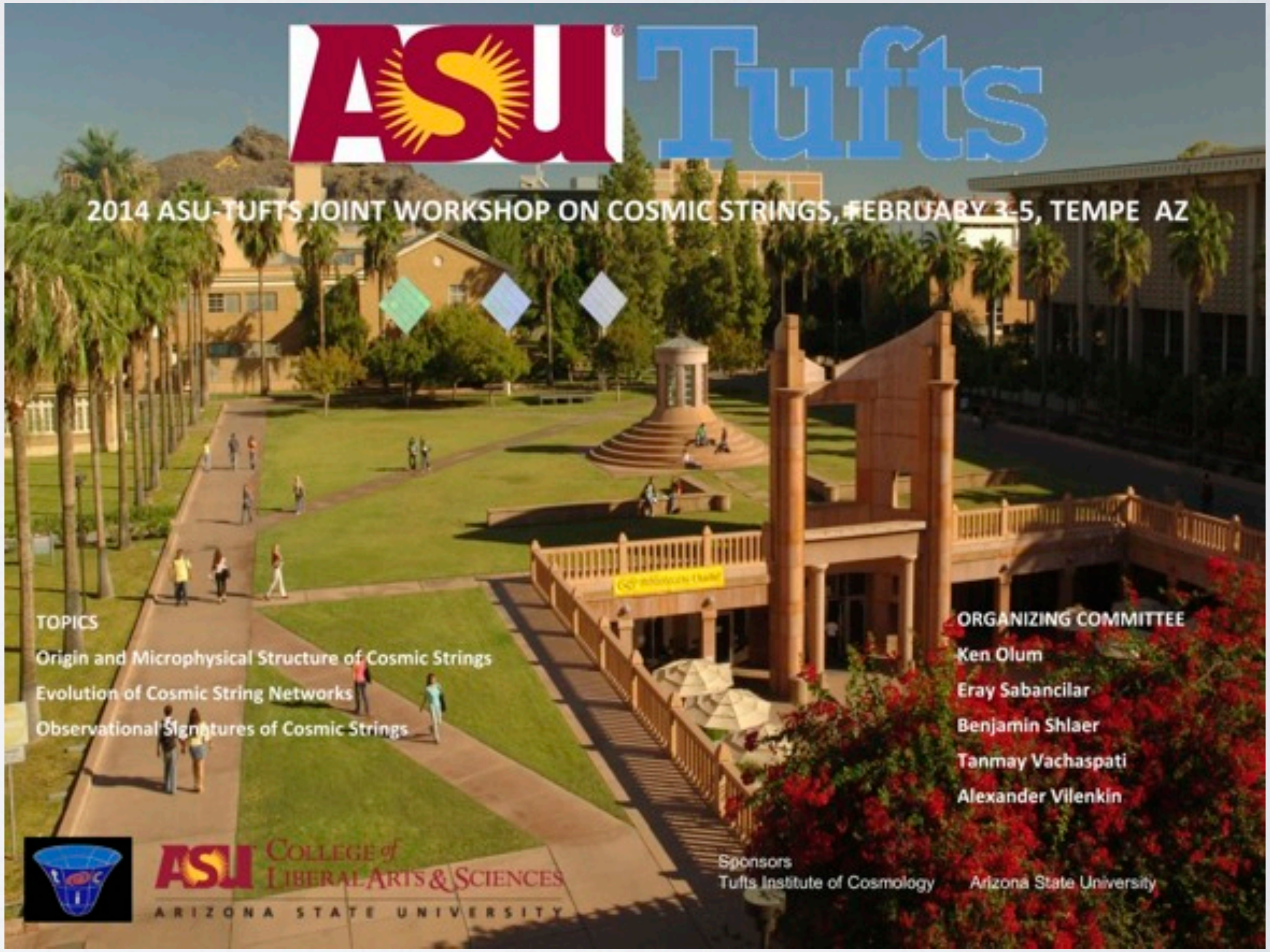
Matter Power Spectrum and Bispectrum from Cosmic String Wakes

Donough Regan
Sussex University

February 4, 2014

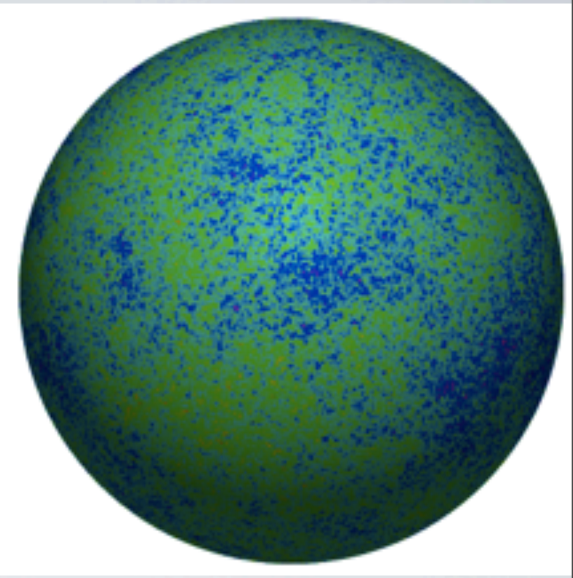
Work done with Mark Hindmarsh... to appear on the arXiv shortly (I hope!)



OUTLINE

- Background and Motivation
- Methodology
- Power Spectrum
- Bispectrum
- Conclusions

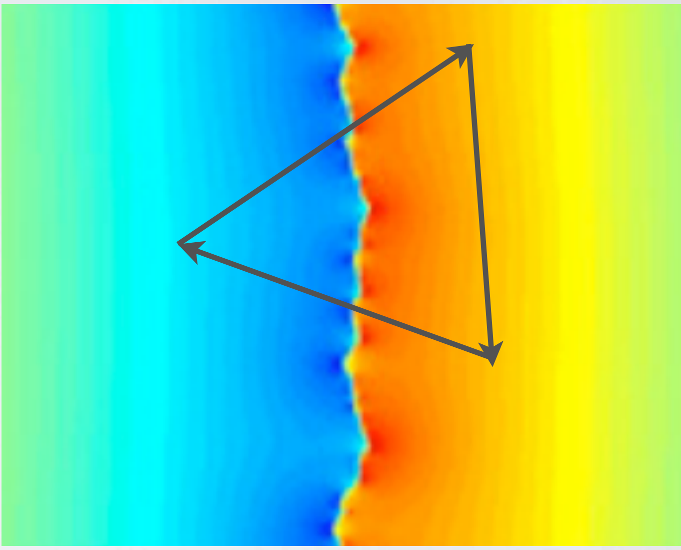
Background and Motivation



- CMB Power Spectrum accepted test bed for cosmic strings $G\mu/c^2 < 1.5 \times 10^{-7}$ $G\mu_{\mathrm{AH}}/c^2 < 3.2 \times 10^{-7}$
- Cosmic strings ARE non-Gaussian. Bispectrum?

Computed in arXiv:0908.0432 (Hindmarsh, Ringeval, Suyama)
arXiv:0911.2491 (DMR, Shellard)

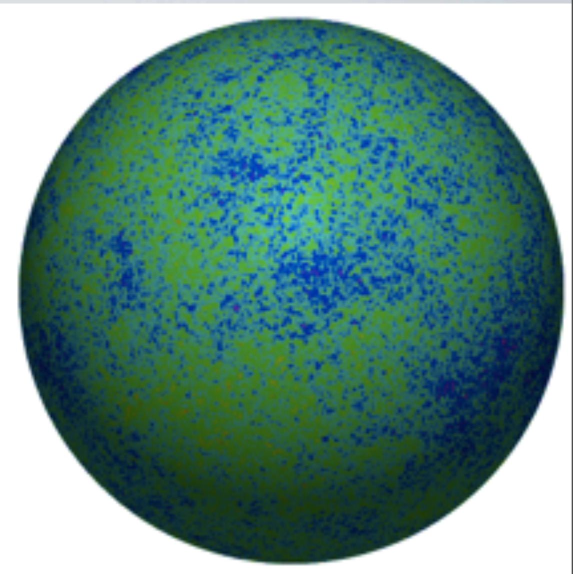
Symmetry suppressed signal but a possibility...



