strings with junctions: cuspy events and unzipping mechanisms

mairi sakellariadou

king's college london
university of london

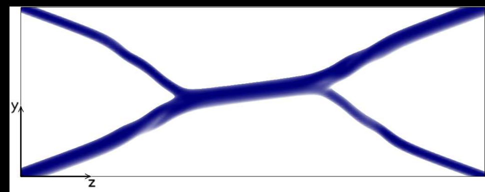
outline

- cusps and pseudo-cusps

elghozi, nelson, sakellariadou (2014)

bursts of GW

elghozi, sakellariadou (in progress)



- zipping and unzipping

stability of junctions

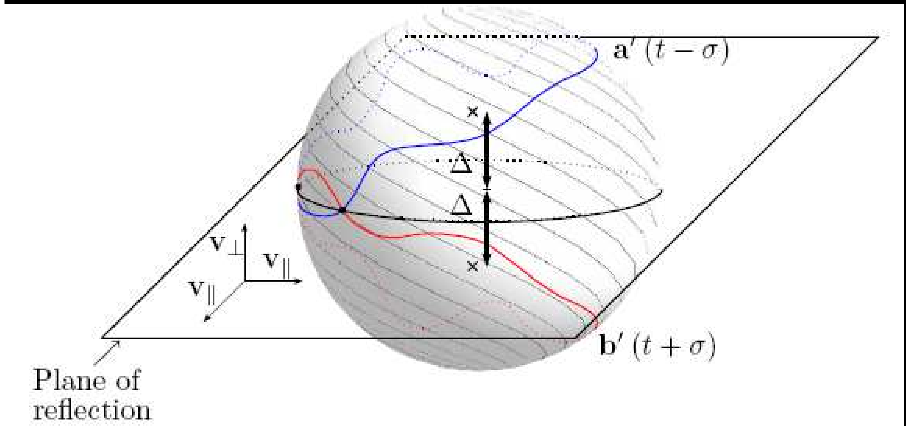
avgoustidis, pourtsidou, sakellariadou (2014)

cusps and pseudo-cusps

cusps and pseudo-cusps

motivation

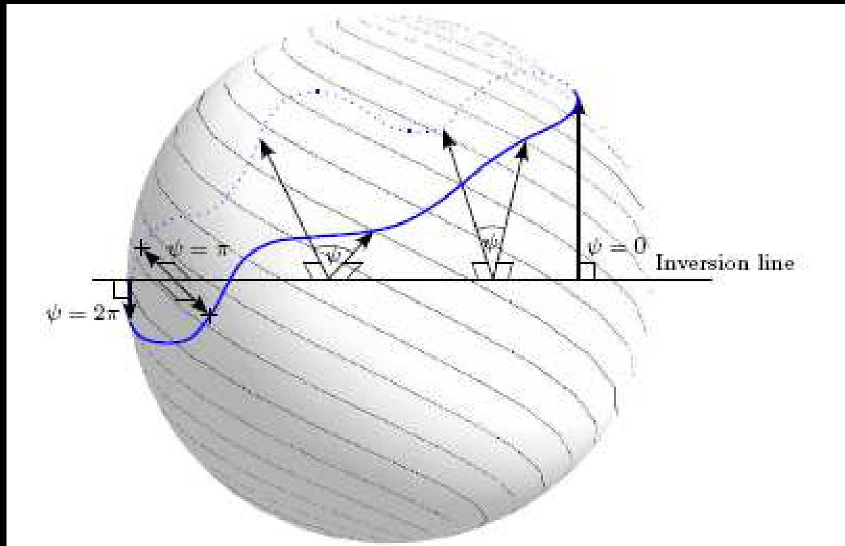
- FD-string junctions generically contain cusps



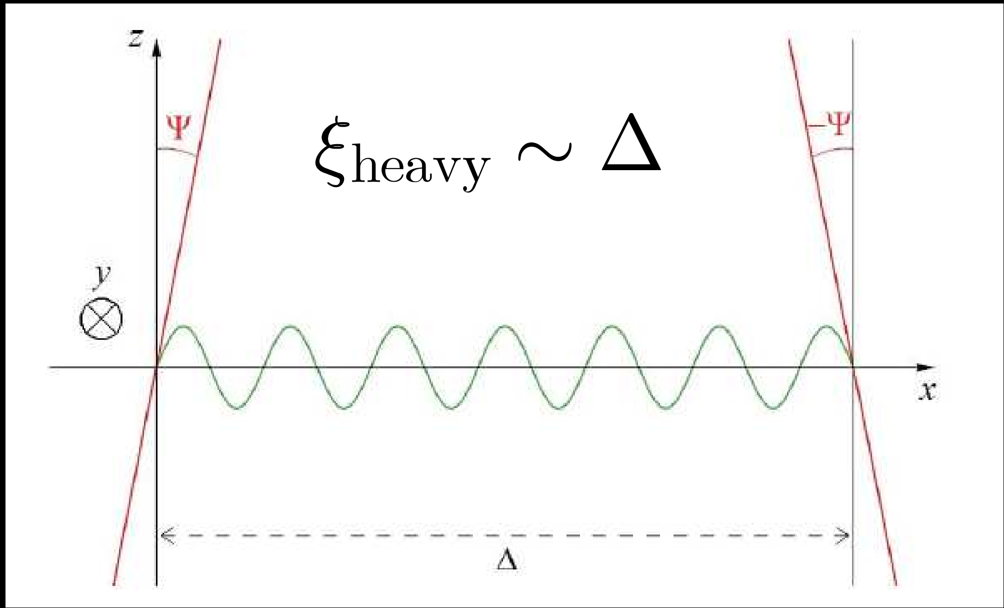
DBI string ending on D2 branes

$$L \gg |\Delta|$$

DBI string ending on D1-branes

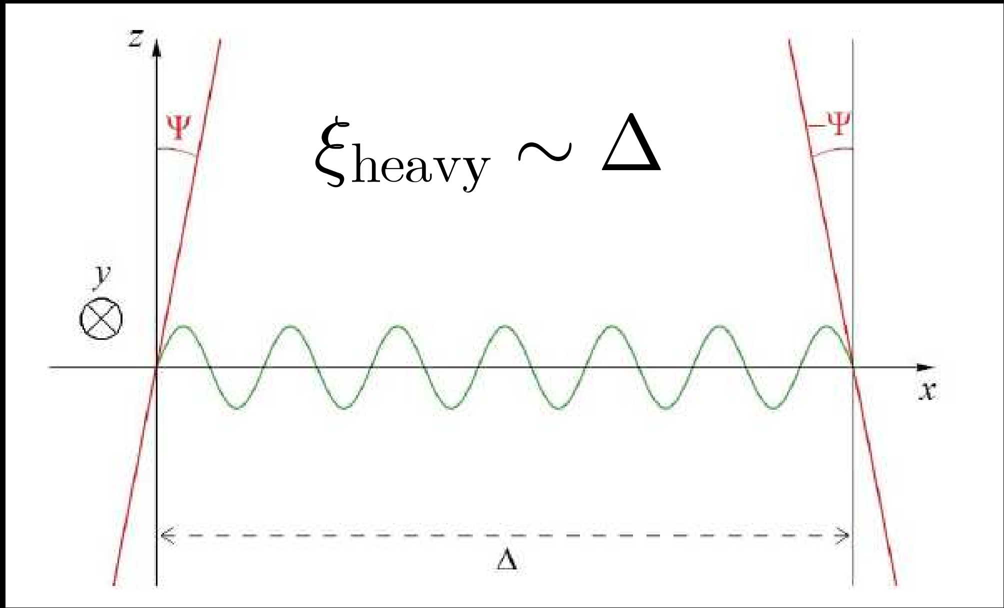


davis, nelson, rajamanoharan, sakellariadou JCAP 0811 (2008) 022



b.c. for light string:

$$
\boxed{
\begin{aligned}
\dot{\mathbf{x}}_{\perp}(t,0) &= \mathbf{x}'_{\parallel}(t,0) = 0\ , \\
\dot{\mathbf{x}}_{\perp}(t,\sigma_m) &= \mathbf{x}'_{\parallel}(t,\sigma_m) = 0\ ,
\end{aligned}
}
$$



