The Power Spectrum of Nambu-Goto Cosmic Strings

Andrei Lazanu and Paul Shellard

Department of Applied Mathematics and Theoretical Physics

UNIVERSITY OF CAMBRIDGE

Outline

- Cosmic String Models
- The USM Model
- The UETC Approach
- The Analytical Model for the USM
- Nambu-Goto Cosmic String Simulations
- Fitting USM to Simulations from UETCs
- The Power Spectrum
- String Tension Constraints

Cosmic String Models

Two approaches for simulating the evolution of cosmic strings have been developed:

- The Abelian Higgs field theory model
  - Strings obtained as solutions to the relativistic generalization of the Ginzburg-Landau action
  - Simulations rely on extrapolation on many orders of magnitude, as their width cannot be resolved with current processing power

- The Nambu-Goto effective field action
  - Obtained as a first-order approximation from the Abelian-Higgs action by considering the string width to be small with respect to its length
  - A further simplification was made in the phenomenological **Unconnected segment model** (USM), where the strings are assumed to be formed from a number of uncorrelated randomly oriented straight string segments which have random velocities

The USM model

Energy-momentum tensor of the strings:

$$\Theta_{\mu\nu}(\mathbf{k},\tau) = \mu\int_{-l/2}^{l/2} d\sigma e^{i\mathbf{k}\cdot\mathbf{X}} \left(\varepsilon\alpha \dot{X}^{\mu}\dot{X}^{\nu} - \frac{1}{\varepsilon\alpha} X'^{\mu} X'^{\nu} \right)$$

where $\varepsilon = \sqrt{\mathbf{X}'^{2} / \left(1-\dot{\mathbf{X}}^{2}\right)}$

α - 'wiggliness' parameter (macroscopic evolution of the strings)

- Energy-momentum tensor: sum of contributions from each of the individual segments
- 3 parameters:
  - v (RMS string velocity)
  - α
  - ξ (correlation length/conformal time)
- Implemented in CMBACT [Pogosian & Vachaspati, arXiv: astro-ph/9903361], based on CMBFAST [Seljak & Zaldarriaga, arXiv:astro-ph/9603033]
- Power spectrum from cosmic strings calculated by averaging the power spectra obtained from each realization (100+)

The UETC Approach

Cosmic strings: active sources, continuously seed perturbations throughout the history of the universe.
Their presence modifies the usual perturbation equations. They are uncorrelated with the primordial fluctuations and hence the total angular power spectrum can be expressed as a sum of the inflationary and the active source contributions.

Einstein equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

metric $g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$

Perturbations due to strings:

$$\delta T_0^0 = -\delta\rho + \Theta_0^0$$

$$\delta T_i^0 = (\rho + P) v_i + \Theta_i^0$$

$$\delta T_j^i = \delta P \delta_j^i + p\Sigma_j^i + \Theta_j^i$$

By considering the first order perturbations to the Einstein equation in synchronous gauge and by splitting the tensors in their scalar, vector and tensor components (in Fourier space), the evolution equations for the metric perturbations are obtained in terms of the matter perturbations and the cosmic strings.

$$k\eta' = 4\pi G a^2 \sum_i (\rho_i + p_i) v_i - \frac{4\pi G}{k}\Theta^D$$

Synchronous gauge equations

$$\ddot{h}^S + 2\frac{a'}{a}\dot{h}^S - 2k^2\eta = 16\pi G\left(a^2 p\Sigma^S + \Theta^S\right)$$

$$\eta = -\frac{h - h^S}{6}$$

$$\ddot{h}^V + 2\frac{a'}{a}\dot{h}^V = 16\pi G\left(a^2 p\Sigma^V + \Theta^V\right)$$

$$\ddot{h}^T + 2\frac{a'}{a}\dot{h}^T + k^2 h^T = 16\pi G\left(a^2 p\Sigma^T + \Theta^T\right)$$

$$\dot{\Theta}^D = \Theta^D\left(-2H - \frac{k^2}{3H}\right) - \frac{k^2}{3}\left(2\Theta^S - \Theta_{00} - \frac{\dot{\Theta}_{00}}{H}\right)$$

I will present the results obtained using this method later.

The UETC method is based on the following diagonalization procedure:

$$\sqrt{\tau\tau'}\langle\Theta_{\mu\nu}(\mathbf{k},\tau)\Theta_{\rho\sigma}(-\mathbf{k},\tau')\rangle = \sum_i \lambda_i v^{(i)}_{\mu\nu}(k\tau) v^{(i)T}_{\rho\sigma}(k\tau')$$

The eigenmodes are coherent and hence each of them can be fed individually into a Boltzmann equation solver like CAMB [Lewis *et al.*, arXiv:astro-ph/9911177], and hence the total result can be expressed as $C_l^{\text{string}} = \sum_i \lambda_i C_l^{(i)}$

Hence, the energy-momentum components in the equations above must be substituted with the corresponding eigenvector: $\Theta(k,\tau) \to \frac{v^{(i)}(k\tau)}{\sqrt{\tau}}$