All-sky maps generated by Ringeval and Bouchet
arXiv:1204.5041 and in Planck arXiv:1303.5085 $G\mu/c^2 < 8.8 \times 10^{-7}$
with bispectrum extracted via modal methods(see Shellard talk)

- Trispectrum? NOT symmetry suppressed signal. Possibly competitive...
See arXiv:0911.1241, arXiv:0911.2491 and arXiv:1012.6039 $G\mu \leq 1.1 \times 10^{-6}$

Background and Motivation



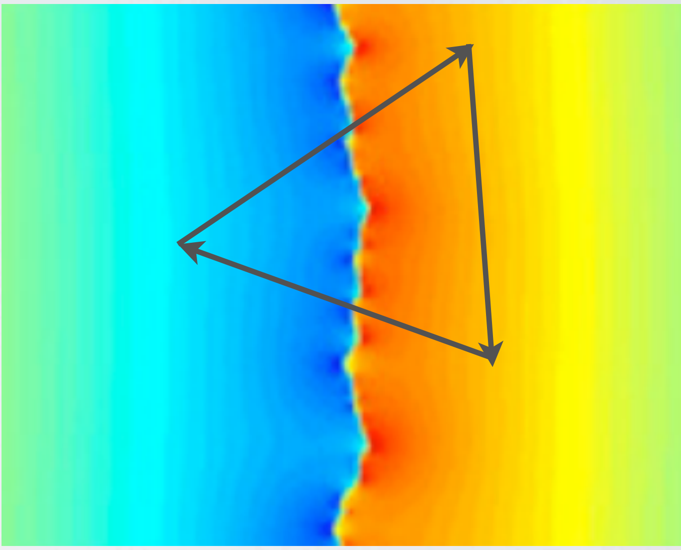
- CMB Power Spectrum accepted test bed for cosmic strings $G\mu/c^2 < 1.5 \times 10^{-7}$ $G\mu_{\rm AH}/c^2 < 3.2 \times 10^{-7}$

- Cosmic strings ARE non-Gaussian. Bispectrum?

Computed in arXiv:0908.0432 (Hindmarsh, Ringeval, Suyama)
arXiv:0911.2491 (DMR, Shellard)

Symmetry suppressed signal but a possibility...

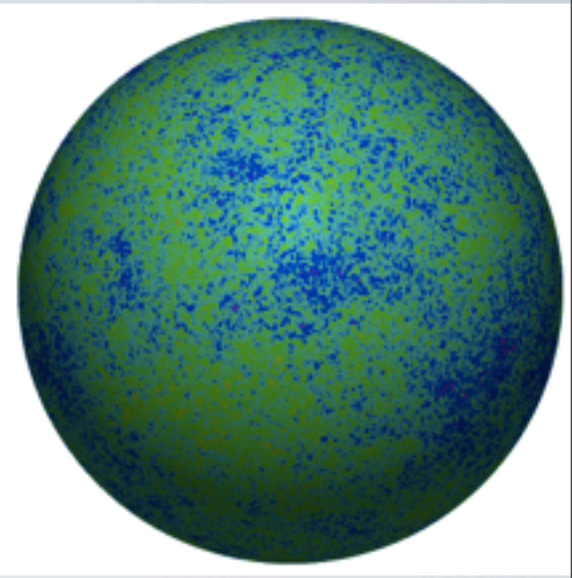
All-sky maps generated by Ringeval and Bouchet
arXiv:1204.5041 and in Planck arXiv:1303.5085 $G\mu/c^2 < 8.8 \times 10^{-7}$
with bispectrum extracted via modal methods(see Shellard talk)



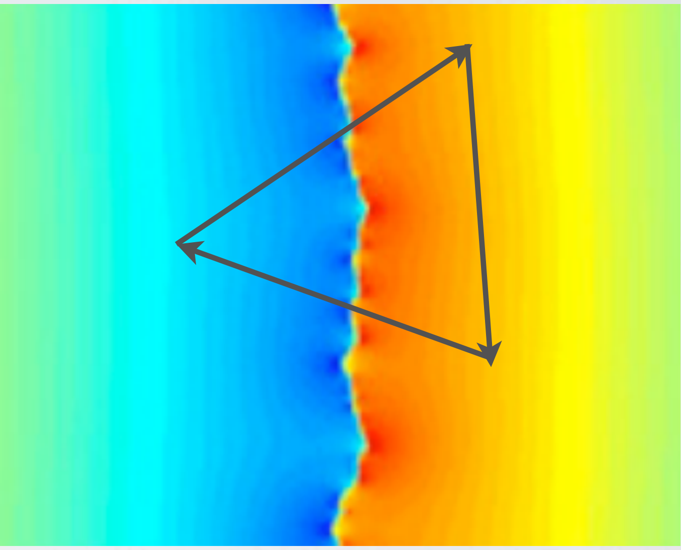
- Trispectrum? NOT symmetry suppressed signal. Possibly competitive...
See arXiv:0911.1241, arXiv:0911.2491 and arXiv:1012.6039 $G\mu \leq 1.1 \times 10^{-6}$

- What about matter spectra? Soon galaxy surveys will have enough modes to exceed CMB sensitivity

Background and Motivation



- CMB Power Spectrum accepted test bed for cosmic strings $G\mu/c^2 < 1.5 \times 10^{-7}$ $G\mu_{AH}/c^2 < 3.2 \times 10^{-7}$
- Cosmic strings ARE non-Gaussian. Bispectrum?



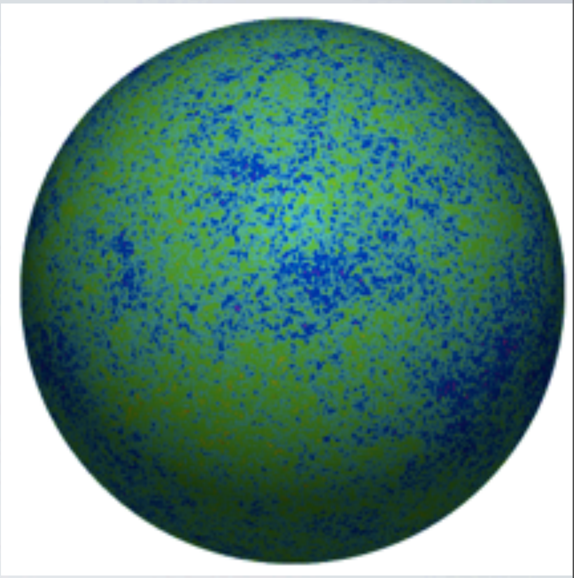
Computed in arXiv:0908.0432 (Hindmarsh, Ringeval, Suyama)
arXiv:0911.2491 (DMR, Shellard)

Symmetry suppressed signal but a possibility...

All-sky maps generated by Ringeval and Bouchet
arXiv:1204.5041 and in Planck arXiv:1303.5085 $G\mu/c^2 < 8.8 \times 10^{-7}$
with bispectrum extracted via modal methods(see Shellard talk)

- Trispectrum? NOT symmetry suppressed signal. Possibly competitive...
  See arXiv:0911.1241, arXiv:0911.2491 and arXiv:1012.6039 $G\mu \leq 1.1 \times 10^{-6}$
- What about matter spectra? Soon galaxy surveys will have enough modes to exceed CMB sensitivity
- Matter Bispectrum will not be symmetry suppressed

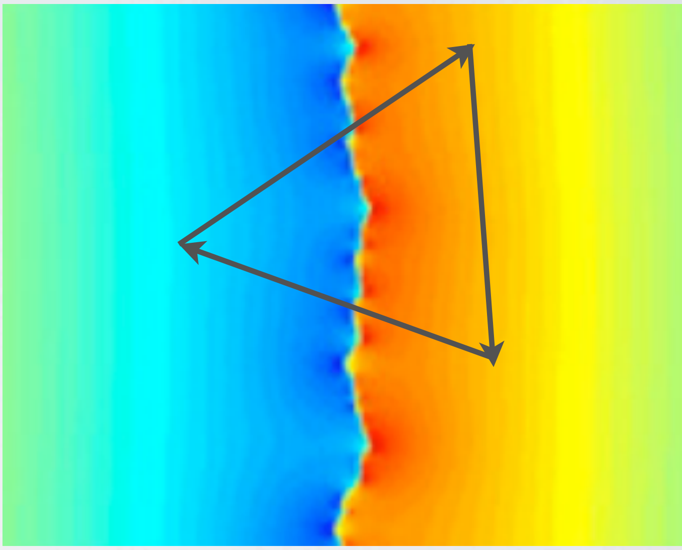
Background and Motivation



- CMB Power Spectrum accepted test bed for cosmic strings $G\mu/c^2 < 1.5 \times 10^{-7}$ $G\mu_{\mathrm{AH}}/c^2 < 3.2 \times 10^{-7}$
- Cosmic strings ARE non-Gaussian. Bispectrum?

