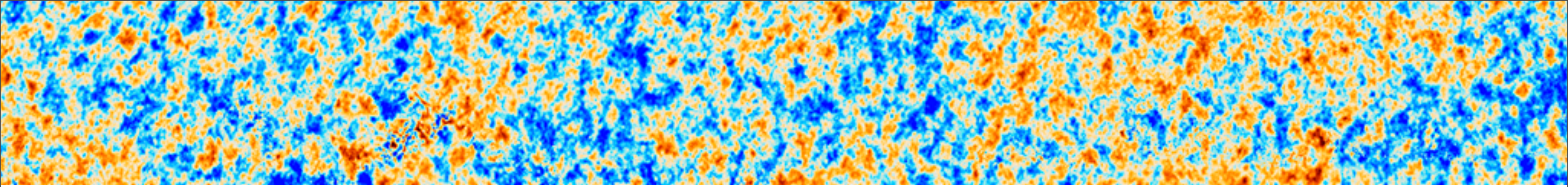


Cosmic Strings post Planck

Adam Moss

University of Nottingham

Observables

- CMB power spectrum. Includes recombination and post-recombination physics. Strongest limit on allowed fraction of strings, improved by Planck
- Non Gaussianity in CMB maps. Search for signatures of post-recombination Doppler shift induced by moving strings. Strongest limit also from Planck (see talk by Paul Shellard)
- CMB B modes. Defects produce comparable scalar, vector and tensor fluctuations. (see talk by Robert Brandenberger)
- 21 cm (see talk by Robert Brandenberger)
- Gravitational lensing (not discussed here)
- Pulsar timing/gravitational waves. Stochastic background from loops, waves from cusps (see talk by Richard Battye)

String Evolution

planck

- Perturbations from strings are **active**, so their evolution is key to understanding CMB anisotropies
- Strings evolve toward self-similar **scaling** regime
- Average properties of network are (nearly) constant with time
- Dynamics can be studied using numerical simulations
- Two approaches - **Nambu** and **Abelian-Higgs** models
- Both have advantages and disadvantages
- Main issue is **dynamical range** - assumptions have to be made in either case
- Will present Planck constraints for each case

esa

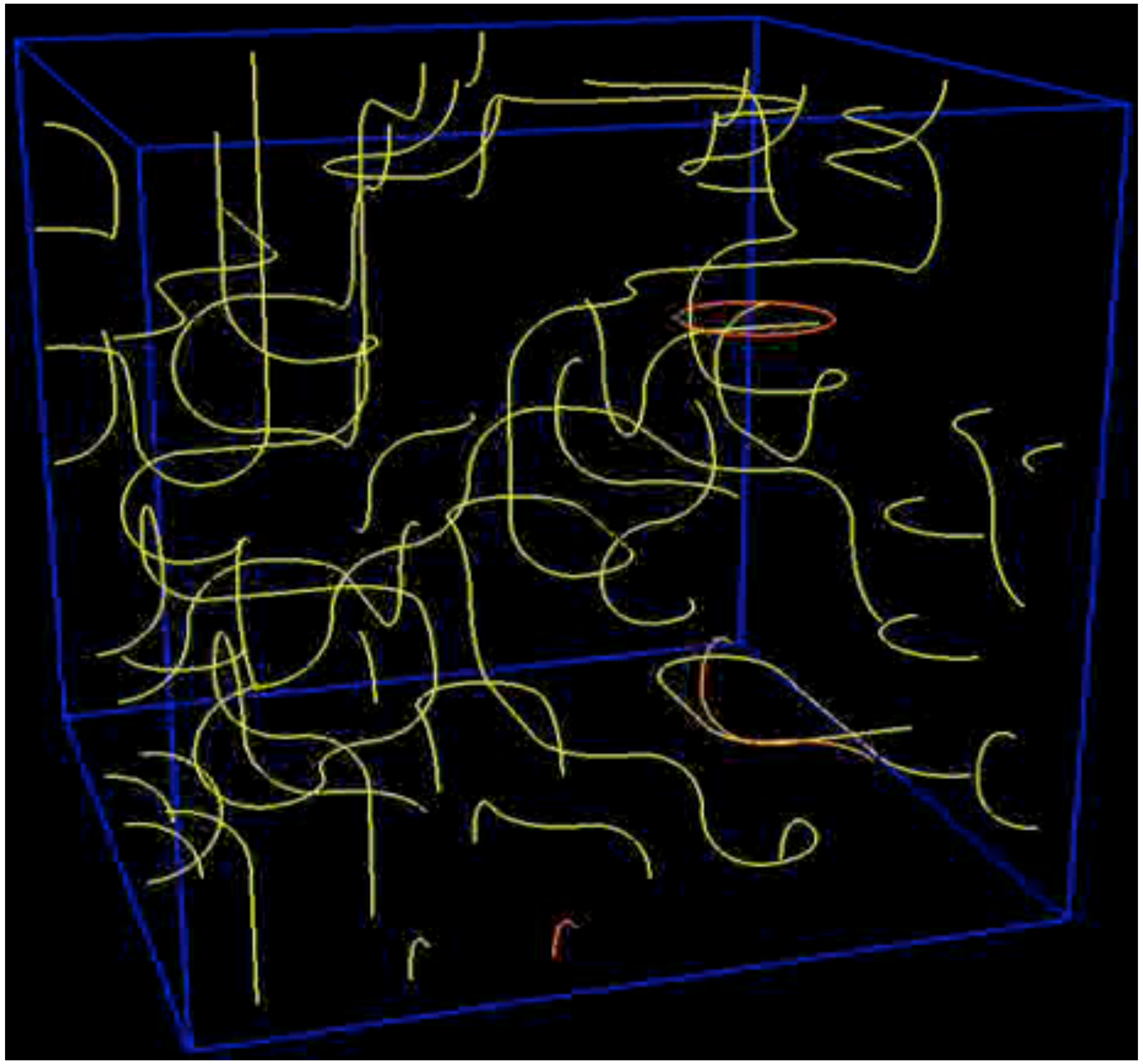
Nambu Model

- Thin string approximation
- Ignore radiation back reaction
- Impose reconnection by hand
- Network characterised by **correlation length** L
- Energy density is $$\rho = \frac{\mu}{L^2},$$
- Observationally, string tension μ is main quantity of interest
- Find scaling solution L ~ t
- Make measurements of correlation length, velocity, small scale structure (wiggliness) ......