The Analytic model for the USM

A model has been devised [Avgoustidis *et al.*, arXiv:1209/2461] for calculating UETCs from the USM model. The UETCs are calculated by integrating over all segments in the network, using the formula:

$$\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi\int_0^{\pi}\sin\theta\, d\theta\int_0^{2\pi} d\psi\int_0^{2\pi} d\chi\,\Theta(k,\tau_1)\Theta(k,\tau_2)$$

where f quantifies the number of strings that decay at each particular time. This can be calculated for all required components of the energy-momentum tensor. The final result only depends on the three parameters from the USM model, as expected and can be expressed in terms of 6 functions A_i:

$$\langle\Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{f(\tau_1,\tau_2,\xi,L_f)\mu^2}{k^2\left(1-v^2\right)}\times\sum_{i=1}^{6} A_i\left[I_i(x_-,\rho)-I_i(x_+,\rho)\right]$$

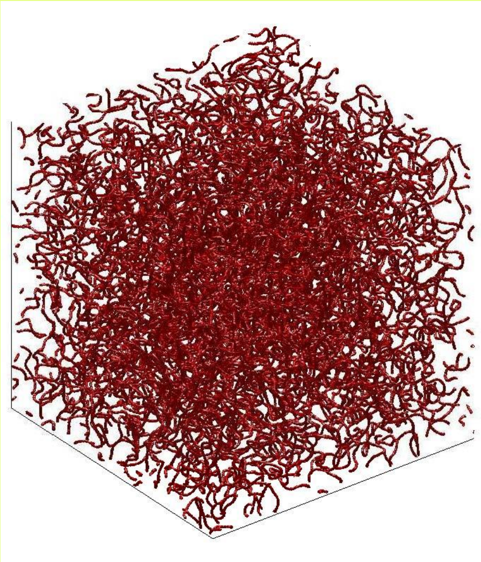
where $\rho=k|\tau_1-\tau_2|v$ and $x_\pm=k\xi(\tau_1\pm\tau_2)/2$. The UETCs used in this case are $<\theta_{00}\theta_{00}>$, $<\theta^S\theta^S>$, $<\theta_{00}\theta^S>$, $<\theta^V\theta^V>$ and $<\theta^T\theta^T>$.

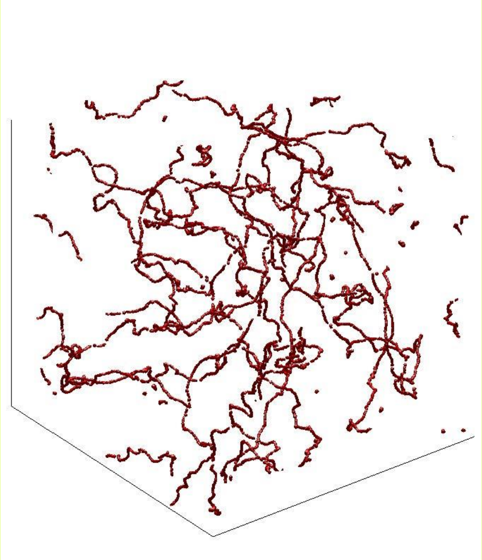
Nambu-Goto Cosmic String Simulations

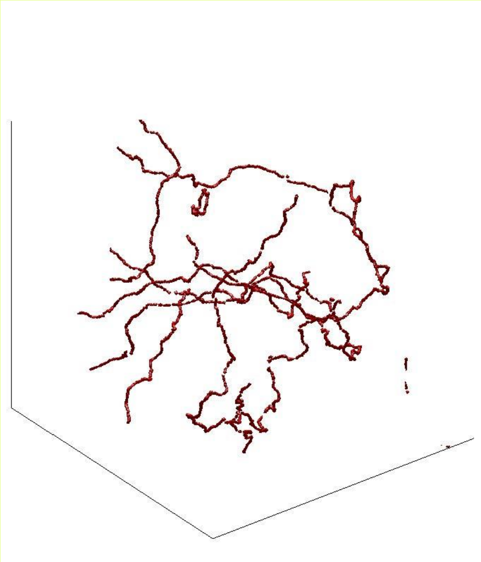
Nambu-Goto cosmic strings simulations generated with the Allen & Shellard code. We have used three simulations, covering in total from the radiation era until Λ domination:

- redshifts: 6348 to 700
- redshifts: 945 to 37.5
- redshifts: 55.4 to 0

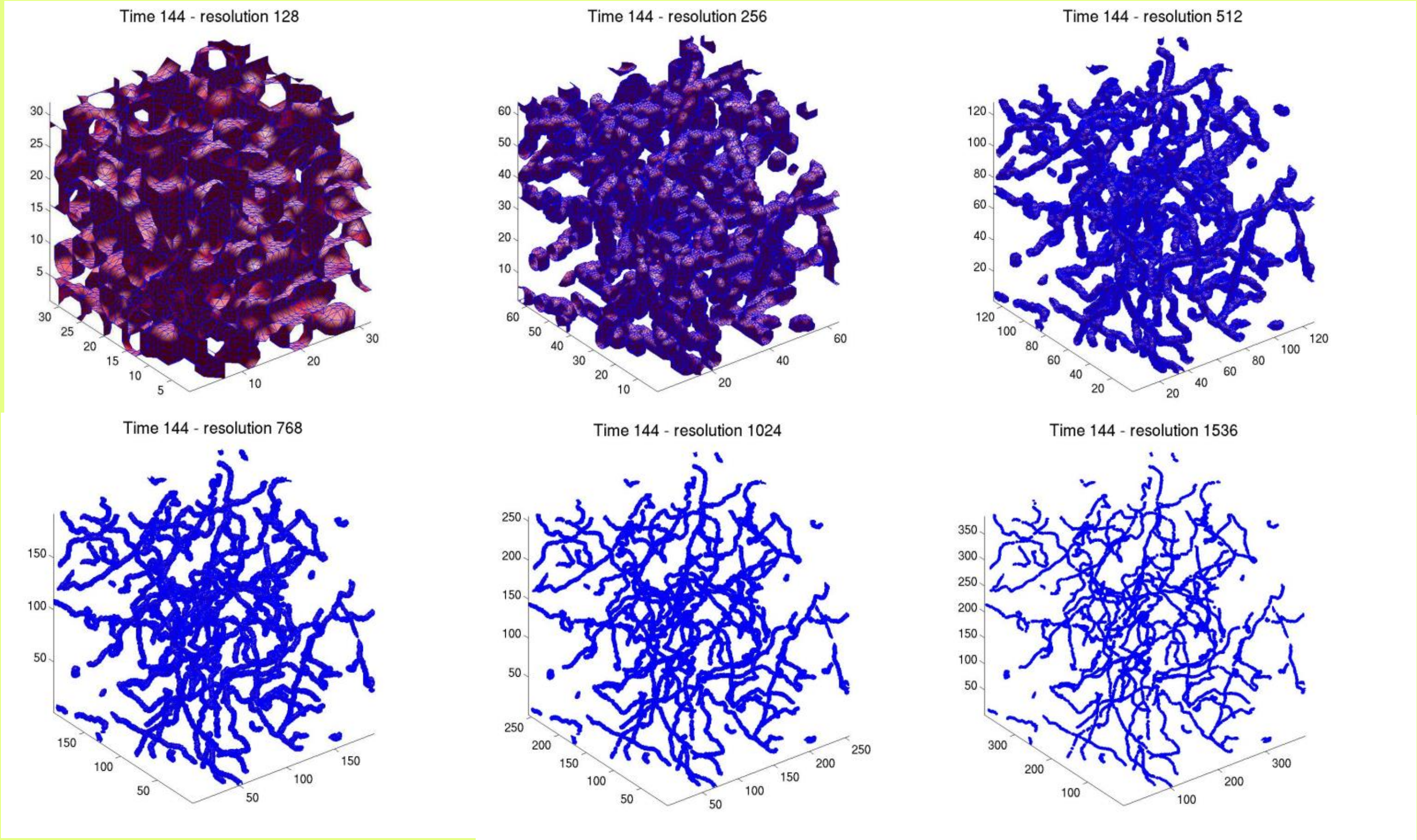
Evolution of string network for redshifts 6348 to 700 (radiation era)





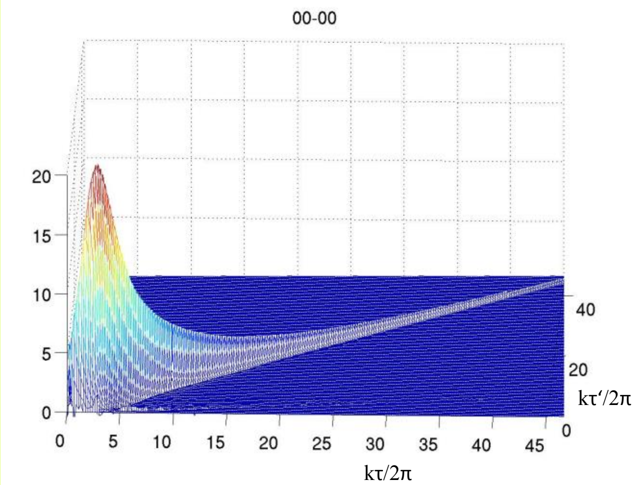


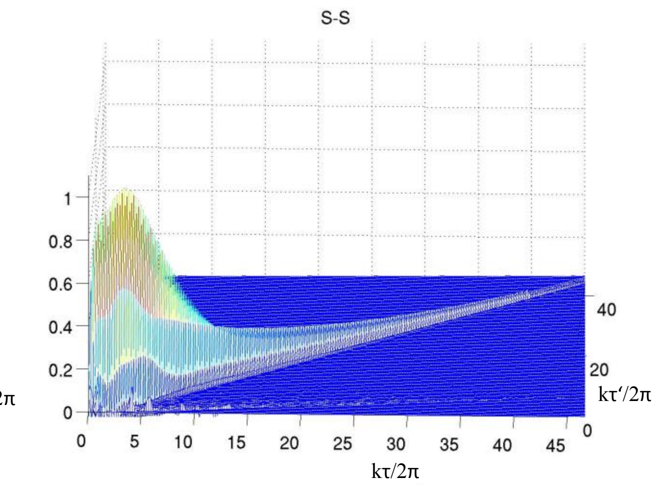
String network in terms of grid resolution