Computed in arXiv:0908.0432 (Hindmarsh, Ringeval, Suyama)
arXiv:0911.2491 (DMR, Shellard)

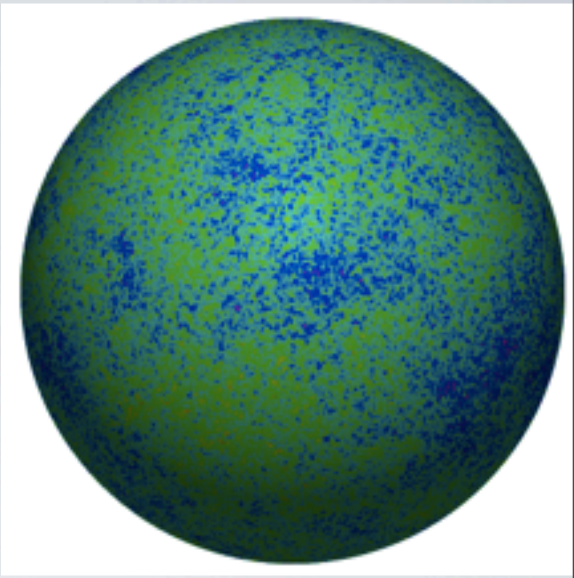
Symmetry suppressed signal but a possibility...



All-sky maps generated by Ringeval and Bouchet
arXiv:1204.5041 and in Planck arXiv:1303.5085 $G\mu/c^2 < 8.8 \times 10^{-7}$
with bispectrum extracted via modal methods(see Shellard talk)

- Trispectrum? NOT symmetry suppressed signal. Possibly competitive...
  See arXiv:0911.1241, arXiv:0911.2491 and arXiv:1012.6039 $G\mu \le 1.1 \times 10^{-6}$
- What about matter spectra? Soon galaxy surveys will have enough modes to exceed CMB sensitivity
- Matter Bispectrum will not be symmetry suppressed
- Wakes could dominate non-linear structures at high z

Background and Motivation

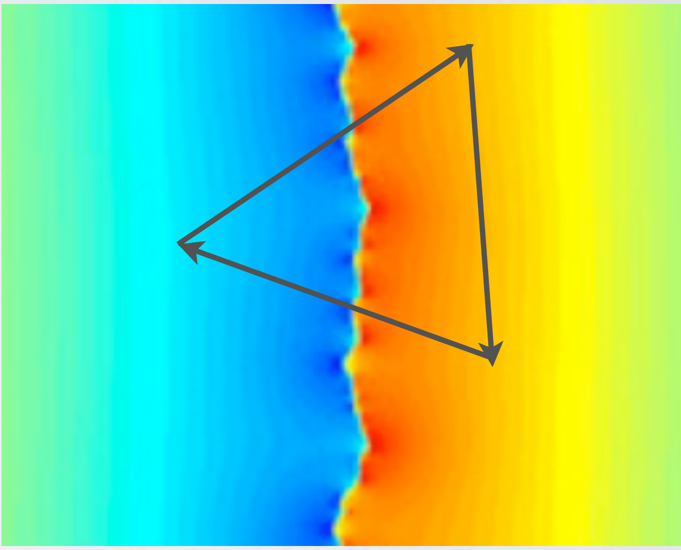


- CMB Power Spectrum accepted test bed for cosmic strings $G\mu/c^2 < 1.5\times10^{-7}$ $G\mu_{\rm AH}/c^2 < 3.2\times10^{-7}$
- Cosmic strings ARE non-Gaussian. Bispectrum?

Computed in arXiv:0908.0432 (Hindmarsh, Ringeval, Suyama)
arXiv:0911.2491 (DMR, Shellard)

Symmetry suppressed signal but a possibility...

All-sky maps generated by Ringeval and Bouchet
arXiv:1204.5041 and in Planck arXiv:1303.5085 $G\mu/c^2 < 8.8\times10^{-7}$
with bispectrum extracted via modal methods(see Shellard talk)



- Trispectrum? NOT symmetry suppressed signal. Possibly competitive...
  See arXiv:0911.1241, arXiv:0911.2491 and arXiv:1012.6039 $G\mu \leq 1.1\times10^{-6}$
- What about matter spectra? Soon galaxy surveys will have enough modes to exceed CMB sensitivity
- Matter Bispectrum will not be symmetry suppressed
- Wakes could dominate non-linear structures at high z

arXiv:1302.3467 Duplessis & Brandenberger

Methodology

- 2 approaches... UETC approach & 'wake model'

UETC

$$\hat{\mathcal{D}}_{ac}(k,a,\dot{a},\rho,...)\tilde{X}_a(\mathbf{k},\tau') \;=\; \tilde{S}_c(\mathbf{k},\tau')$$

$$\left\langle \tilde{X}_a \tilde{X}_b^* \right\rangle = \iint \mathrm{d}\tau' \mathrm{d}\tau'' \mathcal{G}_{ac}(\tau')\mathcal{G}_{bd}^*(\tau'') \left\langle \tilde{S}_c(\tau')\tilde{S}_d^*(\tau'') \right\rangle$$

UETC are the source functions for the Einstein-Boltzmann solver (decomposed into coherent basis functions)

Pen, Spergel, Turok arXiv:9704165

Don't really need full Einstein-Boltzmann hierarchy for our study. Instead just get Green's function from simple matter + radiation equations

$$\ddot{\delta}_c + \frac{\dot{a}}{a}\dot{\delta}_c - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2 \left(\frac{a\delta_c + 2a_{\mathrm{eq}}\delta_r}{a + a_{\mathrm{eq}} + \frac{\Omega_\Lambda}{\Omega_c}\frac{a^4}{a_0^3}}\right) = 4\pi G\Theta_+,$$

$$\ddot{\delta}_r + \frac{1}{3}k^2\delta_r - \frac{4}{3}\ddot{\delta}_c = 0,$$

Albrecht & Stebbins '92
Avelino, Shellard, Wu, Allen arXiv:9712008

$$\delta_N^{\mathrm{S}}(\mathbf{x},\eta) = 4\pi G\int_{\eta_{\mathrm{i}}}^{\eta} d\eta' \int d^3x' \; \mathcal{G}_N(X;\eta,\eta')\Theta_+(\mathbf{x}',\eta'),$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \, \Theta(k,\tau_1)\Theta(k,\tau_2) \,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$ where the coherence time τ_c grows like k^{-1} outside the horizon.

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \, \Theta(k,\tau_1)\Theta(k,\tau_2) \, ,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu\right) \delta^3(\mathbf{x}-\mathbf{X})$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2} \quad \text{where the coherence time } \tau_c \text{ grows like } k^{-1} \text{ outside the horizon.}$$

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu\right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2} \, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi \, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu \right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

$$\epsilon = \sqrt{\mathbf{X}'^2/(1-\dot{\mathbf{X}}^2)}$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu\right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

$$\epsilon = \sqrt{\mathbf{X}'^2/(1-\dot{\mathbf{X}}^2)}$$

Model in terms of 2 pt correlation functions of string network

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} \left[N_d(\tau_i)\right]^{1/2} \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^{\mu}\dot{X}^{\nu} - \epsilon^{-1} X'^{\mu} X'^{\nu}\right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

$$\epsilon = \sqrt{\mathbf{X}'^2/(1-\dot{\mathbf{X}}^2)}$$

Model in terms of 2 pt correlation functions of string network

$$\begin{aligned}
\Gamma(s_-,\eta) &= \langle [\mathbf{X}(s,\eta) - \mathbf{X}(s',\eta)]^2 \rangle\,,\\
\Pi(s_-,\eta) &= \langle (\mathbf{X}(s,\eta) - \mathbf{X}(s',\eta)) \cdot \dot{\mathbf{X}}(s',\eta) \rangle\,,\\
V(s_-,\eta) &= \langle \dot{\mathbf{X}}(s,\eta) \cdot \dot{\mathbf{X}}(s',\eta) \rangle\,.
\end{aligned}$$

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau,\tau_i,L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2}$ where the coherence time τ_c grows like k^{-1} outside the horizon.

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu\right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

$$\epsilon = \sqrt{\mathbf{X}'^2/(1-\dot{\mathbf{X}}^2)}$$

Model in terms of 2 pt correlation functions of string network

$$\begin{aligned}\Gamma(s_-,\eta) &= \langle[\mathbf{X}(s,\eta)-\mathbf{X}(s',\eta)]^2\rangle\,,\\ \Pi(s_-,\eta) &= \langle(\mathbf{X}(s,\eta)-\mathbf{X}(s',\eta))\cdot\dot{\mathbf{X}}(s',\eta)\rangle\,,\\ V(s_-,\eta) &= \langle\dot{\mathbf{X}}(s,\eta)\cdot\dot{\mathbf{X}}(s',\eta)\rangle\,.\end{aligned}$$

$\Gamma(s) \approx \bar{t}^2 s^2$, $\Pi(s) \approx c_0 s/(2\xi)$, $V(s) \approx \bar{v}^2$,

How to calculate the UETC?