(Martins and Shellard)

Nambu Model

- Thin string approximation
- Ignore radiation back reaction
- Impose reconnection by hand
- Network characterised by **correlation length** L
- Energy density is $$\rho = \frac{\mu}{L^2},$$
- Observationally, string tension μ is main quantity of interest
- Find scaling solution L ~ t
- Make measurements of correlation length, velocity, small scale structure (wiggliness) ......



(Martins and Shellard)

VOS Model

planck

- VOS = Velocity One-Scale model (Kibble 1985, Martins and Shellard 1996/2000)
- Expansion stretches strings, long strings reconnect and loops decay
- From Nambu action

Loop production, calibrated from simulations

Curvature term, proportional to RMS velocity v

$$\frac{1}{L}\frac{dL}{dt} = (1+v^2)H + \frac{\tilde{c}v}{2L}, \qquad \frac{dv}{dt} = (1-v^2)\left(\frac{\tilde{k}}{L} - 2Hv\right),$$

- For power law expansion $a(t) \propto t^{\beta}$, find attractor solution with scaling

$$L = \epsilon t, \qquad \epsilon = \sqrt{\frac{\tilde{k}(\tilde{k}+\tilde{c})}{4\beta(1-\beta)}}, \qquad v = \sqrt{\frac{\tilde{k}(1-\beta)}{\beta(\tilde{k}+\tilde{c})}},$$

- More complicated VOS models can be constructed for superstrings, networks with junctions etc.
- Will use comoving correlation length $l = \frac{L}{a} = \xi\tau,$

esa

VOS Parameters

planck

- Simulations find network parameters vary between matter and radiation eras
- Density in radiation era is greater than in matter era
- Simulations of Martins and Shellard find

$$\xi_{\rm rad} = 0.13 \rightarrow \xi_{\rm mat} = 0.21 \qquad v_{\rm rad} = 0.65 \rightarrow v_{\rm mat} = 0.60$$

- Ringeval et al find

$$\xi_{\rm rad} = 0.16 \rightarrow \xi_{\rm mat} = 0.19$$

- Strings also have small scale structure, or 'wiggliness' (Carter 2000)
- Effective coarse grained energy momentum tensor gives rescaled mass per unit length $U = \alpha\mu,$
- Estimated to be

$$\alpha_{\rm rad} = 1.5 \rightarrow \alpha_{\rm mat} = 1.9$$

Health warning: CMBACT v3

esa

Abelian Higgs

- Lagrangian $\mathcal{L} = (D_\mu \Phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi)$
- Solve equations of motion on 3D lattice (Bevis et al)
- Extract unequal time correlator (UETC) of energy momentum tensor

$$\langle \Theta(k, \tau_1) \Theta(k, \tau_2) \rangle$$

- Radiation into propagating modes included
- However, simulation box sizes limited to ~ 300x string core width
- Requires slowing down rate of growth of core (pre 2014, s<1)
- Found only very small loop production - most of energy loss to radiation
- See no observed small scale structure
- Similar network parameters in matter and radiation eras

$$\xi = 0.3 \qquad v = 0.5$$

(Hindmarsh et al 2009)