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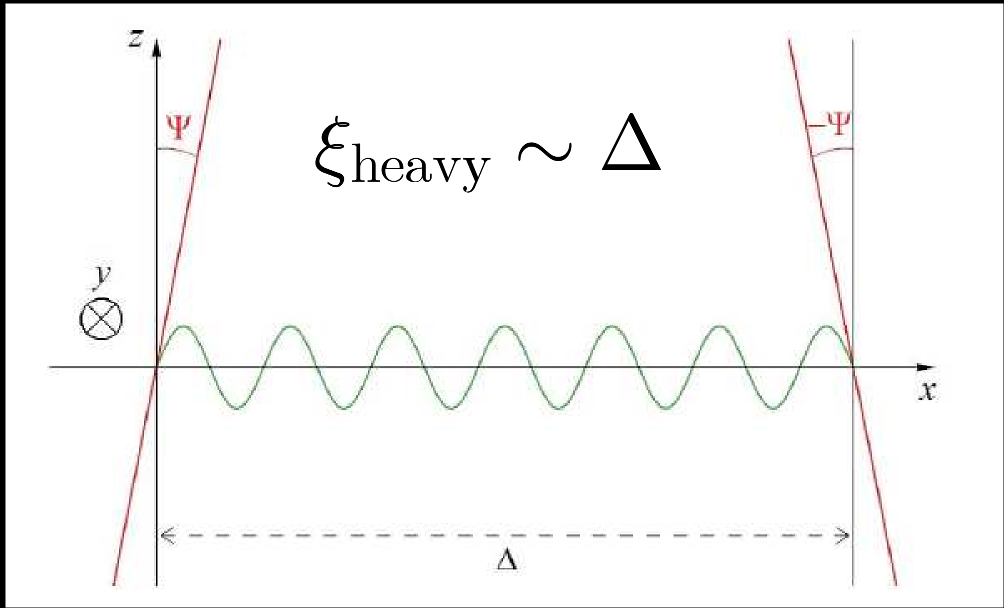
$$\boxed{\mathbf{x}'' - \ddot{\mathbf{x}} = 0}$$

$$\boxed{\mathbf{x}(t,\sigma) \equiv \frac{1}{2}\left[\mathbf{a}(\sigma+t) + \mathbf{b}(\sigma-t)\right]}$$



$$\xi_{\rm heavy} \sim \Delta$$

b.c. for light string:

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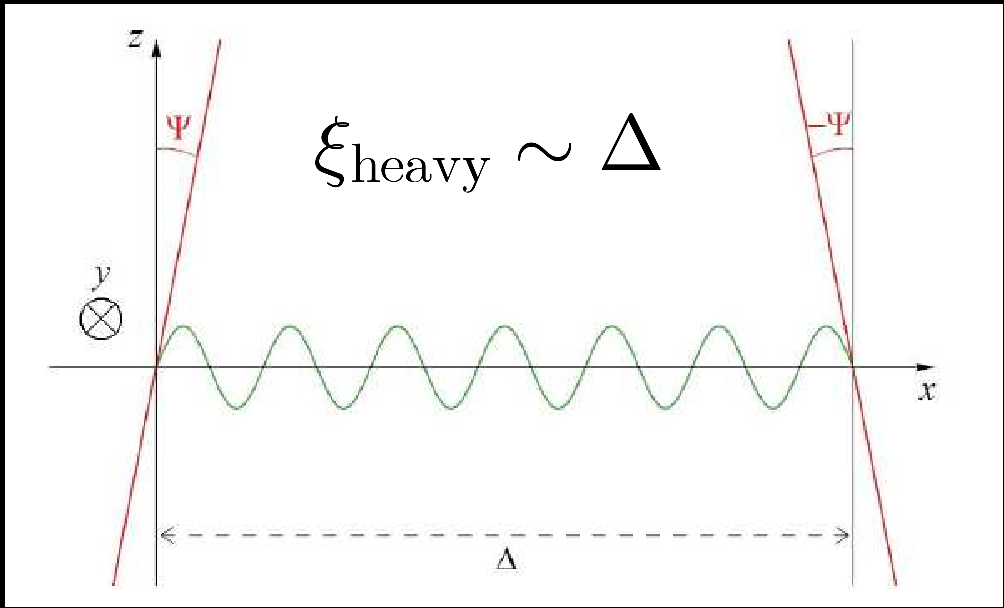
$a'(\sigma+t), b'(\sigma-t)$ periodic for a dense subset of $\Psi \in [-\pi/2, \pi/2]$

decompose them in fourier series



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decompose them in fourier series

symmetries between 2 movers on the string:

b' reflection of a' through axis // to $\sigma = 0$ end of string (z-axis)

analytic estimation of cusps/pseudo-cusps:

average and standard deviation of $\mathbf{a}'$ and $-\mathbf{b}'$ in unit sphere

how many times the standard deviation is larger than the average

cusps when

$$\boxed{\langle a'_x a'_x \rangle_\sigma \gtrsim \frac{1+\alpha}{\alpha} \left(\frac{|\Delta|}{\sigma_m} \right)^2 = \frac{1+\alpha}{\alpha} \, \Delta_a^2}$$

$$\Delta \equiv \mathbf{x}(\sigma_m, t) - \mathbf{x}(0, t)$$

$$\Delta_a \equiv \frac{1}{2\sigma_m} \int_{-\sigma_m}^{\sigma_m} \mathrm{d}\sigma \, a'_x(\sigma + t) \quad , \quad \Delta_b \equiv \frac{1}{2\sigma_m} \int_{-\sigma_m}^{\sigma_m} \mathrm{d}\sigma \, b'_x(\sigma + t)$$

distance in unit sphere between 2 average circles analytic estimation of cusps/pseudo-cusps:

average and standard deviation of $\mathbf{a}'$ and $-\mathbf{b}'$ in unit sphere

cusps when

$$\langle a'_x a'_x \rangle_\sigma \gtrsim \frac{1+\alpha}{\alpha}\left(\frac{|\Delta|}{\sigma_m}\right)^2 = \frac{1+\alpha}{\alpha}\,\Delta_a^2$$

more cusps for a long string with large amplitude waves
than a short straight string or a small-scale structured string

pseudo-cusps

l^μ 4-velocity at point of closest approach

- cusps: null vector
- pseudo-cusps: $l^0 = 1$

theoretical velocity $|l^i| = \sqrt{\frac{1+\cos\theta_c}{2}} \approx 1 - \theta_c^2/8 + \theta_c^4/384$

softness of relativistic part of string

for a cusp: $\theta_c = 0$ and $v = c$

numerical simulation

- string's ends fixed on heavy strings
- position/velocity at t=0 defined by fourier series

numerical simulation

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- position/velocity at t=0 defined by fourier series
- inputs: -- string's length σ_m
  -- interstring distance Δ

  Δ/σ_m

  -- parameters to fix oscillatory behavior of string

  n : # harmonics

  h_m : highest value (amplitudes in $[-h_m, h_m]$)

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  $\Big\} \quad \Delta/\sigma_m$

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numerical simulation

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- decompose strings in large # pieces travelling at velocity of their geometric center and compute their evolution over a period

numerical simulation

237 strings

8719 cusps and 4659 pseudo-cusps

- string's ends fixed on heavy strings
- position/velocity at t=0 defined by fourier series
- inputs: -- string's length σ_m