USM developed by Battye, Albrecht and Robinson... scaling on superhorizon scales emerges due to modelling of decay of string segments

$$\Theta_{\mu\nu} = \sum_{i=1}^{K} [N_d(\tau_i)]^{1/2}\, \Theta^i_{\mu\nu} T^{\rm off}(\tau, \tau_i, L_f)$$

Recent analytical advances have greatly improved the computational speeds

Avgoustidis, Copeland, Moss, Skliros arXiv:1209.2461

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)\,,$$

Superseded work by Vincent, Hindmarsh, Sakellariadou which had an acausal superhorizon behaviour

$$U(k,\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)^2/\tau_c^2} \quad \text{where the coherence time } \tau_c \text{ grows like } k^{-1} \text{ outside the horizon.}$$

Let's revisit this...amongst other things it's good to have a test for USM predictions

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma \left(\epsilon \dot{X}^\mu \dot{X}^\nu - \epsilon^{-1} X'^\mu X'^\nu\right) \delta^3(\mathbf{x}-\mathbf{X})$$

$$\Theta_+(\mathbf{k},\eta) = 2\mu \int ds \dot{\mathbf{X}}_s^2 e^{i\mathbf{k}\cdot\mathbf{X_s}}$$

$$\epsilon = \sqrt{\mathbf{X}'^2/(1-\dot{\mathbf{X}}^2)}$$

Model in terms of 2 pt correlation functions of string network

Perivaropolus '93, Hindmarsh '93

$$\begin{aligned}
\Gamma(s_-,\eta) &= \langle [\mathbf{X}(s,\eta) - \mathbf{X}(s',\eta)]^2\rangle\,,\\
\Pi(s_-,\eta) &= \langle (\mathbf{X}(s,\eta) - \mathbf{X}(s',\eta))\cdot \dot{\mathbf{X}}(s',\eta)\rangle\,,\\
V(s_-,\eta) &= \langle \dot{\mathbf{X}}(s,\eta)\cdot \dot{\mathbf{X}}(s',\eta)\rangle\,.
\end{aligned}$$

$$\Gamma(s) \approx \bar{t}^2 s^2,\ \Pi(s) \approx c_0 s/(2\xi),\ V(s) \approx \bar{v}^2,$$

Power Spectrum

UETC approach

$$\langle \Theta_+(\mathbf{k},\eta)\Theta_+^*(\mathbf{k}',\eta')\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\frac{\phi_0^4}{\sqrt{\eta\eta'}}C_+(k,\eta,\eta')$$

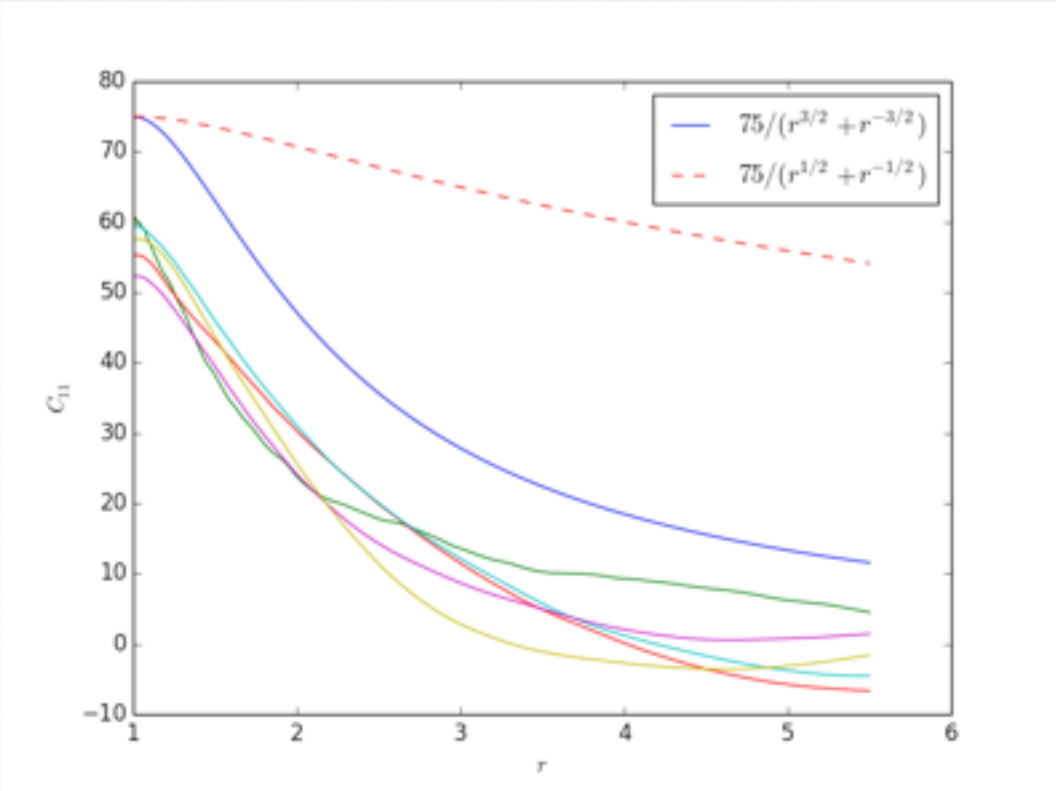
$$P^S(k) = \epsilon^2\int_{\eta_i}^{\eta} d\eta' \int_{\eta_i}^{\eta} d\eta'' \mathcal{G}_c(k;\eta,\eta')\mathcal{G}_c^*(k;\eta,\eta'')\frac{C_+(k\eta',k\eta'')}{\sqrt{\eta'\eta''}},$$

General form of UETC Durrer, Kunz arXiv: 9711133, Bevis, Hindmarsh, Kunz, Urrestilla arXiv:0605018, 1005.2663

$$z = \sqrt{k^2\eta\eta'}, \quad r = \eta'/\eta,$$

$$C_+(z,r) = \begin{cases} \dfrac{2E_+}{r^{\frac{3}{2}}+r^{-\frac{3}{2}}} & z<1 \\ \dfrac{E_+}{z}e^{-z^2\ln(r)^2/2A^2} & z\geq 1 \end{cases}$$

$$C_+(k,\eta,\eta') = \frac{2E_+}{z(r^{\frac{3}{2}}+r^{-\frac{3}{2}})}e^{-z^2\ln(r)^2/2A^2}$$



z= 6.1340e-01,1.0104e+00,1.6112e+00, 2.1668e+00, 2.7643e+00, 3.3984e+00

Thanks to Jon, David

String correlation model gives

$$C_+(k\eta,k\eta') \approx 4\bar{\mu}^2\sqrt{\eta\eta'}\mathcal{L}\int_{-\xi}^{\xi} ds_-\left[\overline{v}^4\exp\left(-\frac{k^2\overline{t}^2s_-^2}{6}\right)+\frac{2}{3}V(s_-)^2\exp\left(-\frac{k^2\overline{t}^2s_-^2}{6}\right)\right]\exp(-k^2\overline{v}^2\eta_-^2/6)$$

$$\longrightarrow \quad E_+ = \frac{20\sqrt{6\pi}}{3}\frac{\bar{\mu}^2\overline{v}^4\gamma_{\overline{v}}}{\alpha^2}\mathrm{erf}(\overline{t}\alpha z/\sqrt{6}), \quad A = \frac{\sqrt{3}}{\overline{v}},$$

Explanation of superhorizon form of UETC

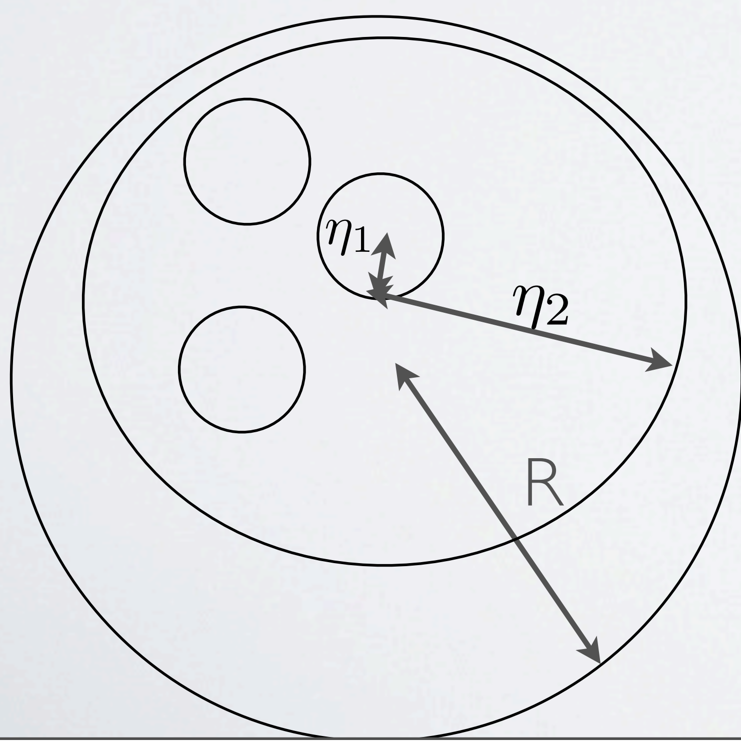
At any scale R the volume in each horizon volume can be regarded as a random process $\rho_R(\mathbf{x},\eta) = \overline{\rho}_R(\eta) + \delta\rho_R(\mathbf{x},\eta)$.
Relative fluctuation of $\mathcal{O}(1)$ on each horizon scale such that $\sqrt{\langle \delta\rho_R(\mathbf{x},\eta)^2 \rangle} = \left(\frac{\eta}{R}\right)^{3/2} \overline{\rho}_R(\eta)$