- More work required to resolve differences with Nambu model

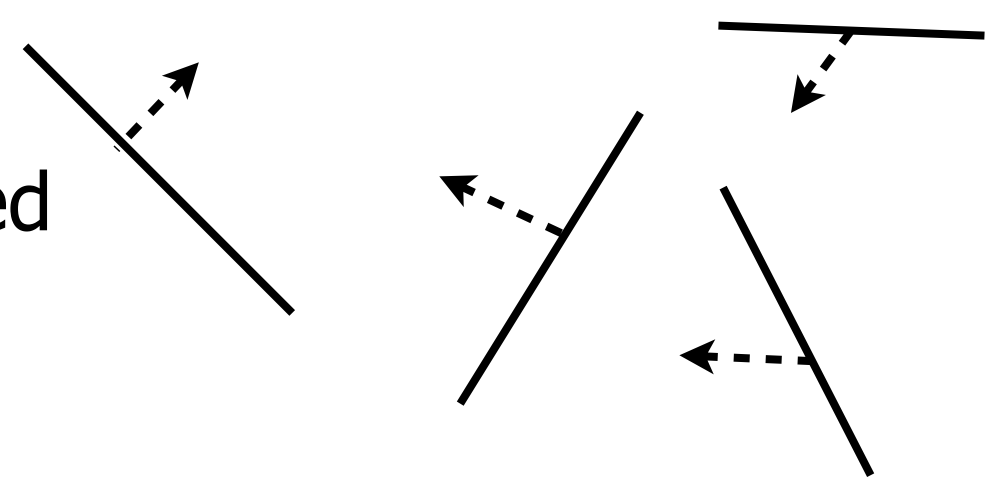
CMB Anisotropies

- **Key** ingredient is the UETC
- Use stiff source approximation

String energy momentum tensor

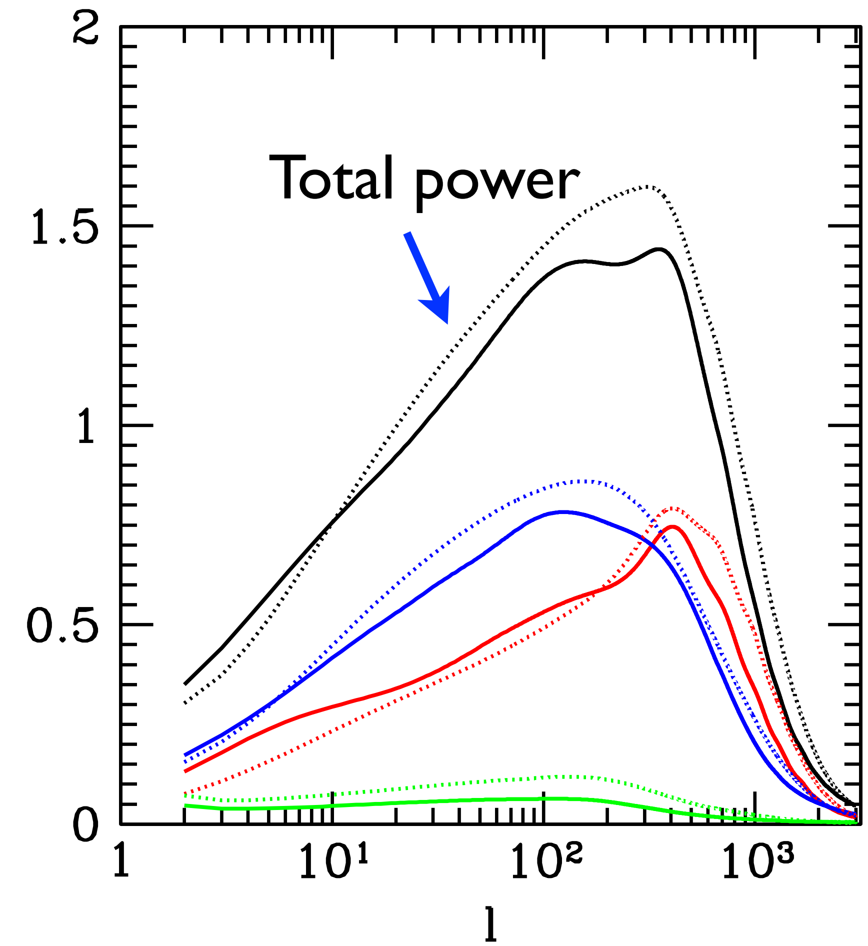
$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \delta G_{\mu\nu} = 8\pi G\left(\delta T_{\mu\nu} + \theta_{\mu\nu}\right)$$

- Can estimate UETC directly from simulations, and use as sources in CMB codes
- For thin strings there is a useful intermediate framework called Unconnected Segment Model (USM) (Vincent et al, Albrecht et al, Pogosian and Vachaspati)
- Model strings are ensemble of uncorrelated straight segments, each moving with random velocity
- Inputs are correlation length, velocity and wiggliness, as measured from simulations



Mimic Model

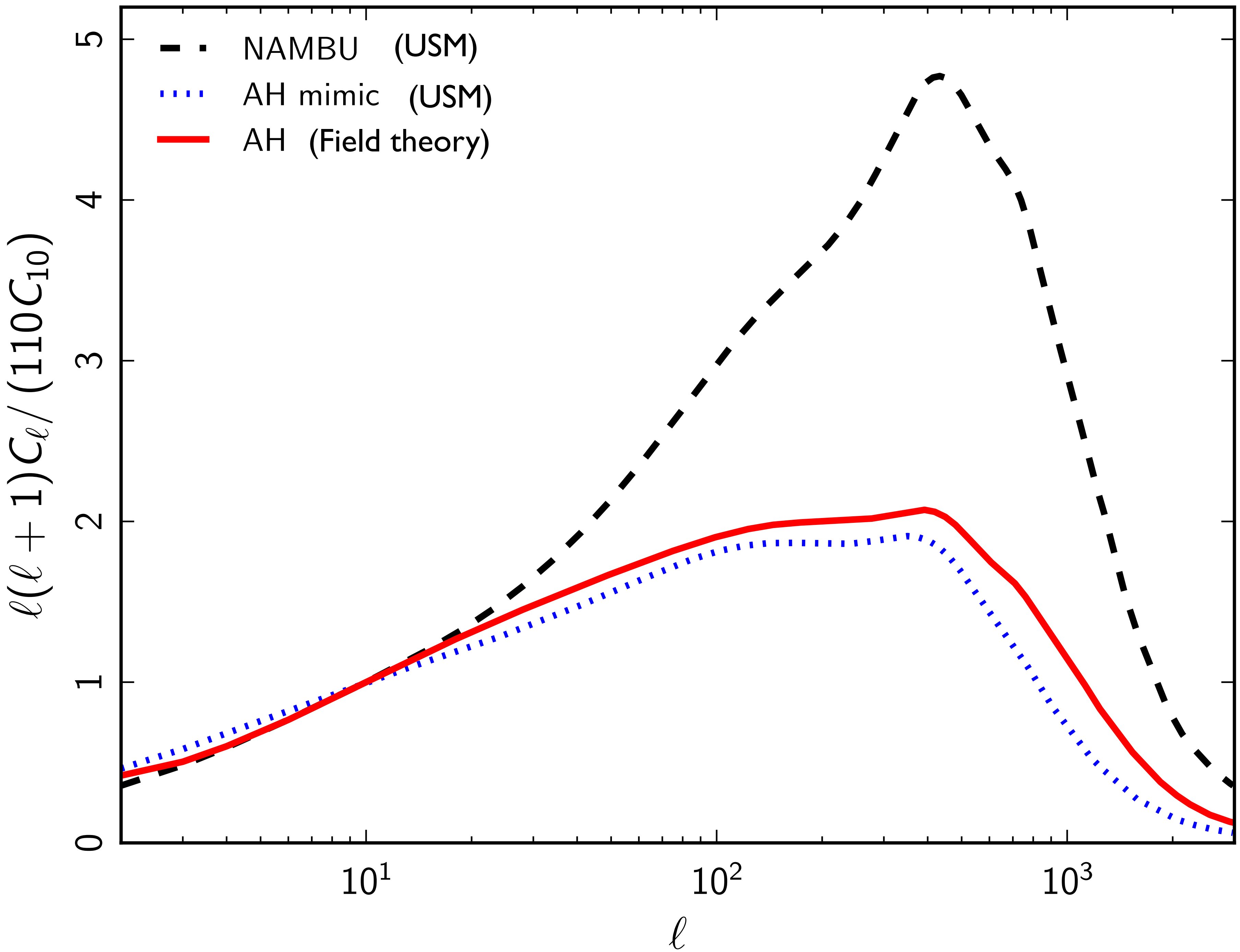
- Mimic model computed with USM, choosing parameters to more closely resemble Abelian-Higgs
- Turn off evolution of network parameters between matter and radiation eras
- Choose $\xi = 0.35 \quad v = 0.4 \quad \alpha = 1.05$
- Encouragingly find closer agreement between spectra
- Remember Nambu suffers less of an issue with dynamical range



- Abelian-Higgs has advantage of including radiation
- **Even with simple physics USM does a good job of fitting both**!