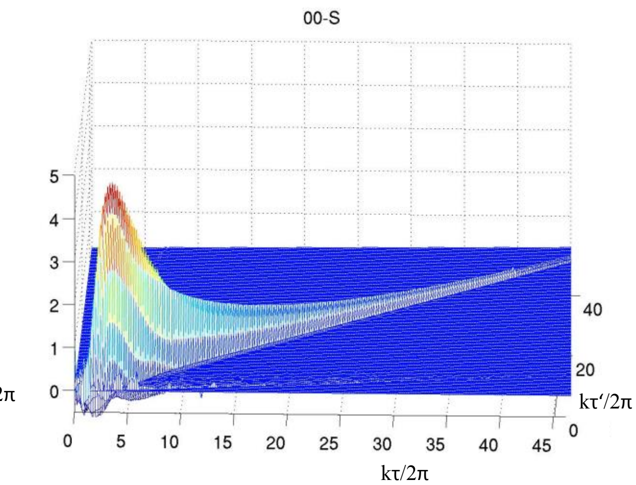


- for 128^3 and 256^3 points resolution, the strings are not resolved, key information being smoothed out.
- 512^3 is the smallest resolution that can be expected to give reliable predictions.
- At higher resolution, the strings only become thinner as the number of grid points increases.

As a balance between computational time and precision, we have chosen the **1024^3 resolution**.