$$\langle \delta\rho_R(\mathbf{x},\eta_1)\delta\rho_R(\mathbf{x},\eta_2)\rangle = \left(\frac{\eta_1}{R}\right)^{3/2} \overline{\rho}_R(\eta_1) \times \left(\frac{\eta_2}{R}\right)^{3/2} \overline{\rho}_R(\eta_2) \times \left(\frac{\eta_1}{\eta_2}\right)^{3/2} = \frac{\eta_1^3}{R^3}\overline{\rho}_R(\eta_1)\overline{\rho}_R(\eta_2)$$

$$\mathrm{RMS}^2 = \langle \delta\rho_R(\mathbf{x},\eta_1)\delta\rho_R(\mathbf{x},\eta_2)\rangle = \int_0^{2\pi/R} d^3k e^{i\mathbf{k}.\mathbf{x}} \langle \delta\rho_R(\mathbf{k},\eta_1)\delta\rho_R(\mathbf{k},\eta_2)\rangle$$

White noise implies that this signal varies as Amp^2/R^3. Since $\rho(\eta_i) \propto 1/\eta_i^2$ (scaling) then

$$\mathrm{RMS}^2 \sim \frac{\eta_1^3}{\eta_1^2\eta_2^2} = \frac{1}{\sqrt{\eta_1\eta_2}}\left(\frac{\eta_1}{\eta_2}\right)^{3/2}$$



Analytic predictions (matter dominated universe)

Superhorizon scales, $k \ll \eta^{-1}$

$$P_{>\mathrm{eq}}(k,\eta) \;\approx\; \epsilon^2 \frac{2\pi E_+^{\mathrm{m}}}{75}\eta^3.$$

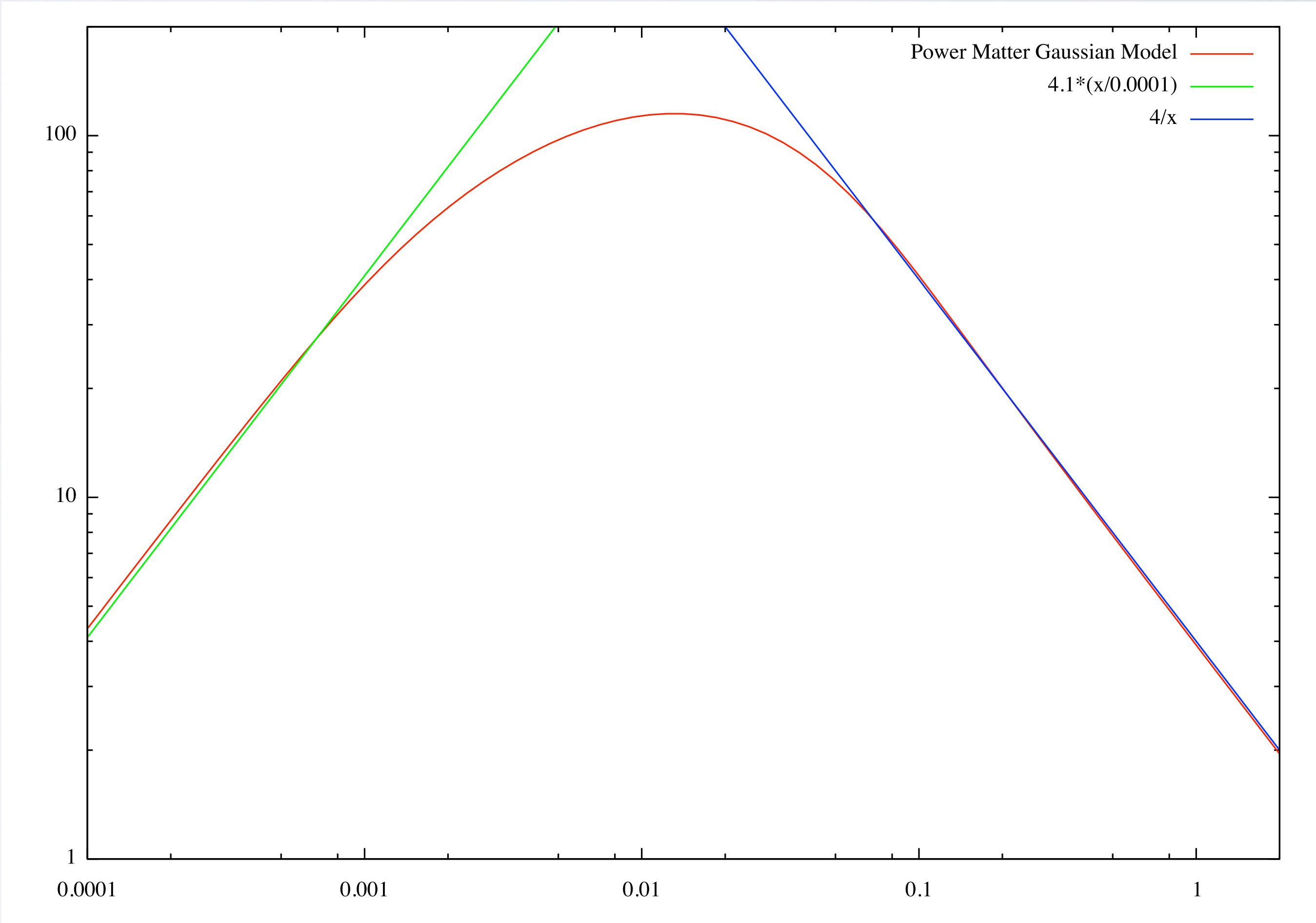
Large scales, $\eta^{-1} \ll k \ll \eta_{eq}^{-1}$

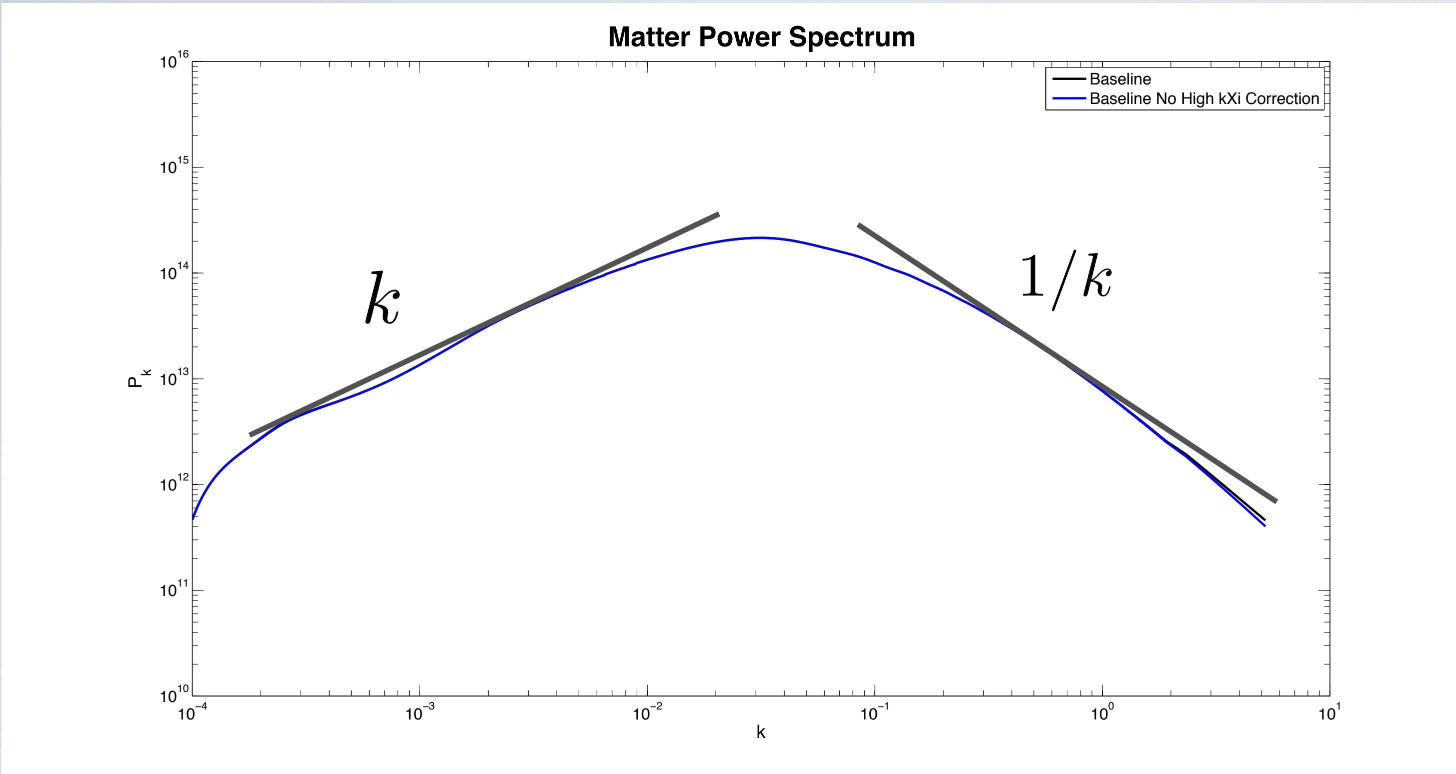
$$P_{>\mathrm{eq}}(k,\eta) \;\approx\; \epsilon^2 k \eta^4 \frac{E_+^{\mathrm{m}}}{75}\left(\sqrt{2\pi}A - 2\pi\right)$$