Health warning: CMBACT v3

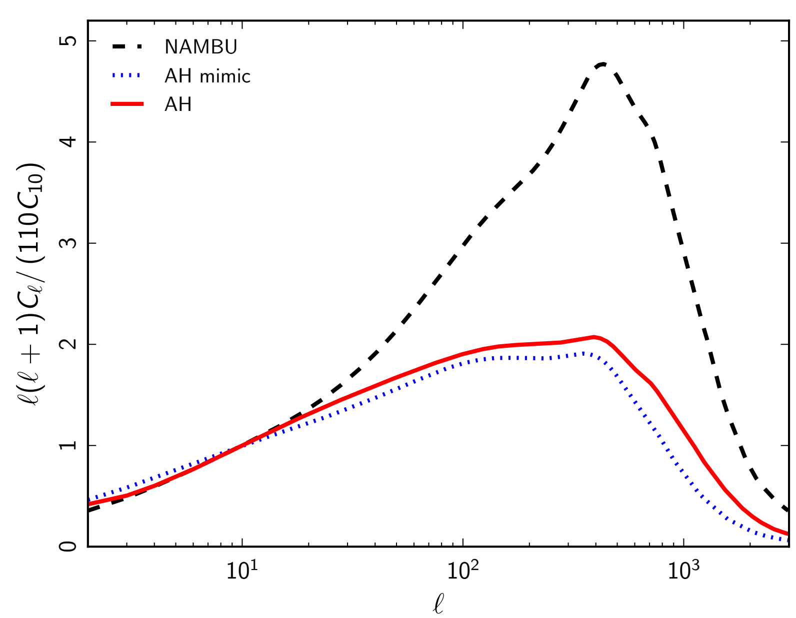
Power Spectrum



Health warning: CMBACT v3

Power Spectrum

- Note: Amplitude set by mass per unit length $C_\ell^{\rm string} \propto (G\mu)^2$
- Two primary contributions to spectrum - density perturbations at last scattering, Kaiser-Stebbins effect along line of sight
- Peak position set by correlation length at last scattering
- Why the difference between Nambu and Abelian-Higgs?
- Normalisation determined by correlation length
- Nambu model has more small scale power - smaller string correlation length in radiation era, by factor of **~1.6**
- Different split into scalar, vector and tensors - small scale structure of strings



USM

- Let's now revisit the USM. In existing codes (CMBACT) the UETC is **not** computed
- Rather an ensemble of source histories are created, then averaged to find power spectra
- The EM tensor of a straight string segment is

$$\Theta_{00} = \frac{\mu\alpha}{\sqrt{1-v^2}} \frac{\sin(k\hat{X}_3\xi\tau/2)}{k\hat{X}_3/2} \cos(\chi + k\hat{\dot{X}}_3 v\tau), \qquad \Theta_{ij} = \left[v^2 \hat{\dot{X}}_i \hat{\dot{X}}_j - \frac{1-v^2}{\alpha} \hat{X}_i \hat{X}_j \right] \Theta_{00},$$

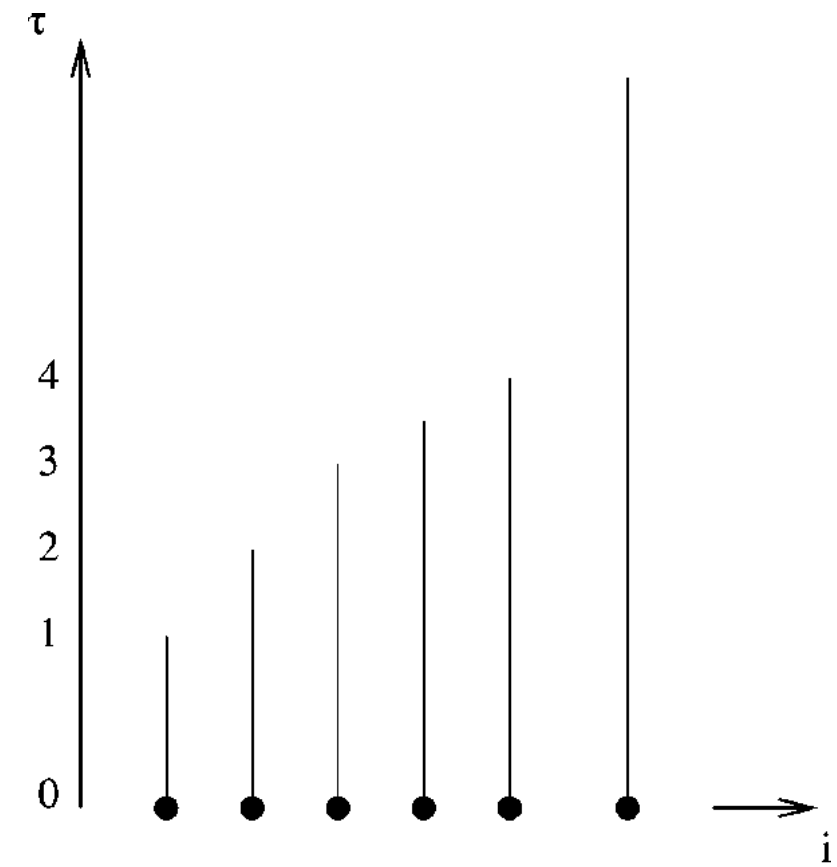
- $\hat{X}$ and $\hat{\dot{X}}$ are randomly orientated unit vectors satisfying $\hat{X} \cdot \hat{\dot{X}} = 0$
- Phase χ set by location of string
- Orientated wave vector as $\mathbf{k} = k\hat{k}_3$, perform scalar, vector, tensor split

$$\Theta^S = (2\Theta_{33} - \Theta_{11} - \Theta_{22})/2, \qquad \Theta^V = \Theta_{13}, \qquad \Theta^T = \Theta_{12}.$$