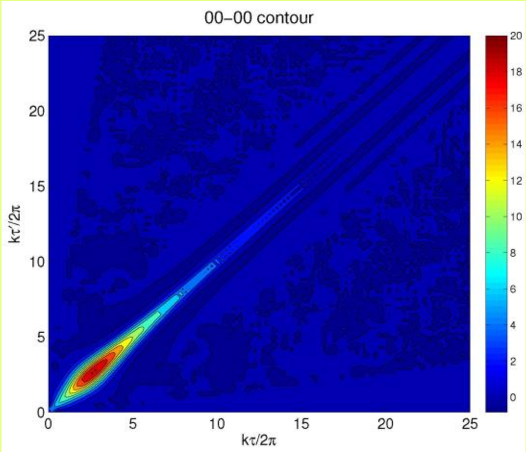
$<\theta_{00}\theta_{00}>$ 3D view

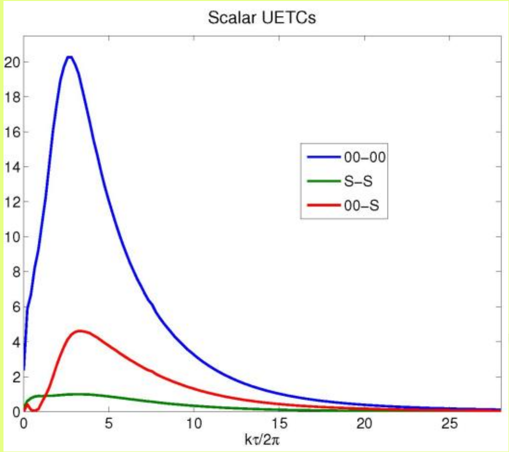
$<\theta^S\theta^S>$ 3D view

$<\theta_{00}\theta^S>$ 3D view

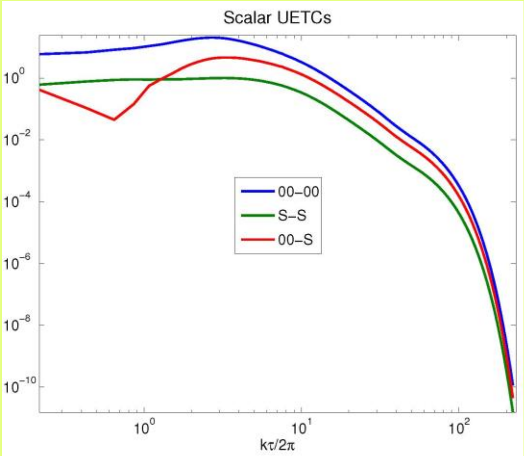
$<\theta_{00}\theta_{00}>$ contour

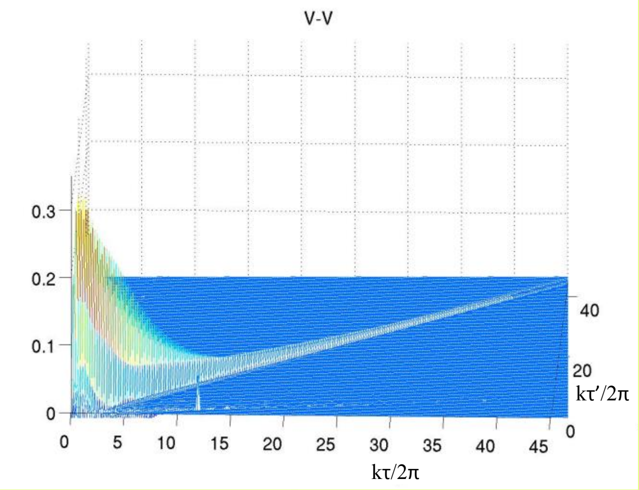
Typical scalar UETCs from simulations



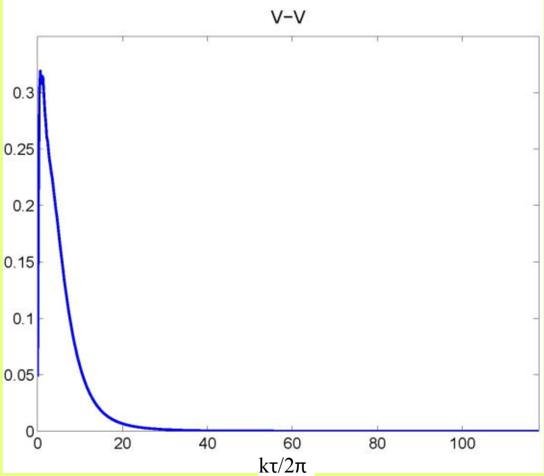


Diagonal views of scalar UETCs





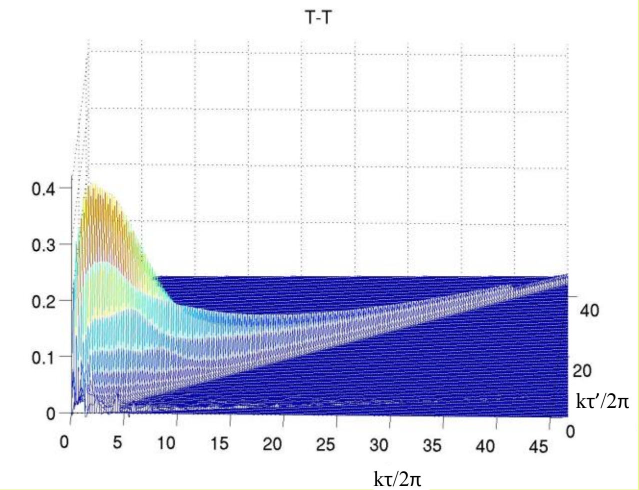
$<\theta^V\theta^V>$



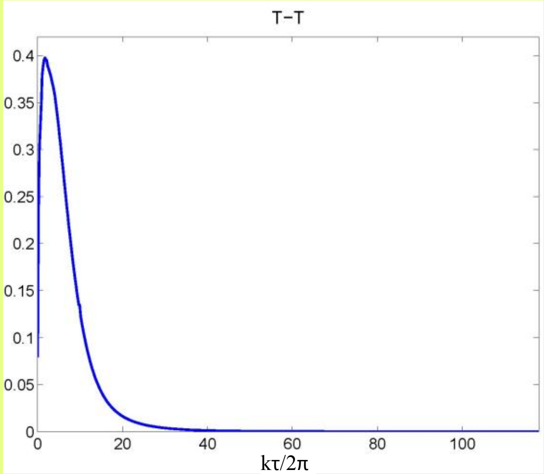
UETC profile views

Vector and tensor UETCs

Equal time correlators

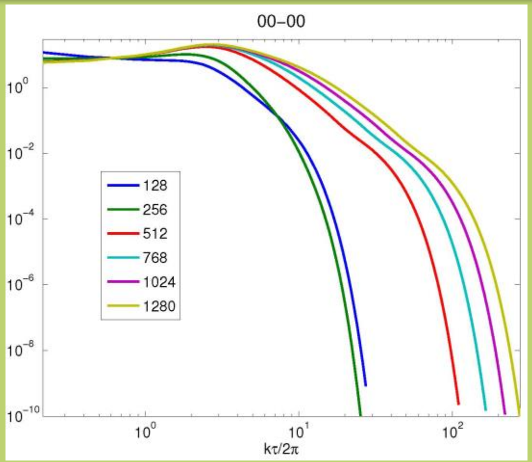


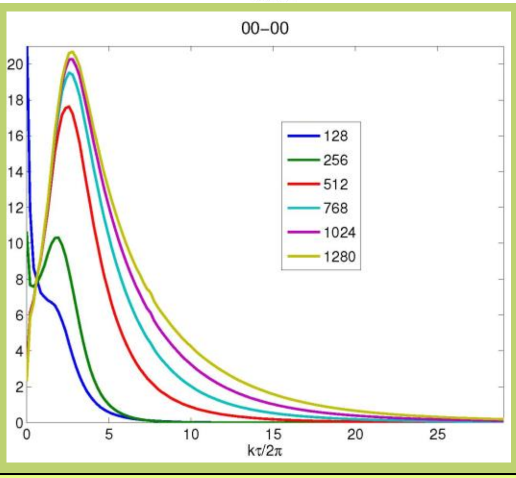
$<\theta^T\theta^T>$



UETC Convergence analysis

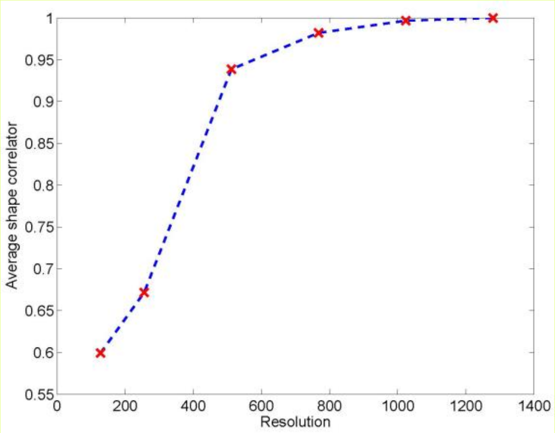
2D section of the $\langle\theta_{00}\theta_{00}\rangle$ UETC for the different resolutions