Small scales, $\eta_{eq}^{-1} \ll k$

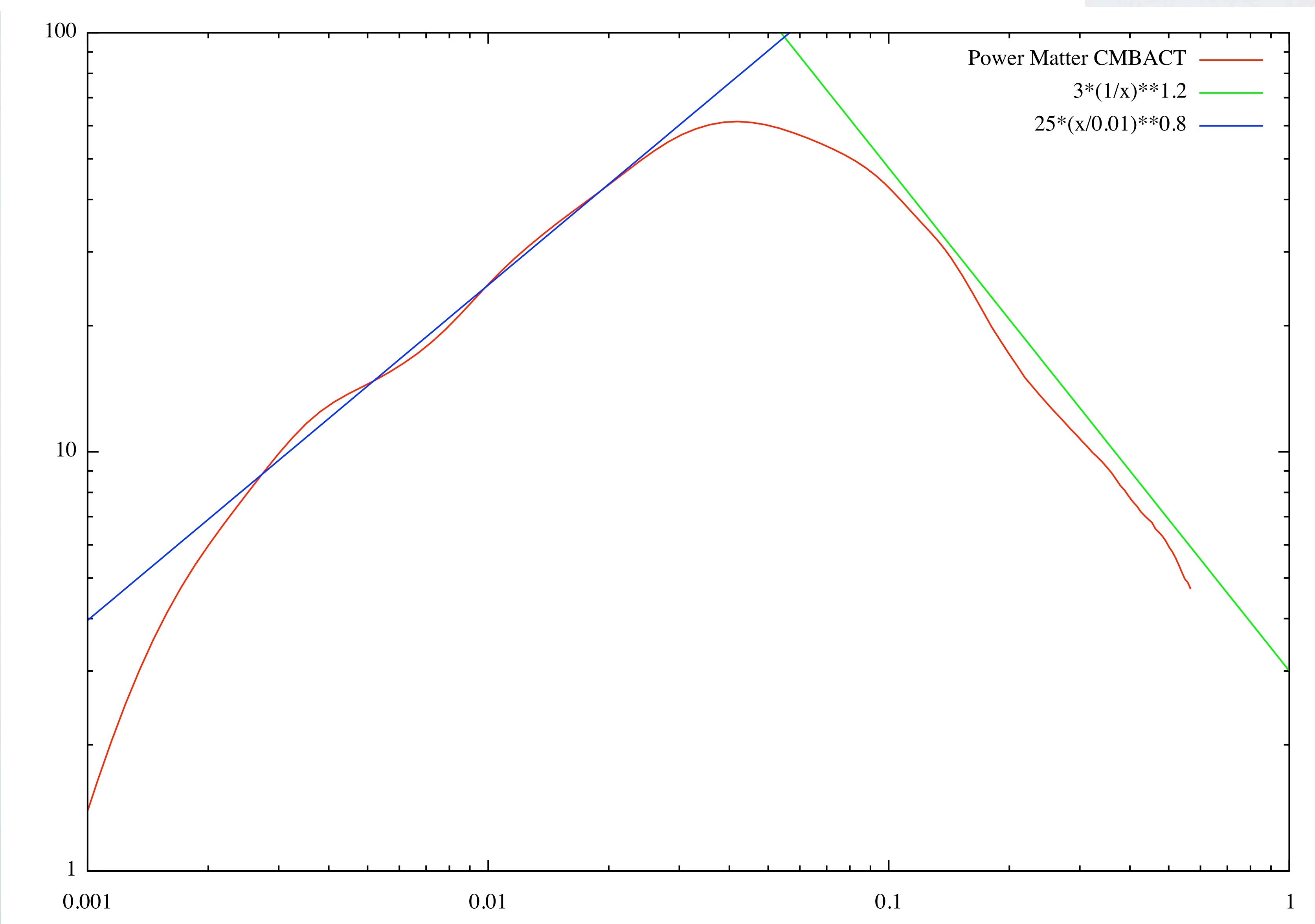
$$P^S_{>\mathrm{eq}}(k,\eta) \;\approx\; \epsilon^2 \frac{\eta_{\mathrm{eq}}}{k^2}\frac{\eta^4}{\eta_{\mathrm{eq}}^4}\frac{\sqrt{2\pi}AE_+^{\mathrm{m}}}{75}.$$

Using numerical Green's functions for matter+radiation





Abelian-Higgs plot...
again thanks to Jon & David

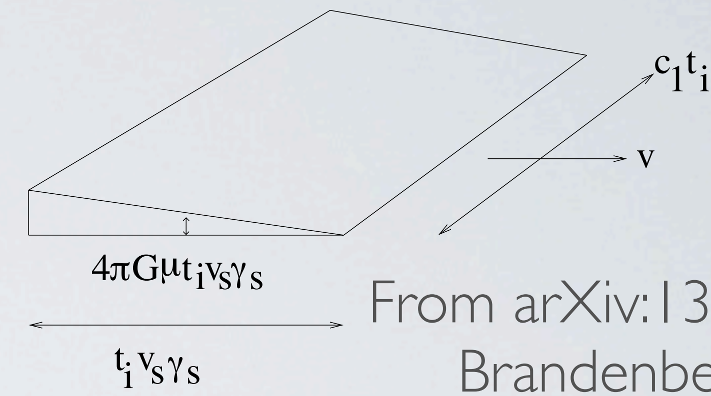


CMBACT plot
thanks to Levon!

'Wake Model'

Variation on wake model by Brandenberger and collaborators

Approach: View the wake as having zero-width but with surface density proportional to the predicted width



From arXiv:1301.2856 Brandenberger

Zel'dovich approx to get the width $r = a(x + \psi)$

$$dr/d\eta = 0 \qquad d \approx |2x| = |4\psi|.$$

$$\psi'' + \frac{2}{\eta}\psi' - \frac{6}{\eta^2}\psi = 0\,, \qquad \text{(matter era)}$$

Initial conditions due to conical deficit angle $\psi(\mathbf{x}, \eta_i) = 0, \quad \psi'(\mathbf{x}, \eta_i) = -4\pi G\mu v_s \gamma_s \Theta(x),$

$$=> \quad \frac{\sigma_w(\eta, \eta_i)}{\rho_c(\eta)} = \frac{4u_i\eta_i}{5}\left(\frac{\eta}{\eta_i}\right)^2$$

$$=> \quad \delta_1(\mathbf{k}, \eta, \eta_i) = \int d^3\mathbf{x} e^{i\mathbf{k}.\mathbf{x}} \frac{\delta\rho(\mathbf{x})}{\rho_c} = \int dz e^{ik_z z}\delta(z)\frac{\sigma_w(\eta, \eta_i)}{\rho_c}\int_0^{\xi(\eta_i)} r dr d\phi e^{i\mathbf{k}_\perp \cdot \mathbf{x}}$$
$$= \frac{\sigma_w(\eta, \eta_i)}{\rho_c}(2\pi\xi(\eta_i))^2 \frac{J_1(k_\perp\xi(\eta_i))}{k_\perp\xi(\eta_i)}\,,$$