USM

- During scaling number density of strings scales as $n(\tau) \propto \tau^{-3}$, requires tracking very large number of segments
- USM consolidates all segments that decay at some time into a **single** segment
- Number of segments that decay between τ_i and τ_{i-1} is

$$N_d(\tau_i) = V[n(\tau_{i-1}) - n(\tau_i)],$$



- EM tensor of network (K consolidated segments) is

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \Theta^i_{\mu\nu} T^{\mathrm{off}}(\tau, \tau_i, L_f),$$

- Note consolidated segment has weight $\sqrt{N_d}$
- T^{off} is segment decay function

Analytic UETC

- From these it is possible to work out the USM UETC **analytically**

(Avogoustidis et al 2012)

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \Theta(k,\tau_1)\Theta(k,\tau_2),$$

- The scaling function can be computed analytically from the segment decay

$$f(\tau_1,\tau_2,\xi,L_f \to 1) = \frac{1}{[\xi \mathrm{Max}(\tau_1,\tau_2)]^3}.$$

- UETC involves several integrals which are doable, e.g.

$$\frac{1}{2}\int_0^{\pi} d\theta \sin^3\theta \cos(x\cos\theta) J_0(\rho\sin\theta) = \left[1+\frac{\partial^2}{\partial x^2}\right]\left(\frac{\sin\sqrt{\rho^2+x^2}}{\sqrt{\rho^2+x^2}}\right),$$

- And only **two** which weren't, e.g.

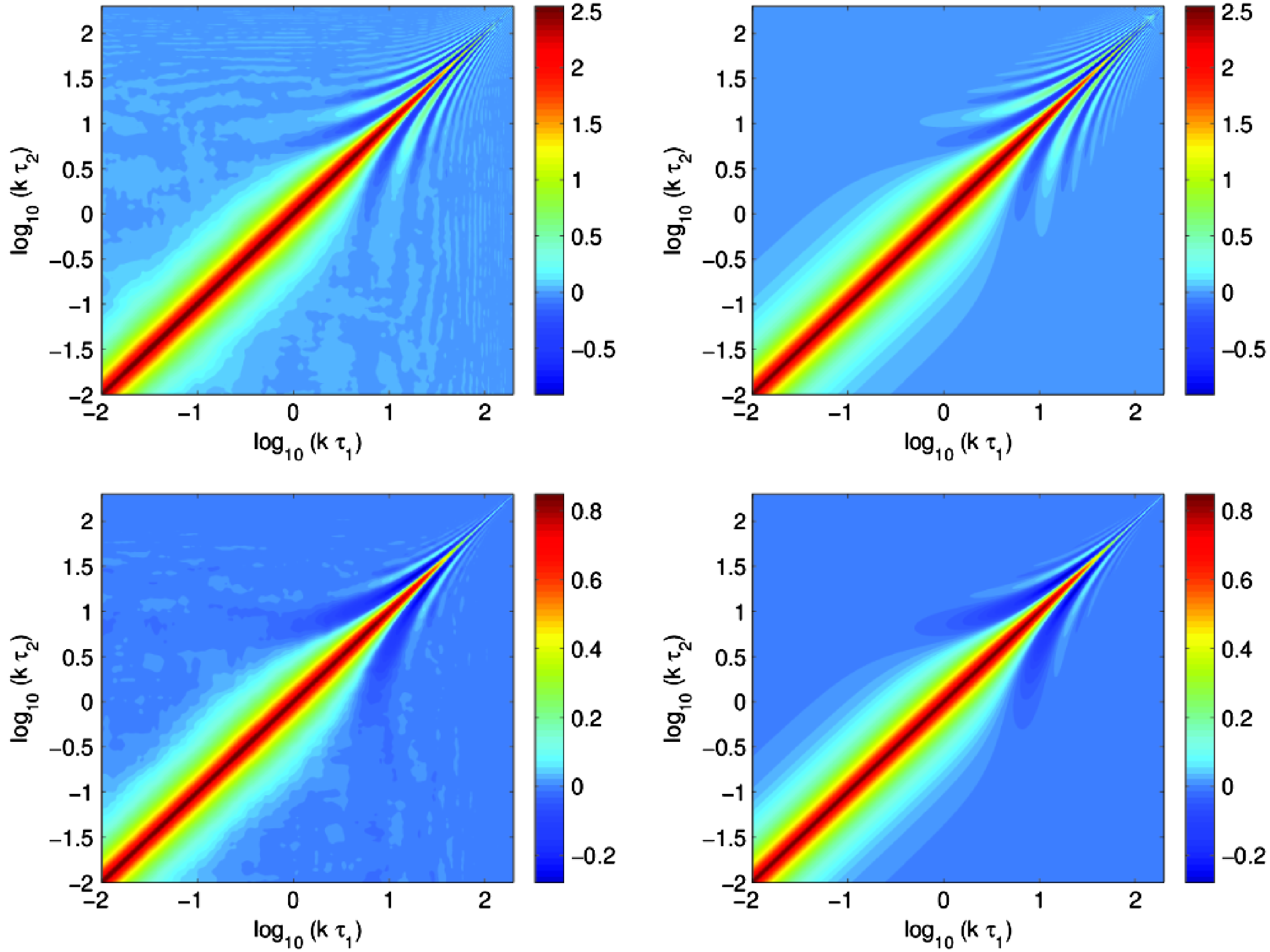
$$\frac{1}{2}\int_0^{\pi} d\theta \sin\theta \cos(x\cos\theta) J_0(\rho\sin\theta)\sec^2\theta = \sum_{c=0}^{\infty}\frac{1}{c!}\frac{\rho}{(2c-1)}\left(-\frac{x^2}{2\rho}\right)^c j_{c-1}(\rho),$$