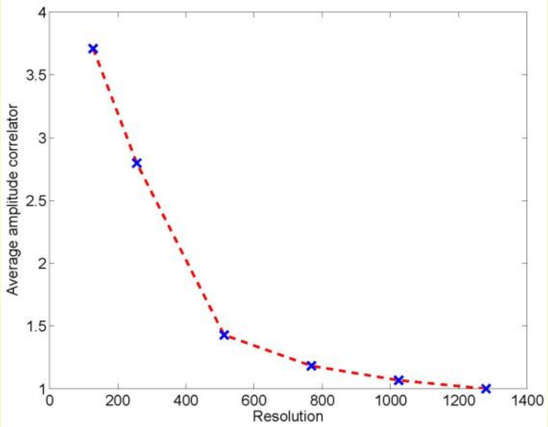
$$s_{A-B} = \frac{\sum_i \sum_j U^{sim1}(i,j) U^{sim2}(i,j)}{\sqrt{\sum_{i,j} (U^{sim1}(i,j))^2} \sqrt{\sum_{i,j} (U^{sim2}(i,j))^2}}$$

Shape correlator



Amplitude correlator

$$r_{A-B} = \frac{\sqrt{\sum_{i,j} (U^{sim1}(i,j))^2}}{\sqrt{\sum_{i,j} (U^{sim2}(i,j))^2}}$$



Fitting USM to Simulations from UETCs

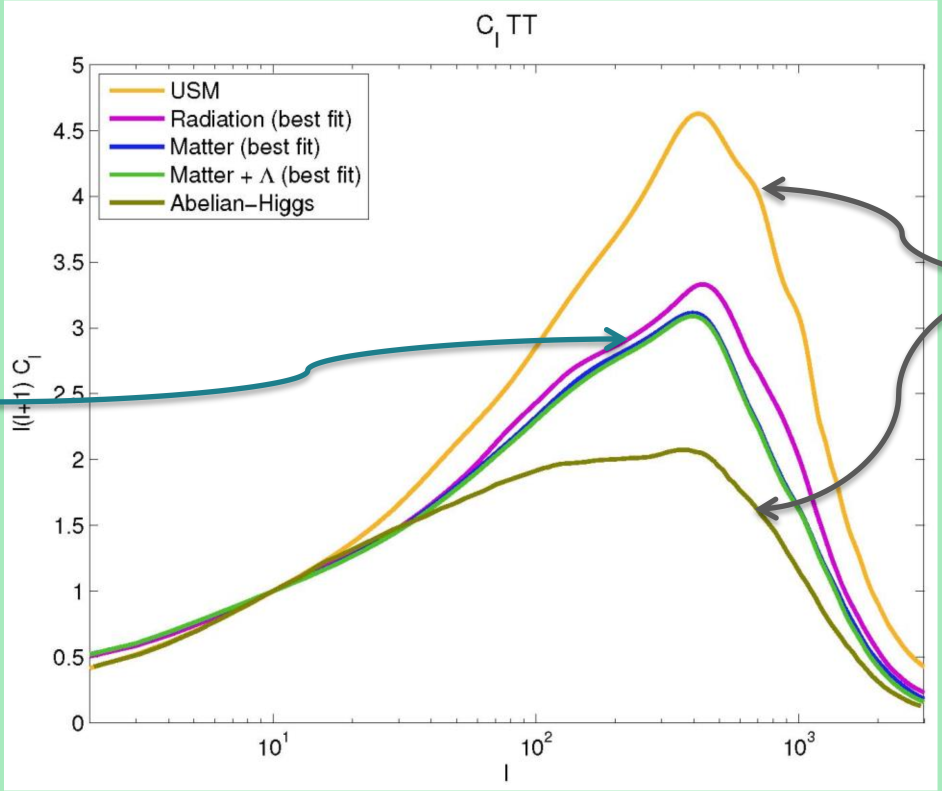
Results for the best fit parameters for shape and amplitude correlators

Simulation	Correlator	00-00	S-S	00-S	V-V	T-T	v	α	ξ
Radiation era	Shape	0.922	0.912	0.769	0.956	0.940	0.4	1.2	0.9
	Amplitude	1	1.733	0.811	1.013	1.010			
Matter era	Shape	0.918	0.912	0.678	0.993	0.906	0.1	1.2	0.9
	Amplitude	1	1.376	0.658	0.841	0.845			
Matter + Λ eras	Shape	0.827	0.923	0.802	0.720	0.928	0.04	1.2	0.8
	Amplitude	1	1.260	0.907	0.508	1.257			

The Power Spectrum

The parameters obtained were used to compute the power spectrum using CMBACT (updated with the last Planck data).