Power Spectrum

'Wake' model approach

$$\langle|\delta_1(\mathbf{k},\eta,\eta_i)|^2\rangle_{\hat{\mathbf{n}}} = \int \frac{d\hat{\mathbf{n}}}{4\pi}|\delta_1(\mathbf{k},\eta,\eta_i)|^2 = (2\pi)^2\left(\frac{\sigma_w(\eta,\eta_i)}{\rho_c}\xi(\eta_i)^2\right)^2\int_{-1}^{1}\frac{d\mu}{2}\frac{\left[J_1(k\xi(\eta_i)\sqrt{1-\mu^2})\right]^2}{k^2\xi(\eta_i)^2(1-\mu^2)}$$

$$P(k,\eta) = \int_{\eta_1}^{\eta} d\eta_i \frac{dn_{\rm w}}{d\eta_i}\langle|\delta_1(\mathbf{k},\eta,\eta_i)|^2\rangle_{\hat{\mathbf{n}}} = \int_{\eta_1}^{\eta} d\eta_i \left[\frac{\nu}{\eta_i^4}\left(2\pi\xi(\eta_i)^2\frac{\sigma_w(\eta,\eta_i)}{\rho_c}\right)^2\int_{-1}^{1}\frac{d\mu}{2}\frac{\left[J_1(k\xi(\eta_i)\sqrt{1-\mu^2})\right]^2}{k^2\xi(\eta_i)^2(1-\mu^2)}\right]$$

$$dn_{\rm w} = \frac{\nu}{\eta_i^4}d\eta_i,$$

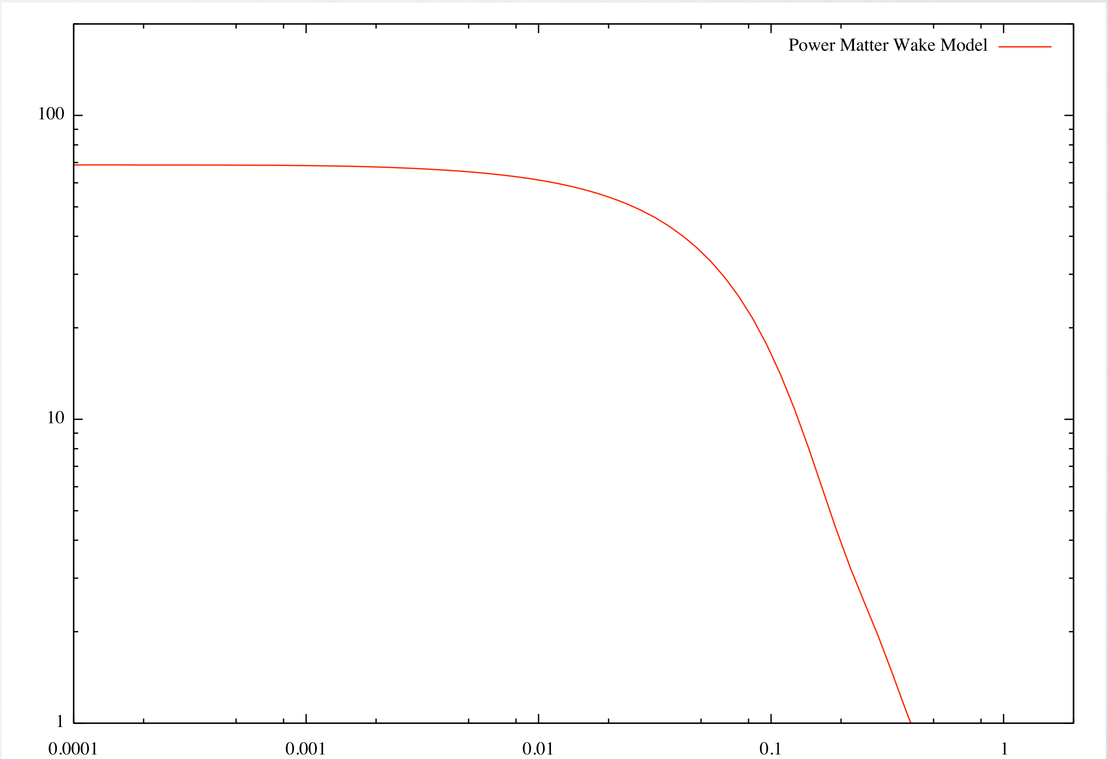
Predictions:

For large scales, such that $k\xi(\eta_i) \ll 1$

$$P(k,\eta) \approx (2\pi)^2\left(\frac{4u_i}{5}\right)^2\nu\alpha^4\left(\frac{a(\eta)}{a(\eta_{\rm eq})}\right)^2\eta_{\rm eq}^3 \propto k^0.$$

For small scales, with $k\xi(\eta_i) \gg 1$

$$P(k,\eta) \approx (2\pi)^2\left(\frac{4u_i}{5}\right)^2\nu\alpha^2\left(\frac{a(\eta)}{a(\eta_{\rm eq})}\right)^2\left(\frac{1}{3}\right)\frac{\eta_{\rm eq}}{k^2} \propto k^{-2}$$



Issues with wake-model:

1) Valid only for matter dominated regime

2) Neglect perturbations outside turnaround surface => do not expect to be accurate on reasonably large scales => appears to be issues on observational scales with such models

Advantages:

Very simple model allowing for a check of calculations for approximation of matter dominated regime

Bispectrum

$$\langle \delta_c(\mathbf{k}_1, \eta)\delta_c(\mathbf{k}_2, \eta)\delta_c(\mathbf{k}_3, \eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B(k_1, k_2, k_3, \eta).$$

UETC approach

$$B(k_1, k_2, k_3, \eta) = \epsilon^3 \int_{\eta_i}^{\eta} d\eta_1 \int_{\eta_i}^{\eta} d\eta_2 \int_{\eta_i}^{\eta} d\eta_3 \Pi_{a=1}^3 \left[\mathcal{G}_c(k_a; \eta, \eta_a)\right] \beta(z_1, z_2, z_3, r_1, r_2).$$

$$\langle \Theta_+(\mathbf{k}_1, \eta_1)\Theta_+(\mathbf{k}_2, \eta_2)\Theta_+(\mathbf{k}_3, \eta_3)\rangle = \phi_0^6 \beta_+(k_1, k_2, k_3, \eta_1, \eta_2, \eta_3)(2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3}, \quad r_2 = \frac{\eta_3}{\eta_1}, \quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1, z_2, z_3, r_1, r_2) = E_+^{(3)}(z_1, z_2, z_3) \exp\left(-\textstyle\sum_a z_a^2 \ln(r_a)^2/2A^2\right), \qquad (x_a \gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+\propto r_1/r_2$

String correl model gives

Bispectrum

$$\langle \delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta} d\eta_1\int_{\eta_i}^{\eta} d\eta_2\int_{\eta_i}^{\eta} d\eta_3 \Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle \Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

Bispectrum

$$\langle \delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3 \int_{\eta_i}^{\eta} d\eta_1 \int_{\eta_i}^{\eta} d\eta_2 \int_{\eta_i}^{\eta} d\eta_3 \Pi_{a=1}^3 \left[\mathcal{G}_c(k_a;\eta,\eta_a)\right] \beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle \Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3}, \quad r_2 = \frac{\eta_3}{\eta_1}, \quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a \gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2 = \sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle=(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta)=\epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle=\phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a=k_a\eta_a,\qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2)=E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right),\qquad (x_a\gg1)$$

What about superhorizon scales...use volume argument to show $\beta_+\propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)\approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\text{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\text{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2=\sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination:

Bispectrum

$$\langle \delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3 \int_{\eta_i}^{\eta} d\eta_1 \int_{\eta_i}^{\eta} d\eta_2 \int_{\eta_i}^{\eta} d\eta_3 \Pi_{a=1}^3 \left[\mathcal{G}_c(k_a;\eta,\eta_a)\right] \beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle \Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3}, \quad r_2 = \frac{\eta_3}{\eta_1}, \quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a \gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\bar{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\bar{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2 = \sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination: *Equilateral limit:* $k_a = k.$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^{3}\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+\propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)\approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\bar{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\bar{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\bar{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2=\sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination: *Equilateral limit:* $k_a = k$. $\qquad B_{\mathrm{E}}(k,k,k,\eta)\approx\frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta} d\eta_1\int_{\eta_i}^{\eta} d\eta_2\int_{\eta_i}^{\eta} d\eta_3 \Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3}, \quad r_2 = \frac{\eta_3}{\eta_1}, \quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2 = \sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination: *Equilateral limit:* $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta) \approx \frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+\propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)\approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\bar{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\bar{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2=\sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination: *Equilateral limit:* $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta)\approx\frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Squeezed limit $k = k_1 = k_2 \gg k_3,$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3},\quad r_2 = \frac{\eta_3}{\eta_1},\quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a \gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\bar{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\bar{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2 = \sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination: *Equilateral limit:* $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta) \approx \frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Squeezed limit $k = k_1 = k_2 \gg k_3,$
$k_3\eta_{\mathrm{eq}} > 1$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+\propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)\approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\bar{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\bar{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2=\sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination:

Equilateral limit: $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta)\approx\frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Squeezed limit $k=k_1=k_2\gg k_3,$ $k_3\eta_{\mathrm{eq}}>1$

$$B_{\mathrm{S}}(k,k,k_3,\eta)\approx\frac{B_0}{375}\frac{\eta_{\mathrm{eq}}^3}{k^2k_3}\frac{2\alpha}{\sqrt{6\pi}}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6.$$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta}d\eta_1\int_{\eta_i}^{\eta}d\eta_2\int_{\eta_i}^{\eta}d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1=\frac{\eta_2}{\eta_3},\quad r_2=\frac{\eta_3}{\eta_1},\quad r_3=\frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a\gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)\approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2=\sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination:

Equilateral limit: $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta)\approx\frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Squeezed limit $k = k_1 = k_2 \gg k_3,$
$k_3\eta_{\mathrm{eq}} > 1$

$$B_{\mathrm{S}}(k,k,k_3,\eta)\approx\frac{B_0}{375}\frac{\eta_{\mathrm{eq}}^3}{k^2k_3}\frac{2\alpha}{\sqrt{6\pi}}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6.$$

Folded limit, $k = k_2 \approx k_3 \approx k_1/2,$

Bispectrum

$$\langle\delta_c(\mathbf{k}_1,\eta)\delta_c(\mathbf{k}_2,\eta)\delta_c(\mathbf{k}_3,\eta)\rangle = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3,\eta).$$

UETC approach

$$B(k_1,k_2,k_3,\eta) = \epsilon^3\int_{\eta_i}^{\eta} d\eta_1\int_{\eta_i}^{\eta} d\eta_2\int_{\eta_i}^{\eta} d\eta_3\Pi_{a=1}^3\left[\mathcal{G}_c(k_a;\eta,\eta_a)\right]\beta(z_1,z_2,z_3,r_1,r_2).$$

$$\langle\Theta_+(\mathbf{k}_1,\eta_1)\Theta_+(\mathbf{k}_2,\eta_2)\Theta_+(\mathbf{k}_3,\eta_3)\rangle = \phi_0^6\beta_+(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3).$$

$$x_a = k_a\eta_a, \qquad r_1 = \frac{\eta_2}{\eta_3}, \quad r_2 = \frac{\eta_3}{\eta_1}, \quad r_3 = \frac{\eta_1}{\eta_2}.$$

$$\beta_+(z_1,z_2,z_3,r_1,r_2) = E_+^{(3)}(z_1,z_2,z_3)\exp\left(-\textstyle\sum_a z_a^2\ln(r_a)^2/2A^2\right), \qquad (x_a \gg 1)$$

What about superhorizon scales...use volume argument to show $\beta_+ \propto r_1/r_2$

String correl model gives

$$\beta(k_1,k_2,k_3,\eta_1,\eta_2,\eta_3) \approx 8\bar{\mu}^3\frac{1}{\xi^2}\frac{35}{9}\frac{\overline{v}^6}{\overline{t}^2}\frac{6\pi}{\sqrt{k_2^2k_3^2-\kappa_{23}^2}}\mathrm{erf}\left(\frac{\overline{t}k_2\xi}{\sqrt{6}}\right)\mathrm{erf}\left(\frac{\overline{t}K_2\xi}{\sqrt{6}}\right)\exp\left(-\frac{\overline{v}^2}{6}(k_2^2\eta_{12}^2+K_2^2\eta_{13}^2)\right).$$

$$K_2 = \sqrt{k_2^2k_3^2-\kappa_{23}^2}/k_2.$$

Assuming matter domination:

Equilateral limit: $k_a = k.$

$$B_{\mathrm{E}}(k,k,k,\eta) \approx \frac{B_0}{500}\frac{4\eta_{\mathrm{eq}}^2}{3k^4}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6$$

$$B_0 = 6\pi\frac{\epsilon^3\beta_0}{\alpha^2\overline{v}^2} = (8\pi G\mu)^3\frac{35}{9}36\pi^2\frac{\overline{v}^4}{\alpha^2\overline{t}^2}.$$

Squeezed limit $k = k_1 = k_2 \gg k_3,$ $k_3\eta_{\mathrm{eq}} > 1$

$$B_{\mathrm{S}}(k,k,k_3,\eta) \approx \frac{B_0}{375}\frac{\eta_{\mathrm{eq}}^3}{k^2k_3}\frac{2\alpha}{\sqrt{6\pi}}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6.$$

Folded limit, $k = k_2 \approx k_3 \approx k_1/2,$

$$B_{\mathrm{F}}(2k,k,k,\eta) \approx \frac{B_0}{375}\frac{\eta_{\mathrm{eq}}^3}{k^2K_2}\frac{2\alpha}{\sqrt{6\pi}}\left(\frac{\eta}{\eta_{\mathrm{eq}}}\right)^6.$$

Bispectrum

'Wake' model approach

Valid for matter domination, use to check against UETC results

$$B(k_1,k_2,k_3,\eta) = \int_{\eta_1}^{\eta} d\eta_i \frac{\nu}{\eta_i^4}\left(\frac{\sigma_w(\eta,\eta_i)}{\rho_{\rm CDM}}\xi(\eta,\eta_i)\right)^3 \int \frac{d\hat{\mathbf{n}}}{4\pi}\frac{J_1(k_{1\perp}\xi(\eta,\eta_i))J_1(k_{2\perp}\xi(\eta,\eta_i))J_1(k_{3\perp}\xi(\eta,\eta_i))}{k_{1\perp}\,k_{2\perp}\,k_{3\perp}}\,,$$

Small Scales - Squeezed Limit:

$$B(k,k,k_3,\eta)^{k_3\ll k} \approx \nu\alpha^4\left(\frac{4u_i}{5}\right)^3\eta^6\int_{\eta_1}^{\eta} d\eta_i\frac{1}{\eta_i^3}\int_{-1}^{1}\frac{d\mu}{2}\frac{J_1(k\sqrt{1-\mu^2}\alpha\eta_i)^2}{k^2(1-\mu^2)} \approx 2\nu\alpha^4\left(\frac{2u_i}{5}\right)^3\left(\frac{a(\eta)}{a(\eta_{\rm eq})}\right)^3\frac{\eta_{\rm eq}^4}{k^2}$$

Small Scales - Folded Limit: $B(k,k,2k,\eta) \approx 2\left(\frac{u_i}{5}\right)^3\alpha^4\nu\left(\frac{a(\eta)}{a(\eta_{\rm eq})}\right)^3\frac{\eta_{\rm eq}^4}{k^2}$

Small Scales - Equilateral Limit:

$$B(k,k,k,\eta) \approx \left(\frac{4u_i\alpha}{5}\right)^3\frac{3\nu\alpha^3\ln(2\pi)}{2\pi^2\beta^3}\frac{\cos^2(\beta z-3\pi/4)}{z^5} \approx \left(\frac{4u_i}{5}\right)^3\frac{3\alpha\nu\ln(2\pi)}{4\pi^2\beta^3}\left(\frac{a(\eta)}{a(\eta_{\rm eq})}\right)^3\frac{\eta_{\rm eq}}{k^5}$$

Different behaviour in equil limit. Why?

Is it measurable?

$$B_\delta^{\rm grav}(k_1,k_2,k_3) = 2P_\delta^L(k_1)P_\delta^L(k_2)F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + 2 \text{ perms},$$

$$P_\delta^L(k) = \begin{cases} A_s^2 k\eta_{\rm eq}^4 \left(\dfrac{a}{a_{\rm eq}}\right)^2, & (k_a\eta_{\rm eq} \ll 1), \\ A_s^2 \frac{1}{k^3}\left(\frac{a}{a_{\rm eq}}\right)^2, & (k_a\eta_{\rm eq} \gg 1). \end{cases}$$

$$F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$$

$$\begin{aligned} B_E(k,k,k)/B^{\rm grav}(k,k,k) =& \mathcal{O}(1)\frac{(4\pi G\mu)^3}{A_s^4}\frac{a_{\rm eq}}{a}(\eta_{\rm eq}k)^2 \\ \approx& \frac{1+z}{1+z_{\rm eq}}(\eta_{\rm eq}k)^2 \end{aligned}$$

So possibly will dominate for $k \gtrsim 1/\sqrt(1+z).$

Conclusions

Revisited analytic method to calculate UETC

Comparison to simple wake model

Calculation of Matter bispectrum carried out for both approaches

Preliminary investigations indicate the bispectrum may be competitive with the CMB (but probably not better)

TO DO:

Can refine approach using One-Scale model instead of keeping parameters fixed (not valid for radiation-matter transition)

Bispectrum probably underestimated by using Green's function for matter domination