  -- interstring distance Δ

  $\left.\begin{array}{l} \sigma_m \\ \Delta \end{array}\right\} \Delta/\sigma_m$

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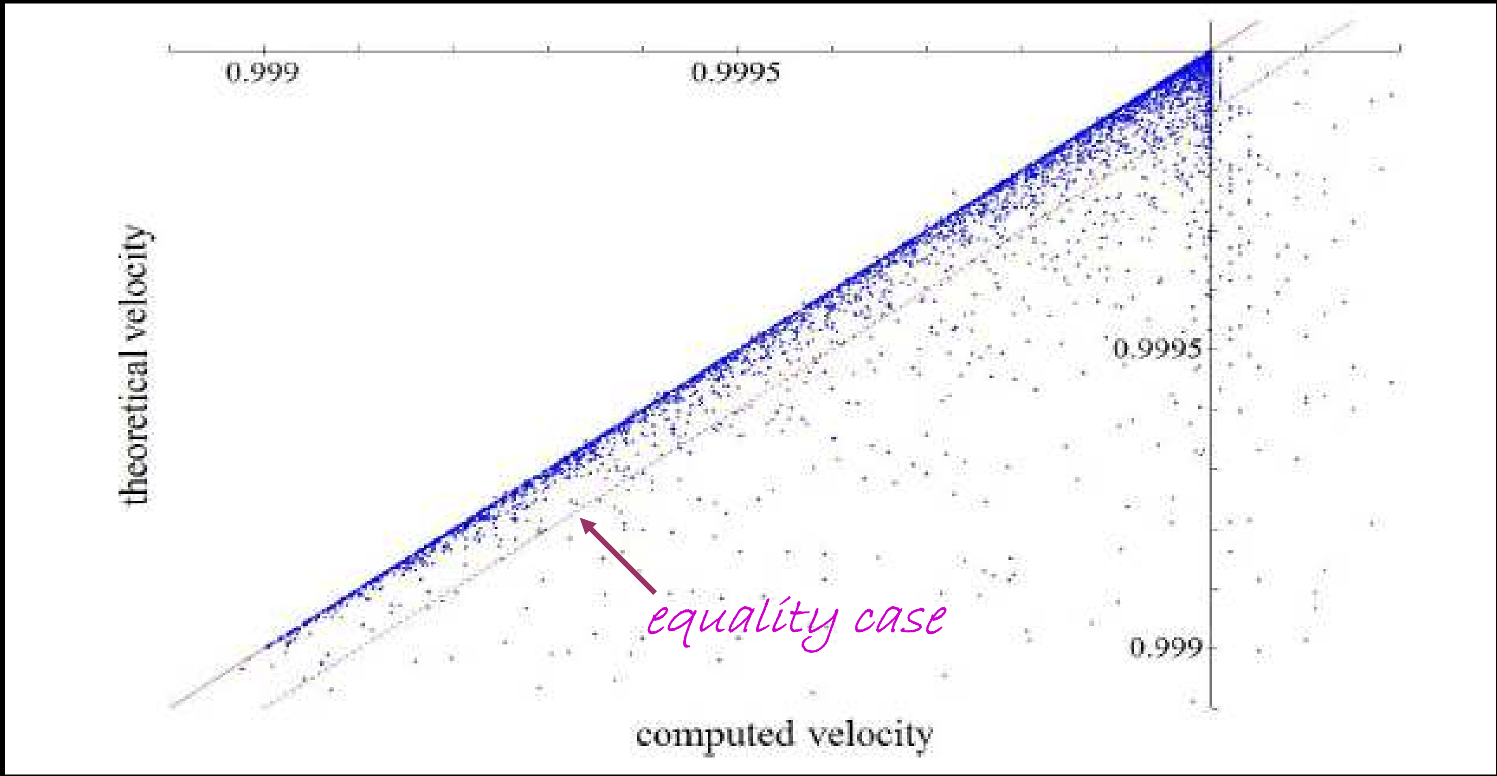
about 4300 pseudo-cusps

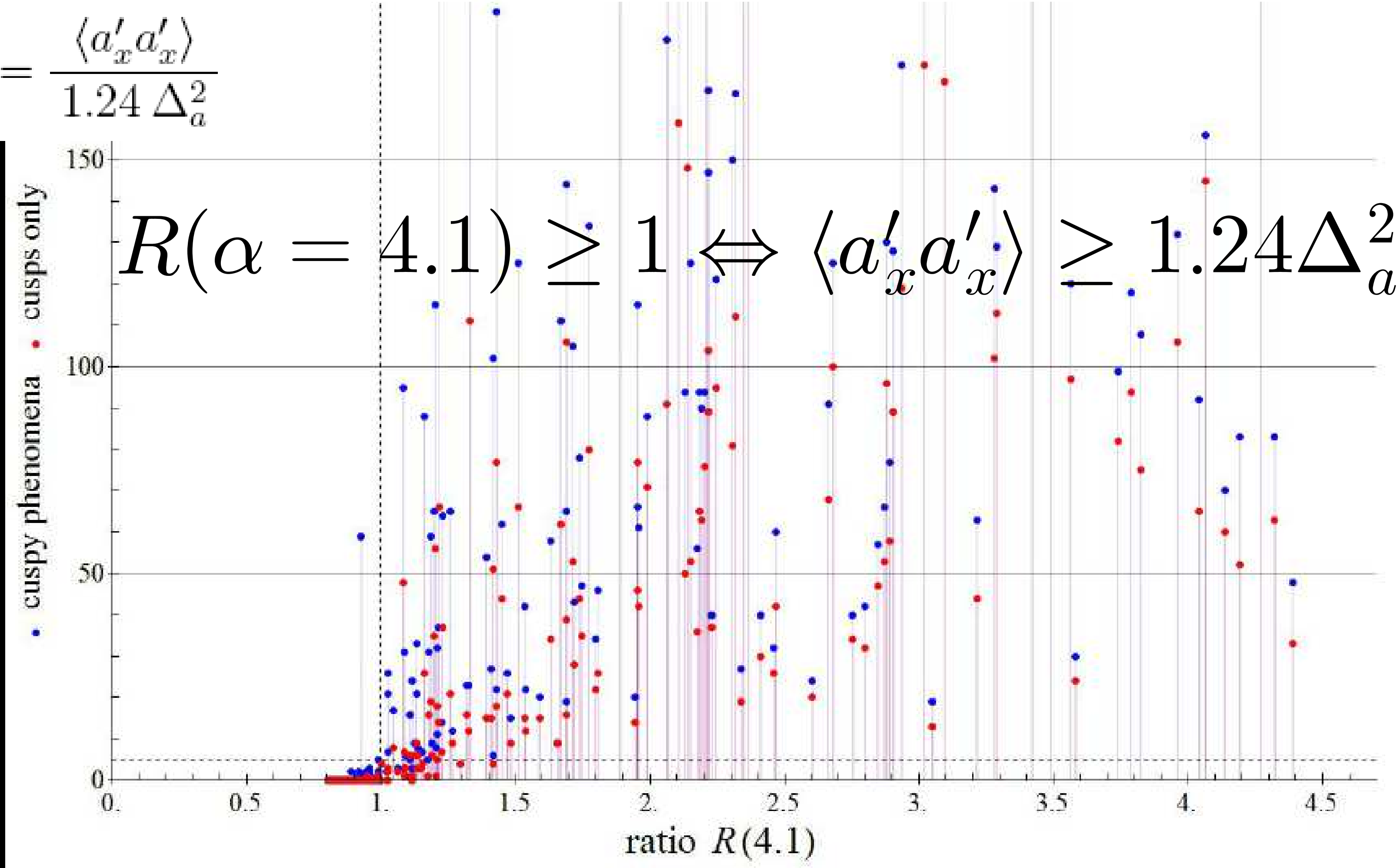


80 % of pseudo-cusps with a difference between two velocities below 10^{-4}

- cusps: $v > (1 - 10^{-6})c$
- pseudo-cusps $v \in [(1 - 10^{-3})c, (1 - 10^{-6})c]$

to check criterion: $\langle a'_x a'_x \rangle \geq \frac{1+\alpha}{\alpha} \Delta_a^2$ cuspy phenomena

$$R(\alpha = 4.1) = \frac{\langle a'_x a'_x \rangle}{1.24\, \Delta_a^2}$$



$$R(\alpha = 4.1) \geq 1 \Leftrightarrow \langle a'_x a'_x \rangle \geq 1.24 \Delta_a^2$$