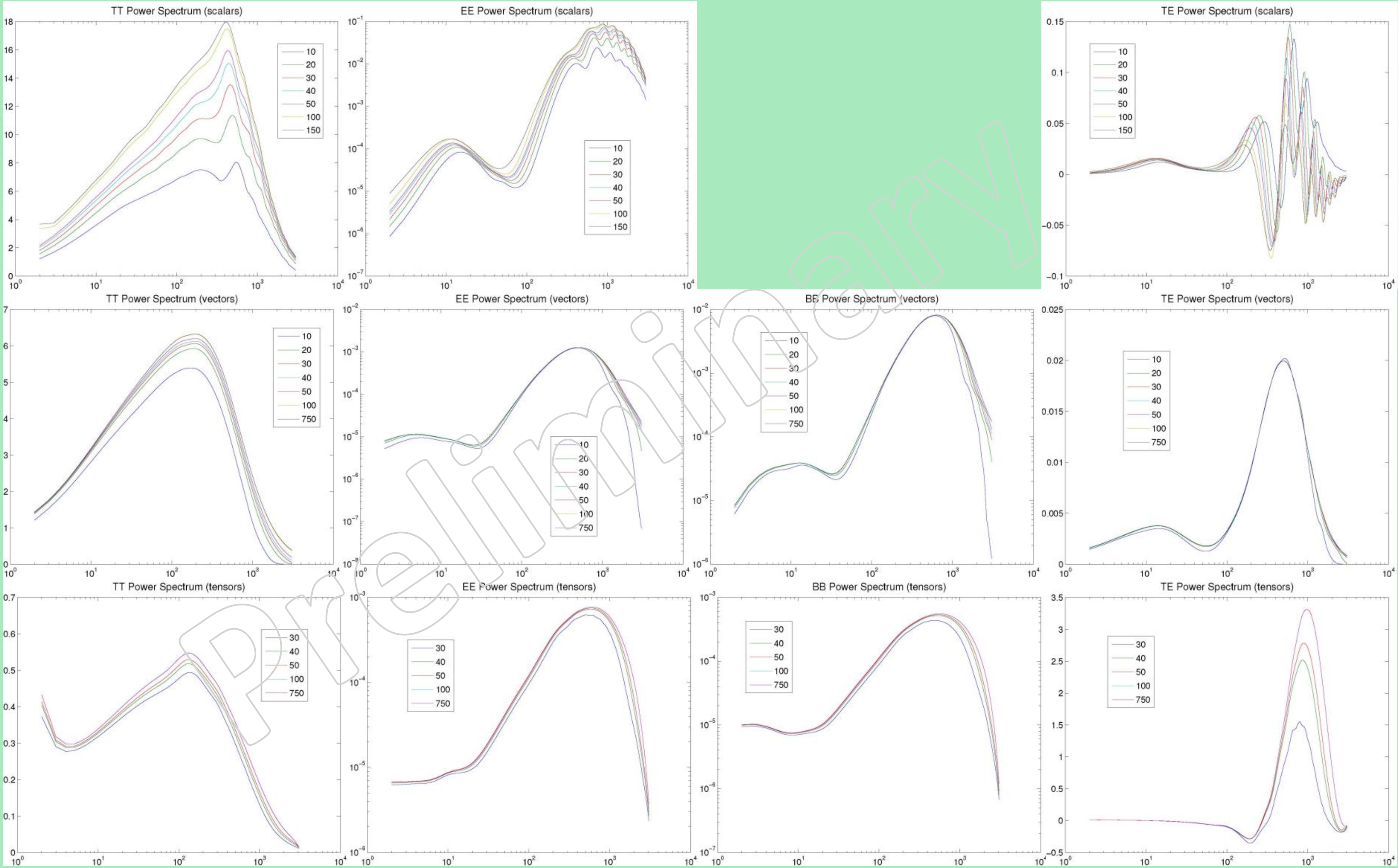
Results obtained with CMBACT with the 'best fit' parameters



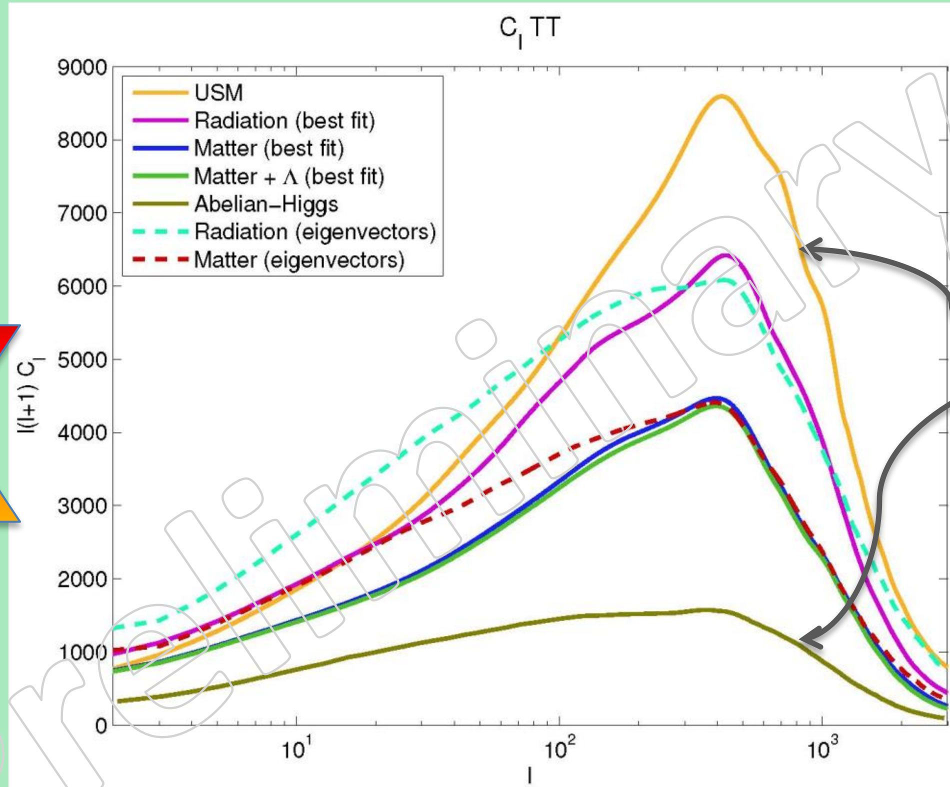
Planck Collaboration, Paper XXV arXiv:1303.5085

Comparison between the cosmic string power spectra obtained with the 'best fit' method and the standard USM and Abelian-Higgs methods.

Power spectra obtained using UETCs from Nambu-Goto cosmic string simulation 2 (matter era, $G\mu = 1.5\text{x}10^{-7}$)

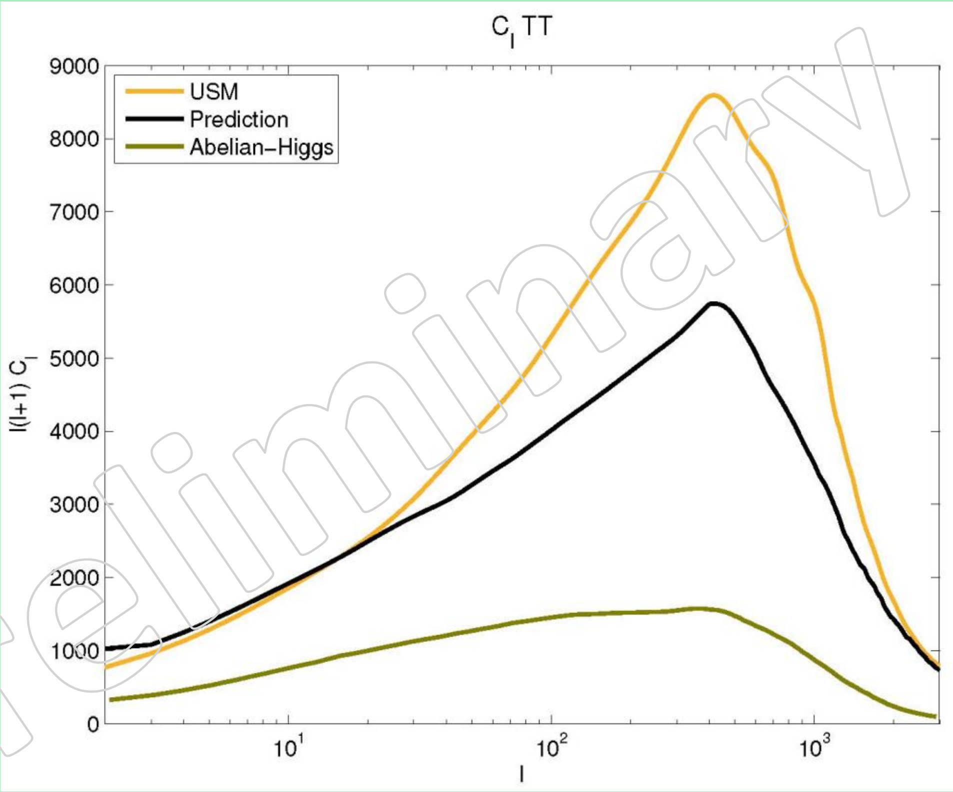


Power spectrum with UETCs



Planck Collaboration, Paper XXV arXiv:1303.5085

Comparison between the cosmic string power spectra obtained with UETCs (eigenvectors) and the standard USM and Abelian-Higgs methods.



Prediction for the string power spectra and the standard USM and Abelian-Higgs methods.

String Tension Constraints

Constraints on the maximum values of the cosmic string tension obtained with COSMOMC [Lewis & Bridle, *Phys. Rev.* D **66** (2002) 103511] using the **'best fit'** power spectrum at 95% confidence level with the 6 standard parameters ($\Omega_b h^2$, $\Omega_c h^2$, τ, θ, A_s, n_s) and strings.

String Tension	WMAP	WMAP+SPT	Planck
$G\mu/c^2$	5.89×10^{-7}	5.44×10^{-7}	2.36×10^{-7}

Planck Collaboration, