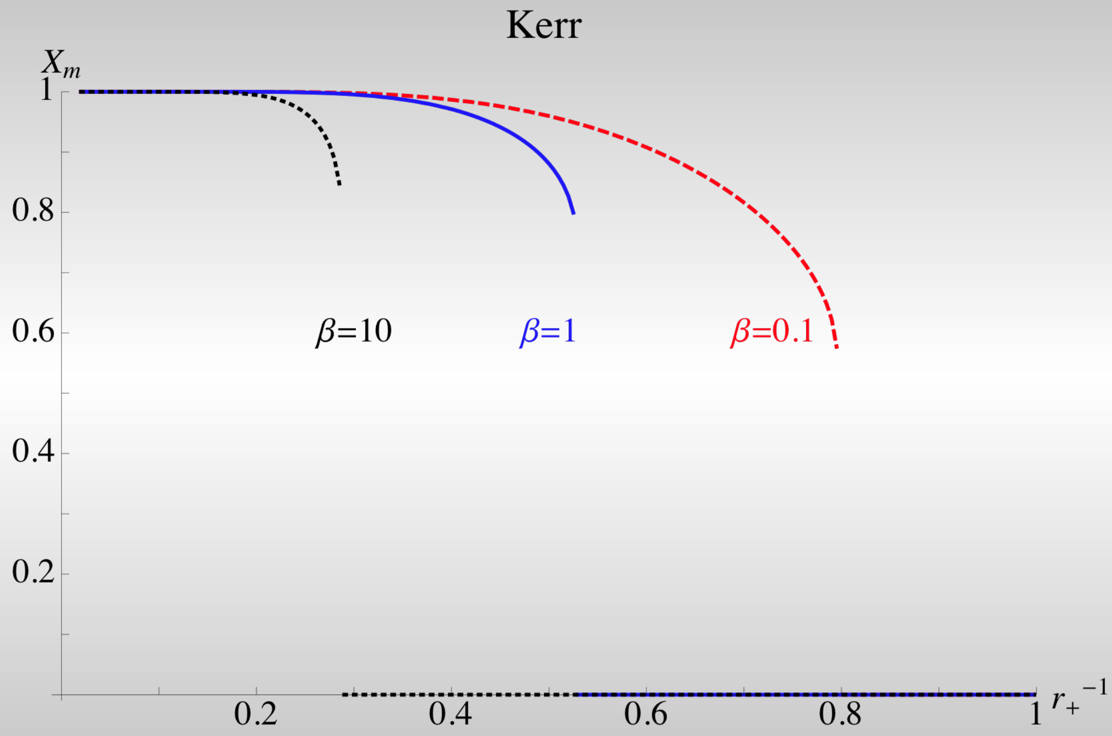


Meissner Effect?



Small extremal black holes seem to expel flux – but very likely unstable.

Extremal Kerr

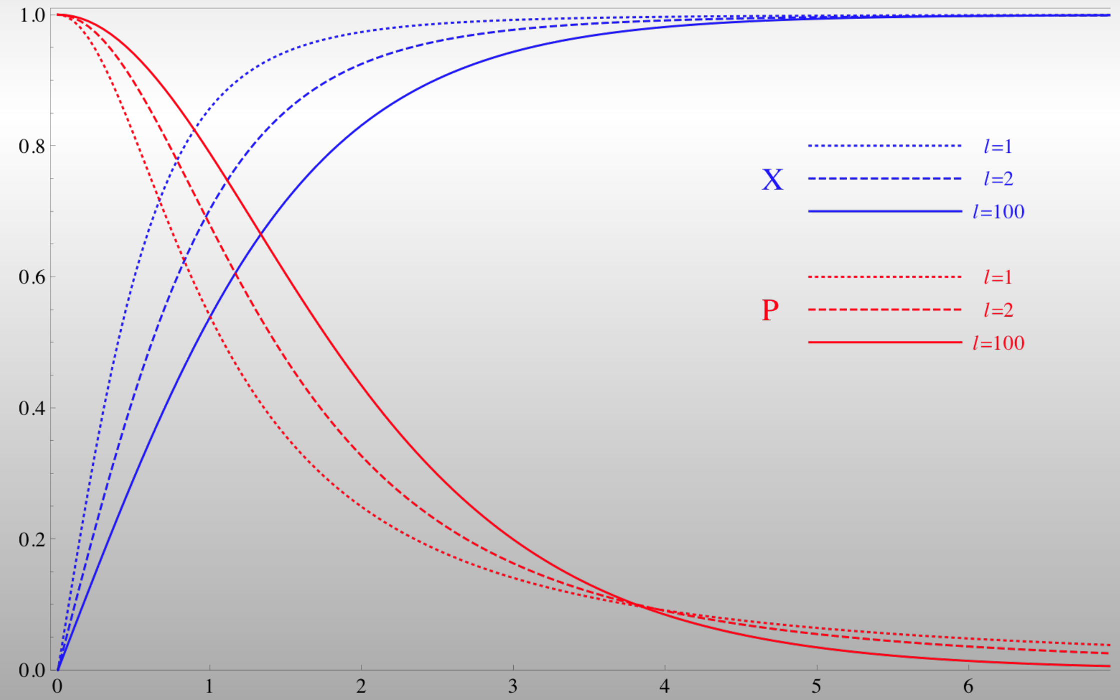


Sharp, apparently discontinuous, phase transition, supported by analytic analysis.

ADS VORTEX

$$\left(1+\frac{R^2}{l^2}\right)P_0''+\left(\frac{2R}{l^2}-\frac{1}{R}\right)P_0'-\frac{X_0^2P_0}{\beta}=0$$

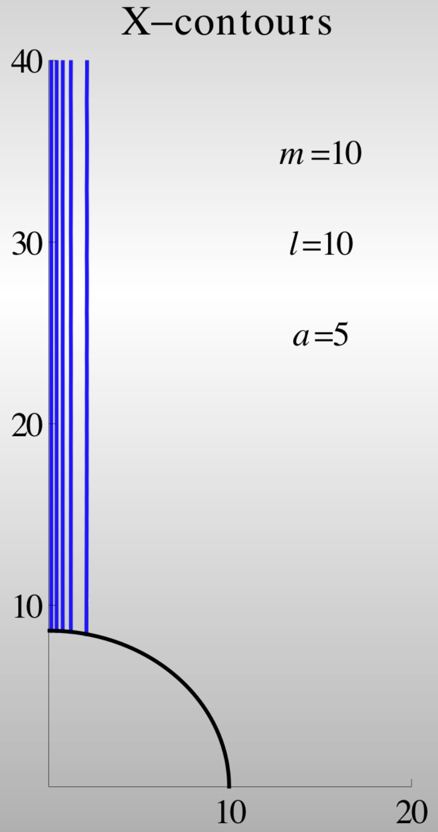
$$\left(1+\frac{R^2}{l^2}\right)X_0''+\left(\frac{4R}{l^2}+\frac{1}{R}\right)X_0'-\frac{P_0^2X_0}{R^2}-\frac{1}{2}X_0(X_0^2-1)=0\,.$$

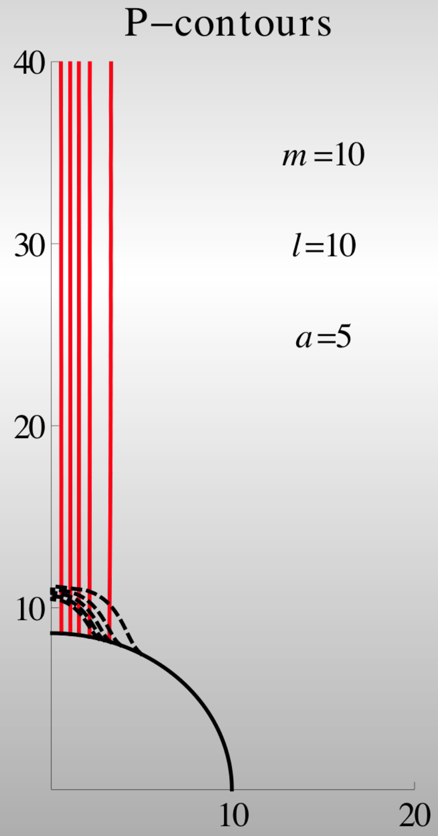


RG, Gustainis, Kubiznak, Mann, Wills

Kerr-ADS

$$X \simeq X_0(R)\,, \quad P_\phi \simeq P_0(R)\,, \quad P_t \simeq \frac{a}{\rho^2}\Big(\frac{\Delta}{\rho^2} - \Xi\Big) P_0(R)\,, \quad R \equiv \frac{\rho}{\sqrt{\Xi}} \sin\theta,$$





RG, Gustainis, Kubiznak, Mann, Wills

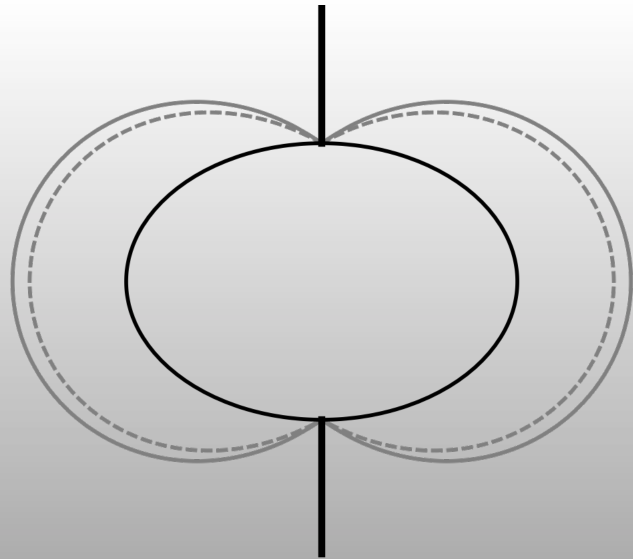
Implications?

STAR TREK
THE NEXT GENERATION

Kerr with String

$$ds^2 = \left(1 - \frac{2GMr}{\Sigma} + \frac{8(GMar\sin\theta)^2}{\Gamma\Sigma}\epsilon\hat{\mu}\right) dt^2 - \Sigma d\theta^2 - \frac{\Sigma}{\Delta} dr^2$$
$$- \frac{\Gamma}{\Sigma}(1 - 2\epsilon\hat{\mu})\sin^2\theta\, d\varphi^2 + \frac{4GMar\sin^2\theta}{\Sigma}(1 - 2\epsilon\hat{\mu})\, dt d\varphi$$

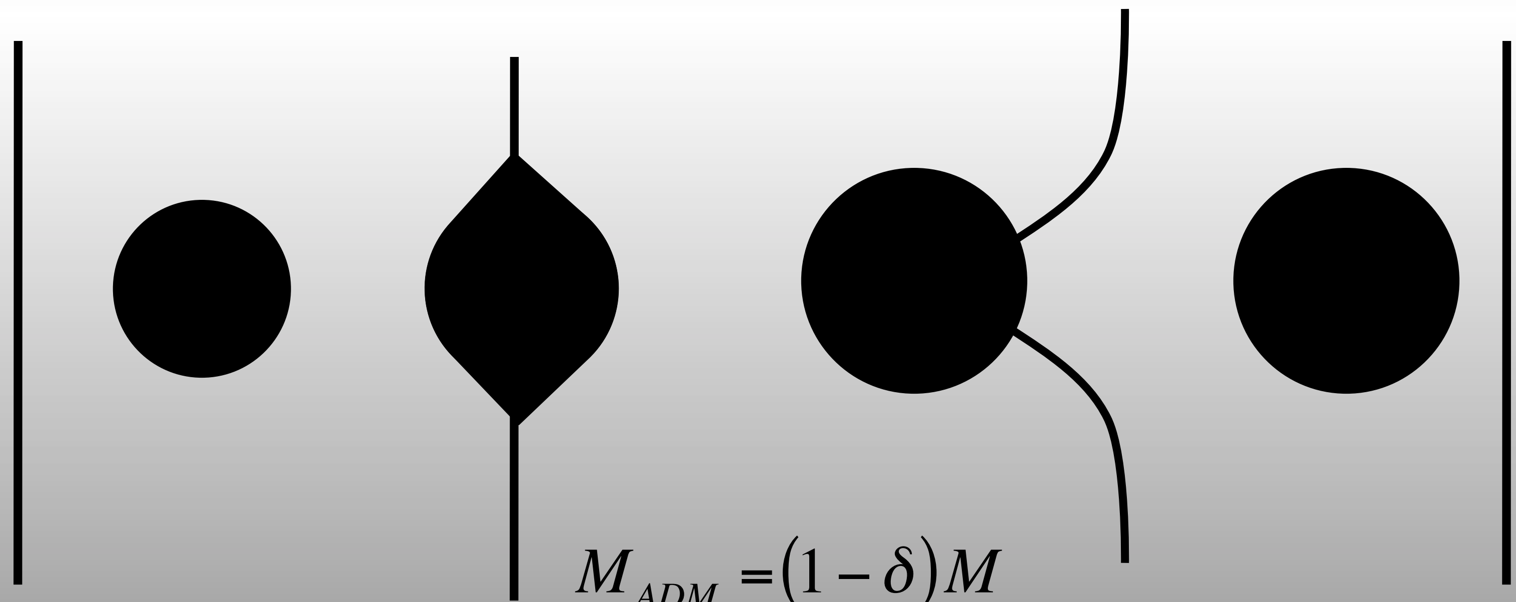
The string produces a deficit, but not quite the usual cone. The deficit is with respect to a local co-rotating frame: Orbits are affected, as is the ergosphere.



$$\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 - 2GMr + a^2 \qquad \Gamma = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta\,.$$

Black Holes & Strings

String capture increases gravitational mass and entropy by $4G\mu$. After string has passed, ADM mass increased back to gravitational mass, entropy again increases by $4G\mu$.



$$M_{ADM} = (1-\delta)M$$

$$Area = 16\pi(1-\delta)M^2$$

Summary

- Existence of hair and regular solutions confirms horizon is not "special"
- Black holes will capture slower strings
- Near horizon metric is altered - vortex cuts out a deficit in a co-rotating frame – could affect geodesics.
- Possible applications in holographic superconductivity