number of cusps and pseudo-cusps

Δ/σ_m and $\langle a'_x a'_x \rangle, \langle b'_x b'_x \rangle$

$\xi \sim \Delta/\sigma_{\mathrm{m}}$

$\bar{\xi} \sim \frac{1}{2}(\text{ripple's size})$

$\bar{\xi} \sim \frac{1}{4}(\text{smallest } \lambda)$

$\bar{\xi} \sim \frac{\sigma_m}{2n_{\mathrm{eff}}}$

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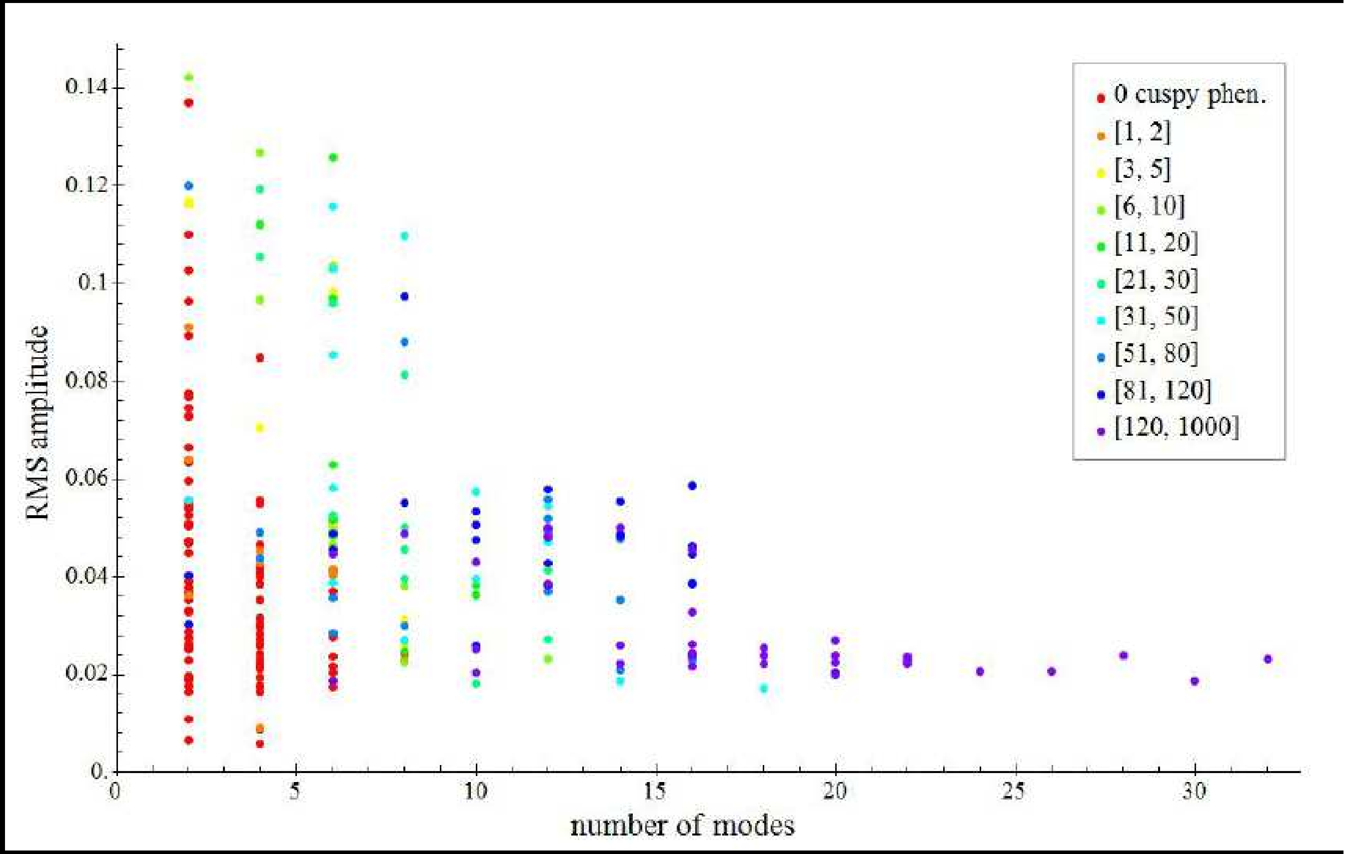
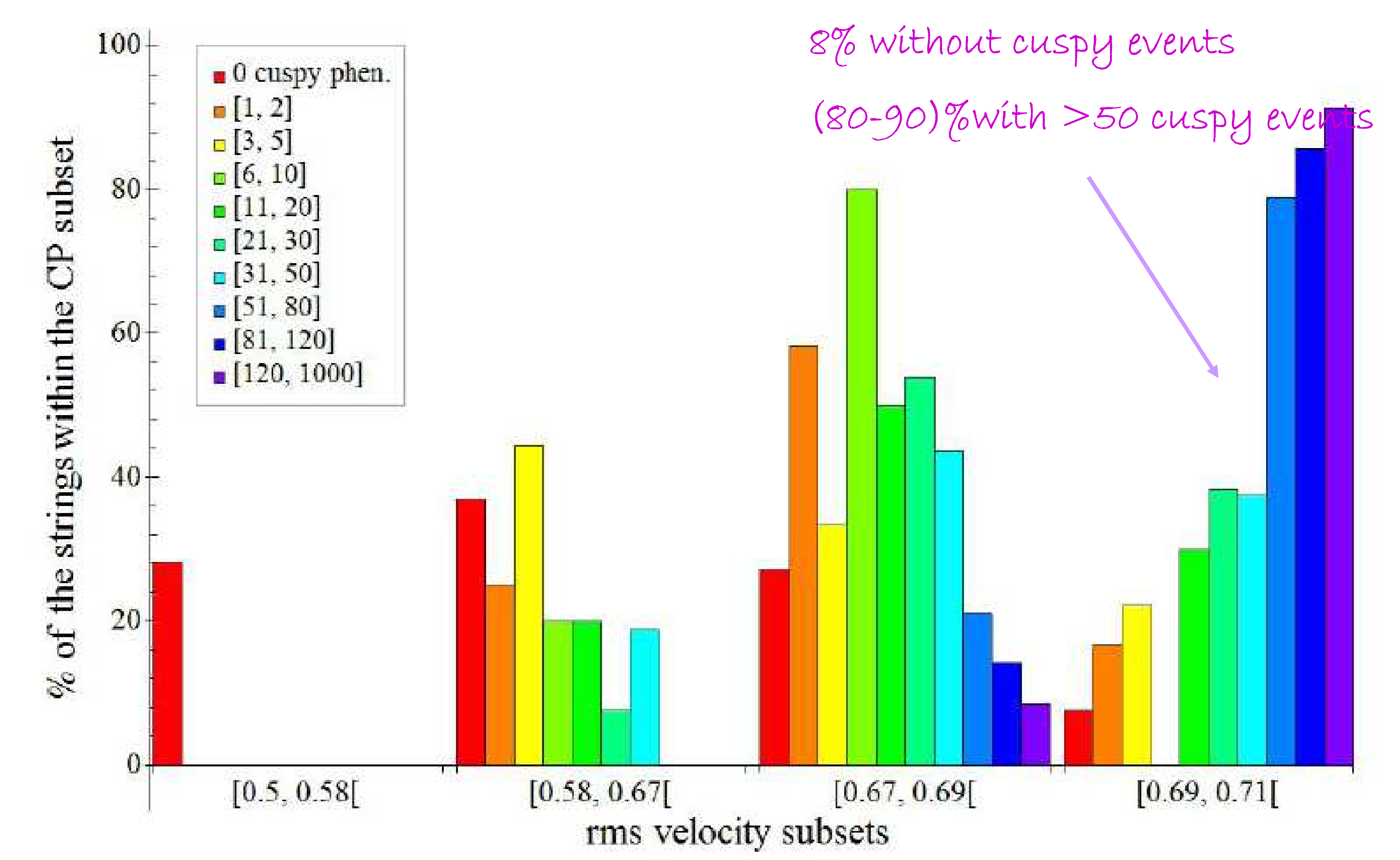
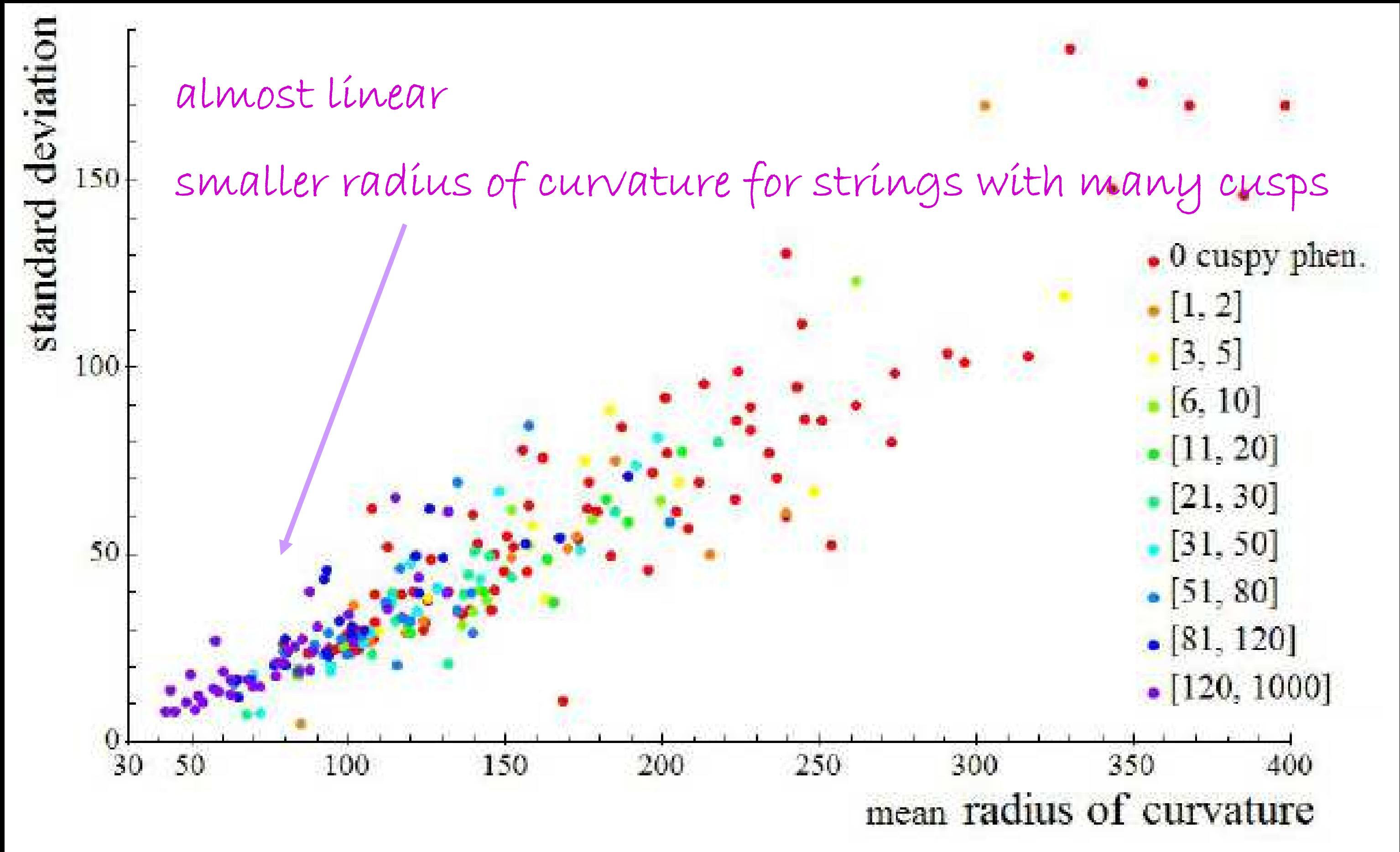


the more energy is on the string, the more cusps appear



8% without cuspy events
(80-90)%with >50 cuspy events
0 cuspy phen.
[1, 2]
[3, 5]
[6, 10]
[11, 20]
[21, 30]
[31, 50]
[51, 80]
[81, 120]
[120, 1000]
% of the strings within the CP subset
rms velocity subsets
[0.5, 0.58[
[0.58, 0.67[
[0.67, 0.69[
[0.69, 0.71[
0
20
40
60
80
100

deviation from average over a period in time



time-average of space-averaged radius of curvature of each string

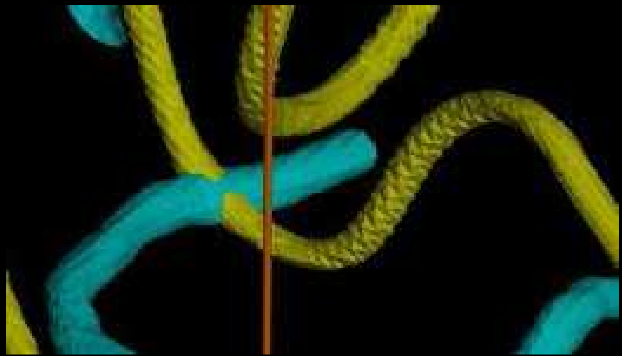
a smaller radius of curvature means more waves, hence more cusps

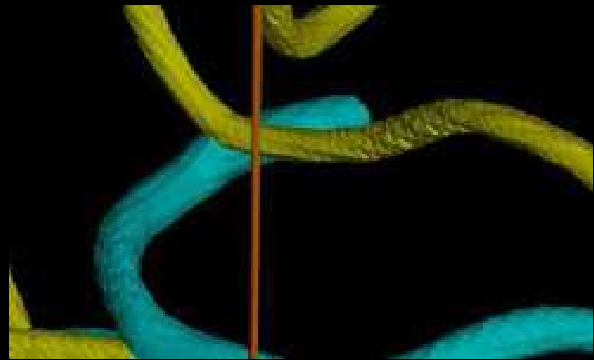
unzipping mechanisms

unzipping mechanisms

motivation

$$\mathcal{S} = \int \mathrm{d}^3x\mathrm{d}t\Big[-\frac{1}{4}F^2 - \frac{1}{2}\left(D_\mu\phi\right)\left(D^\mu\phi\right)^* - \frac{\lambda_1}{4}\left(\phi\phi^* - \eta_1^2\right)^2 - \frac{1}{4}H^2 - \frac{1}{2}\left(D_\mu\chi\right)\left(D^\mu\chi\right)^* - \frac{\lambda_2}{4}\phi\phi^*\left(\chi\chi^* - \eta_2^2\right)^2 \Big]$$



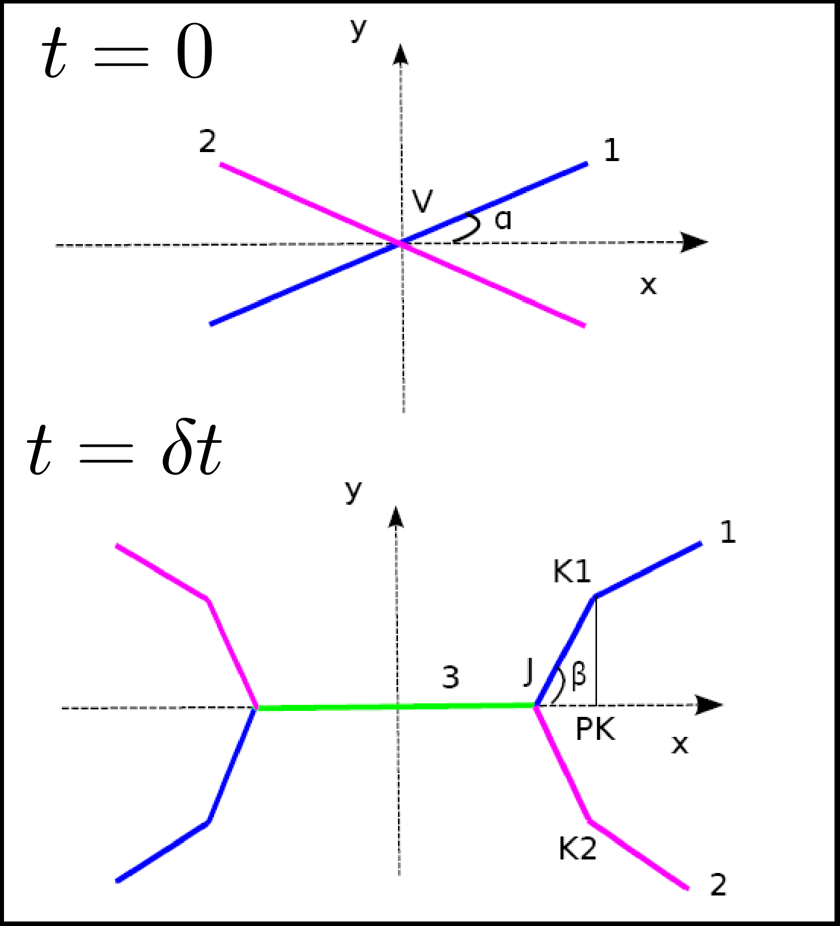


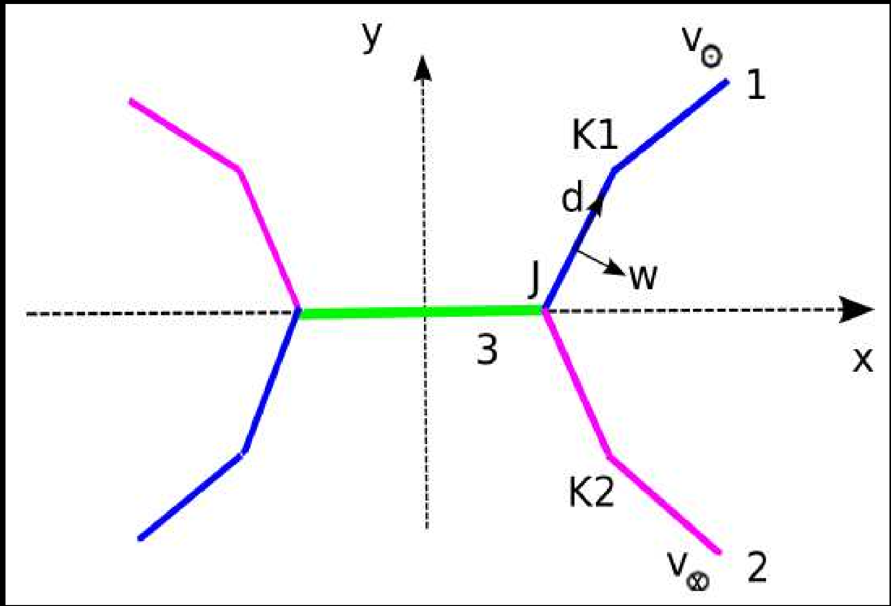
rajantie, sakellariadou, stoica, JCAP 0711 (2007) 021

sakellariadou, stoica, JCAP 0808 (2008) 022

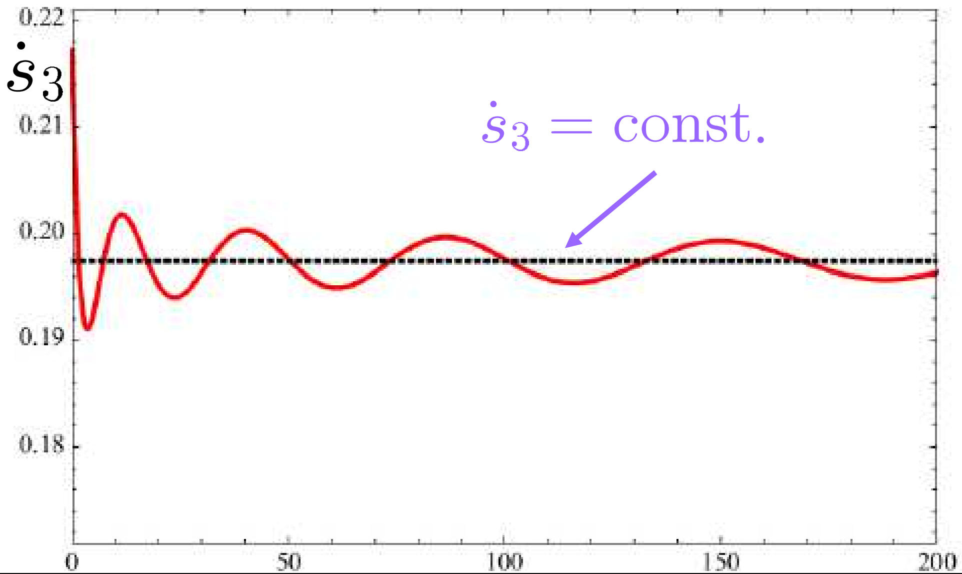
zipping/unzipping in NG approximation

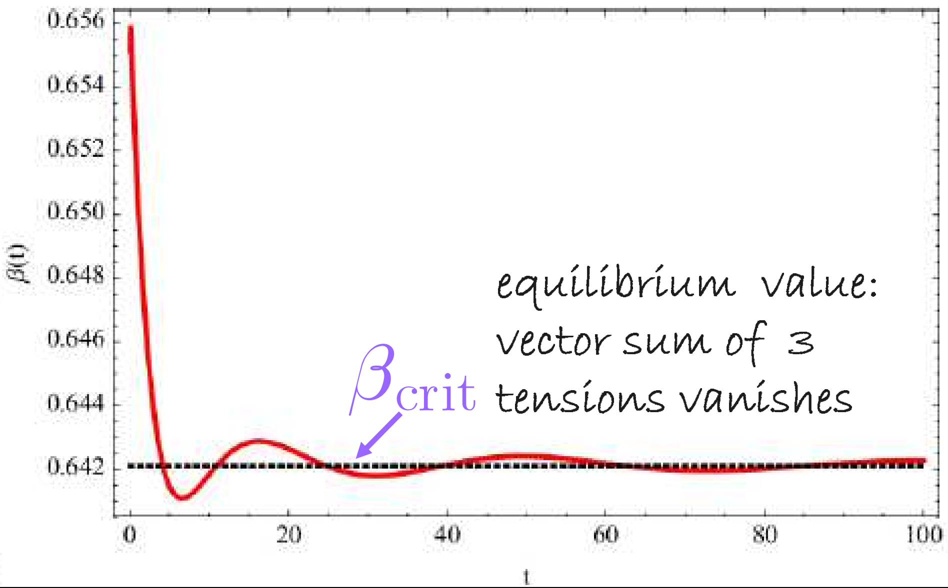
evolution of local angle β
at a massive junction

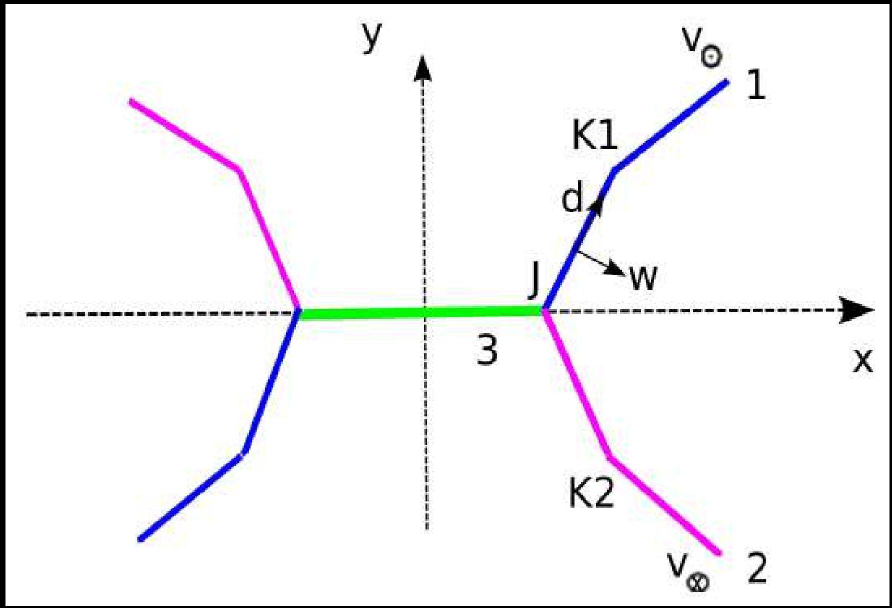




$$\ddot{s}_3 = -\frac{1}{m}(1-\dot{s}_3^2)^{3/2}[\mu_3 - 2\mu\gamma_w(t)d_x(t)]$$





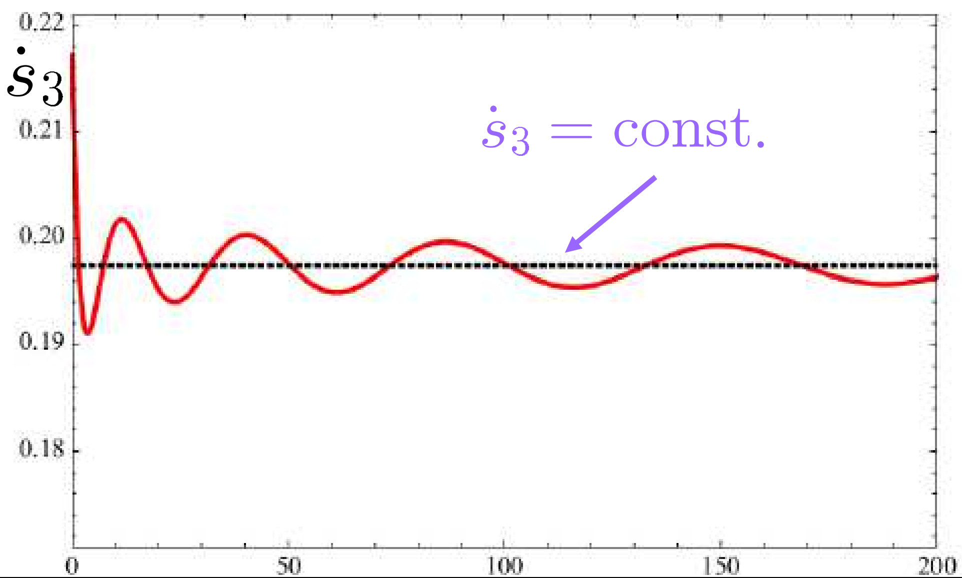


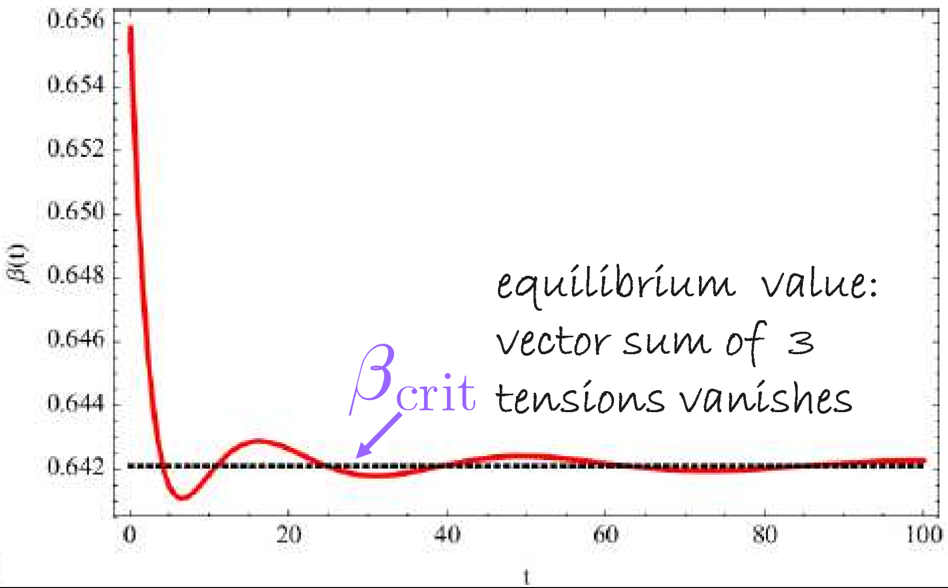
even though the constant zipper growth prediction is based on special configurations:

$$2\mu \cos\beta\gamma_w - \mu_3 = 0$$

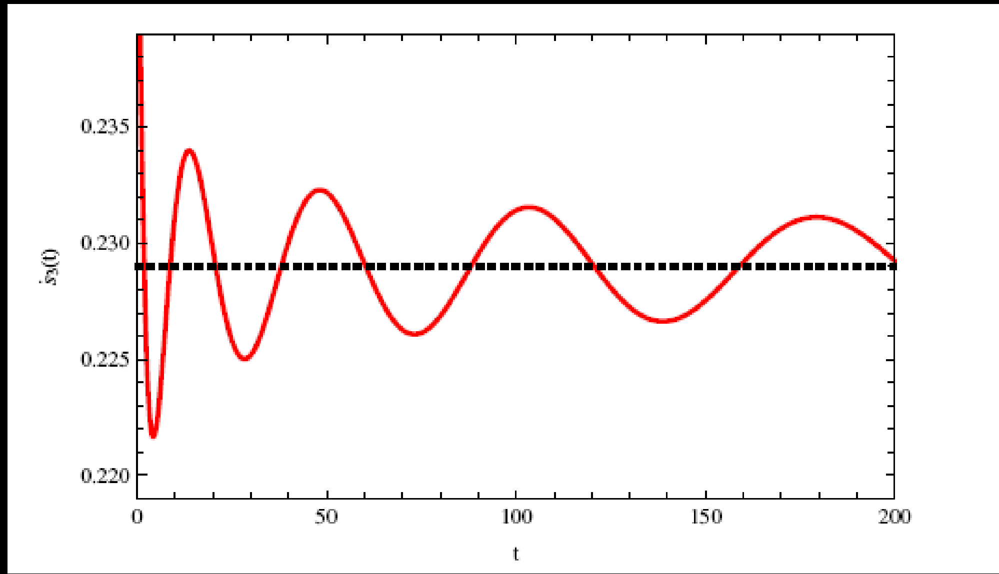
these configurations are stable

Unzipping cannot happen by simply perturbing $\beta_{\rm crit}$

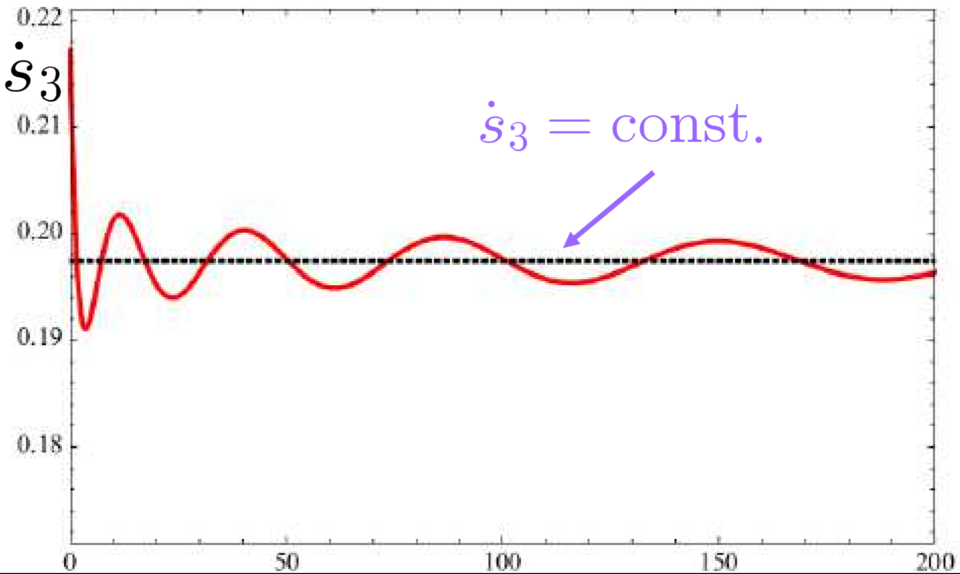


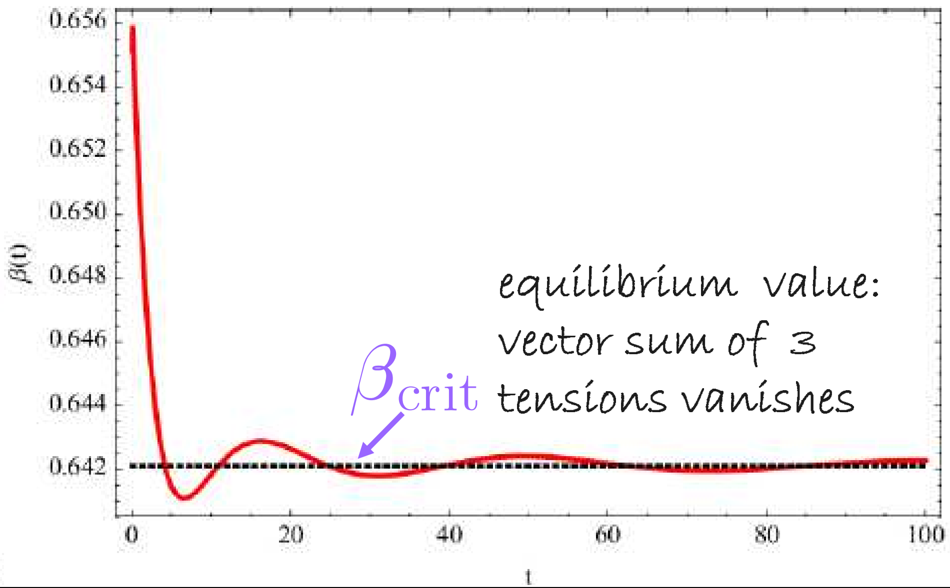


unequal tensions



equal tensions



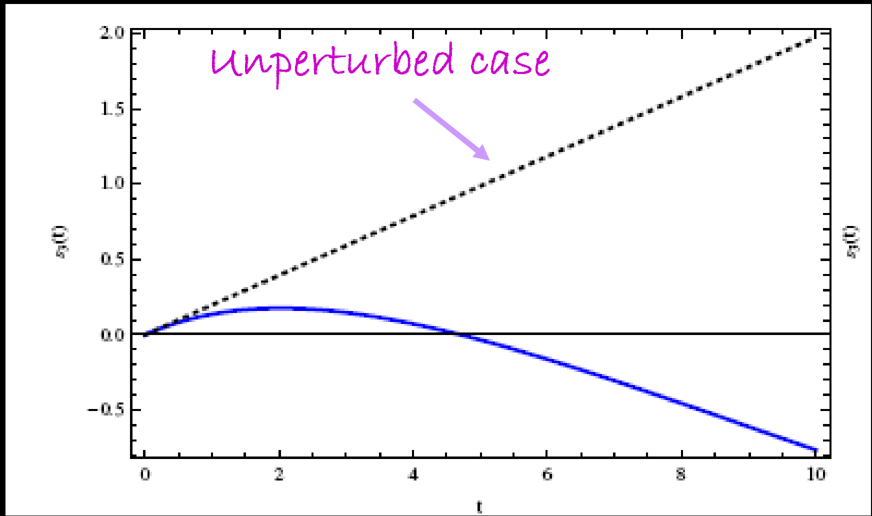


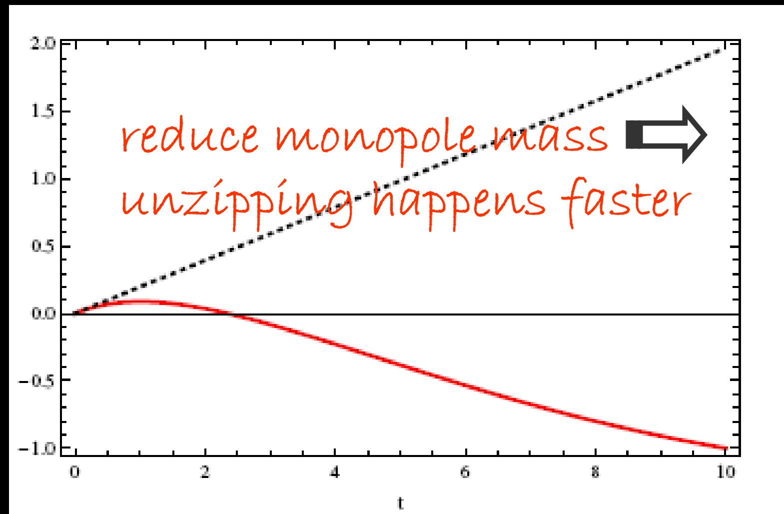
possible unzipping mechanisms:

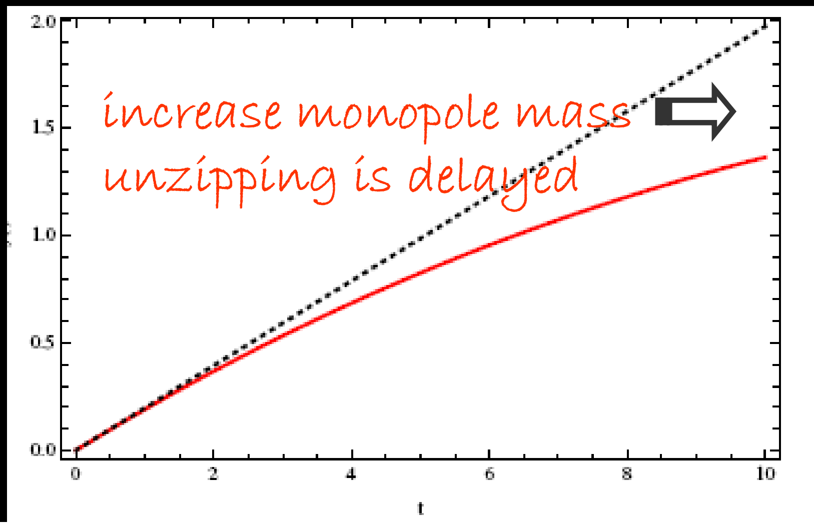
- velocity damping in expanding background
- monopole/string forces
- string curvature (loops with junctions)

monopole force

$$
\begin{aligned}
S &= -\sum_i \mu_i \int d\tau d\sigma_i \Theta(s_i(\tau)-\sigma_i)\sqrt{-x_i'^2 \dot{x}_i^2} \\
&\quad + \sum_i \int d\tau\, f_{i\mu} \cdot [x_i^\mu(\tau, s_i(\tau)) - X_m^\mu(\tau)] - m \int d\tau \sqrt{\dot{X}_m^2} \\
&\quad - q \int d\tau A_\nu(X_m^\mu(\tau))\dot{X}_m^\nu \,,
\end{aligned}
$$





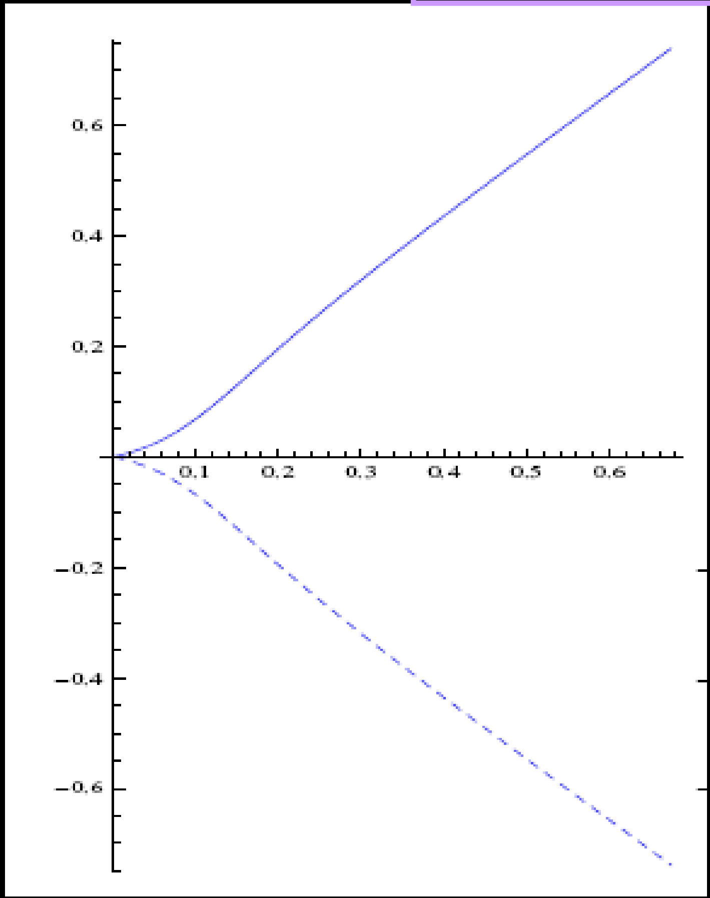


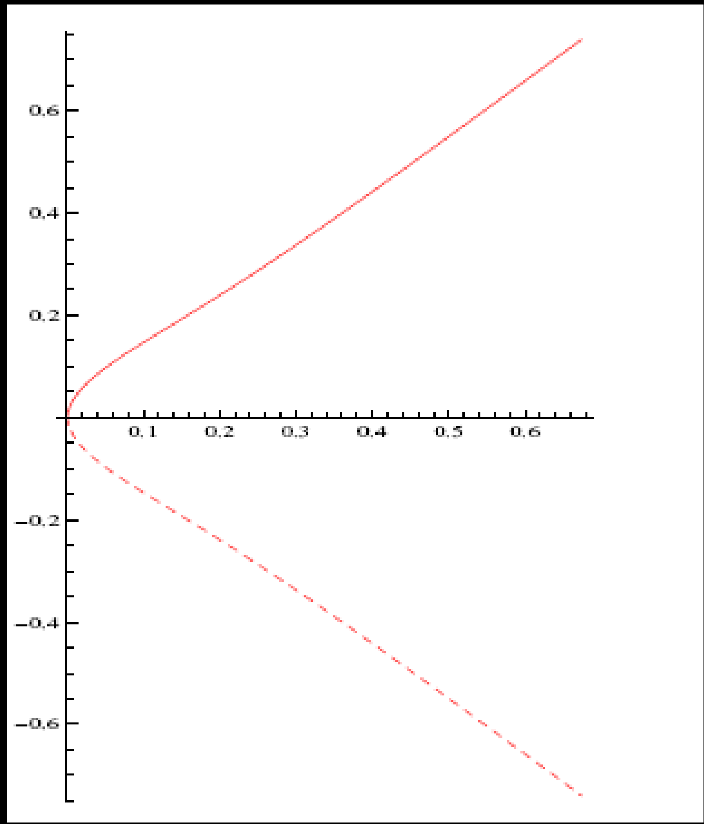
results depend on monopole mass and magnitude of force

string force

close to junction: linear force

away from junction: force exponentially damped





attractive force

$\beta < \beta_{\mathrm{crit}}$

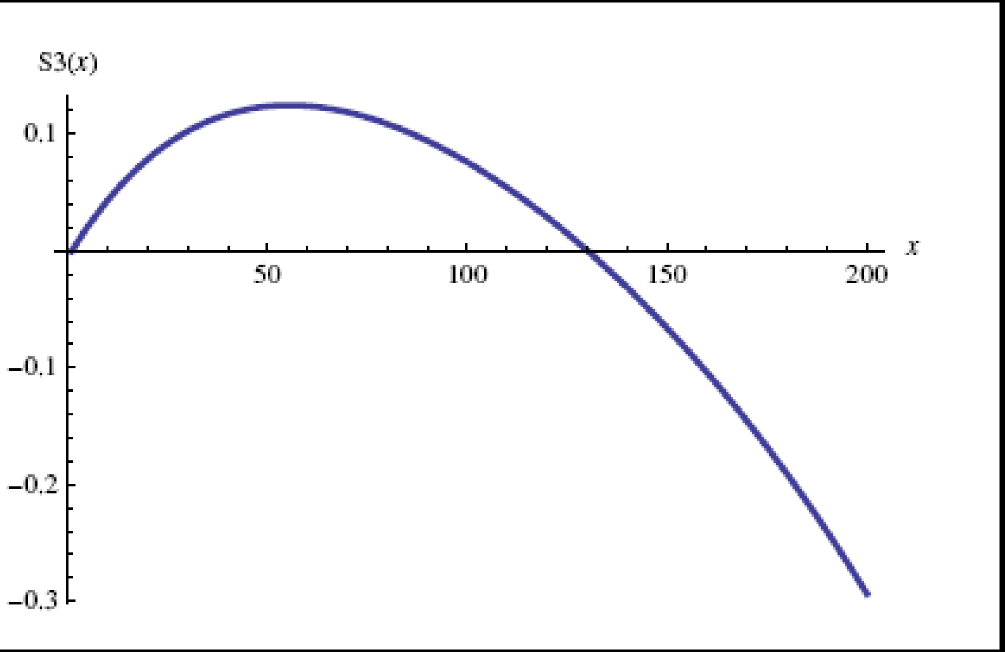
junction accelerates

repulsive force

$\beta > \beta_{\mathrm{crit}}$

junction decelerates

string curvature (loops with junctions)



heavier bound states can actually unzip, leading to a lower abundance of heavy strings in the network

in agreement with numerical simulations