Comparison

planck

Scalar anisotropic stress, numerical simulations

Scalar anisotropic stress, analytic



Vector anisotropic stress, numerical simulations

Vector anisotropic stress, analytic

(Hours)

(Seconds)

esa

Eigenmodes

- UETC can be decomposed into eigenmodes - each of these are **coherent**, can be used as source functions in CMB code

(Pen et al 1997)

$$(k^2\tau_1\tau_2)^\gamma(\tau_1\tau_2)^{1/2}\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \sum_{i=1}^{n}\lambda_i u_i(k\tau_1)\otimes u_i(k\tau_2),$$

- Diagonalization introduces a change of basis, but since modes are orthogonal and perturbations are linear, CMB sources are

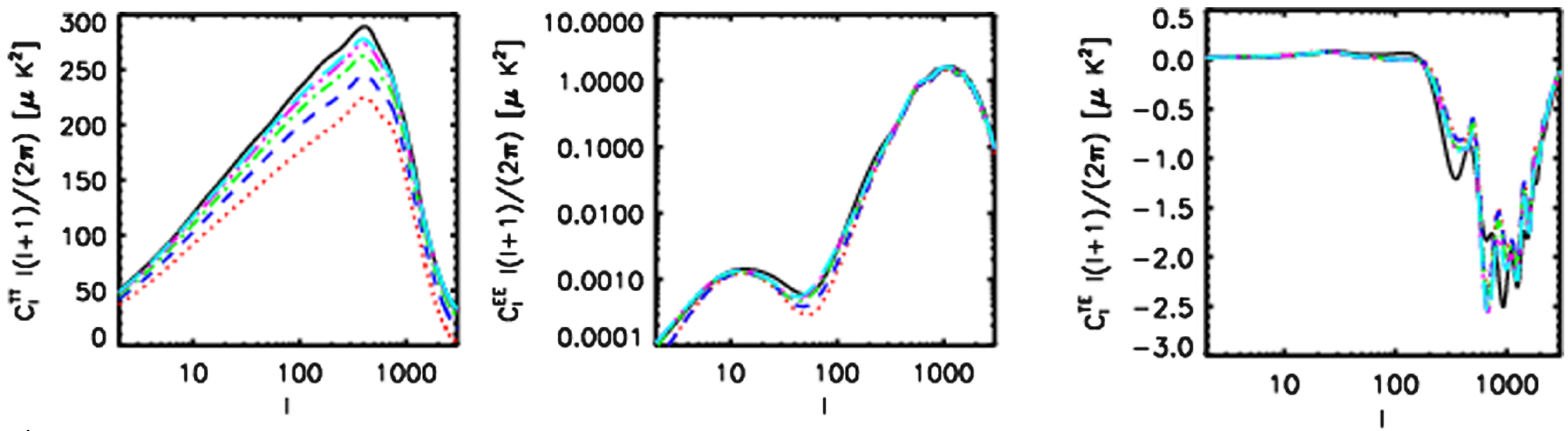
$$\Theta(k\tau) \longrightarrow \frac{u(k\tau)}{(k\tau)^\gamma \tau^{1/2}}.$$

- Power spectra found by summing over eigenmodes (ordered from highest to lowest) and truncated at some number

$$C_\ell = \sum_{i=1}^{n}\lambda_i C_\ell^i,$$

CMB Spectra

- CMB spectra are $$C_\ell^{i(I)} = \frac{2}{\pi} \int k^2 dk \Delta_\ell^{i(I)}(k, \tau_0) \Delta_\ell^{i(I)}(k, \tau_0),$$
- Incorporated into CAMB - CAMBACT (Avogoustidis et al 2012)
- E.g. scalar spectra



$$C_\ell^{\text{string}} \propto (G\mu)^2$$

Black - CMBACT, 2000 realizations
Red - CAMBACT, 16 eigenmodes
Green - CAMBACT, 64 eigenmodes
Magenta - CAMBACT, 128 eigenmodes

Health warning: CMBACT v3

Some Numbers

planck

- Running CMBACT for 2000 source realisations takes ~ 20 hours
- Impossible to run any sort of MCMC with Planck (other than fitting for the overall amplitude of a given spectrum)
- With CAMBACT we need somewhere between 50-100 eigenmodes for reasonable accuracy
- Each mode requires running CAMB for scalars, vectors and tensors (i.e. 150-300 CAMB evaluations)
- CAMB takes ~ 1 second to run, hence total CAMBACT computational time is several minutes
- A big improvement, Planck MCMC implementation **work in progress**
- Philosophy - use USM, marginalize over string parameters

esa

Abelian Higgs

planck

- So far focused on USM case. Idea is to use simulations (both Nambu and Abelian-Higgs) to inform network parameters
- State of the art Abelian-Higgs simulations have also been used for the Planck analysis (Bevis et al)
- Fields evolved on 1024^3 grid, starting from random initial conditions designed to mimic a phase transition
- Brief diffusive period ensures system rapidly reaches scaling
- String cores are partially fattened to enlarge dynamical range
- Various runs performed to check results insensitive to string fattening parameter
- UETC's calculated at regular intervals and used in CMBEASY code

esa

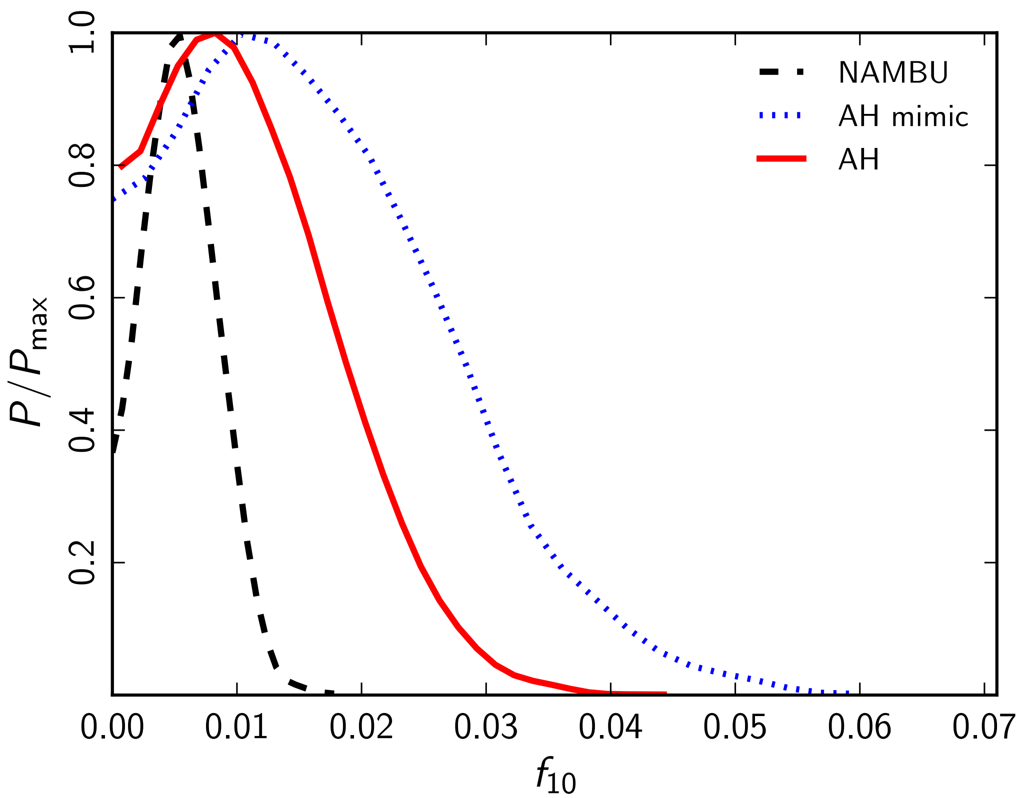
Planck Constraints

- Additional parameter added to LCDM, fraction of strings at $\ell = 10$

$$f_{10} = \frac{C_\ell^{\text{string}}}{C_\ell^{\text{string}} + C_\ell^{\text{inflation}}}$$

- Use same dataset/priors as in main cosmology paper

Defect type	*Planck*+WP		*Planck*+WP+highL	
	f_{10}	$G\mu/c^2$	f_{10}	$G\mu/c^2$
NAMBU	0.015	1.5×10^{-7}	0.010	1.3×10^{-7}
AH-mimic	0.033	3.6×10^{-7}	0.034	3.7×10^{-7}
AH	0.028	3.2×10^{-7}	0.024	3.0×10^{-7}
SL	0.043	11.0×10^{-7}	0.041	10.7×10^{-7}
TX	0.055	10.6×10^{-7}	0.054	10.5×10^{-7}

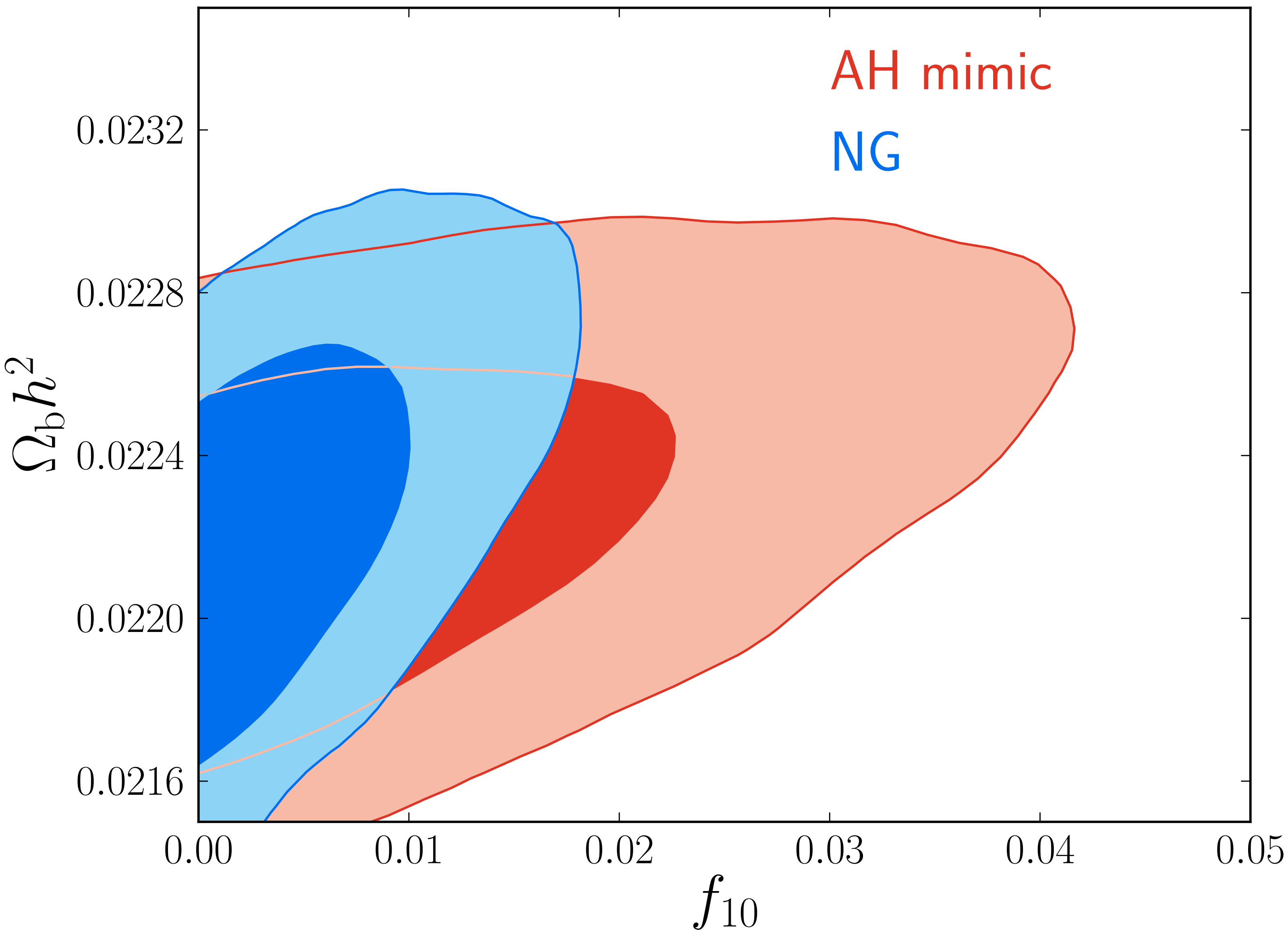


Planck + WP + highL

- Limits on string tension significantly improved, e.g. in Nambu model string fraction <1%

Planck Constraints

- Strings exhibit no significance correlations with any other cosmological parameter



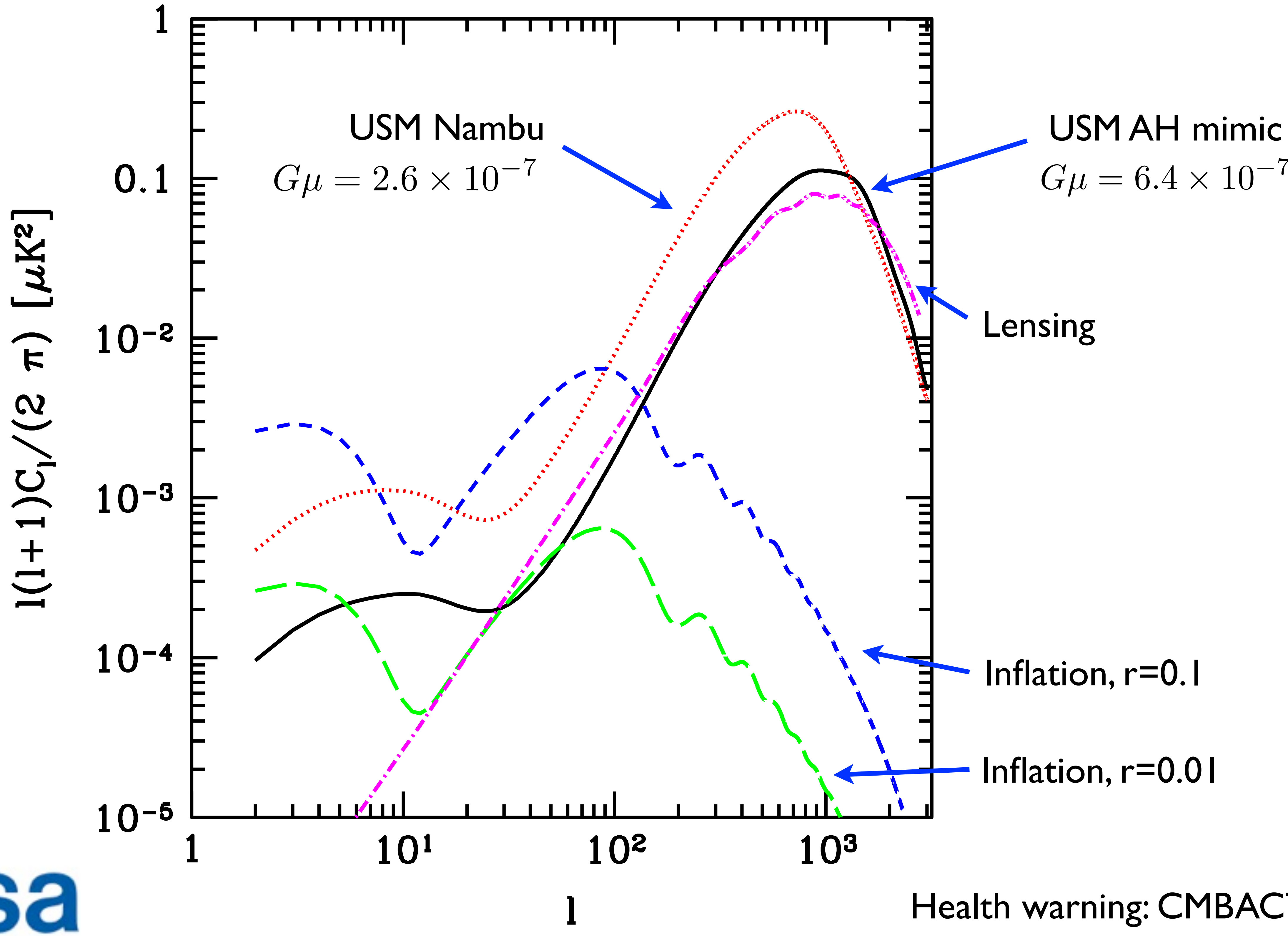
Conclusions

- Planck has provided improved limits on strings
- Power spectrum gives

$$G\mu/c^2 < 1.5 \times 10^{-7}\,, \qquad f_{10} < 0.015\,, \qquad \text{(Nambu)}$$
$$G\mu_{\rm AH}/c^2 < 3.2 \times 10^{-7}\,, \qquad f_{10} < 0.028\,. \qquad \text{(Abelian-Higgs)}$$

- String evolution is crucial for understanding CMB anisotropies
- Still some uncertainty in modelling approaches - main problem is dynamical range
- Improved USM model will enable MCMC search of string parameter space
- Is the best solution to use USM and marginalize?

B Modes



Health warning: CMBACT v3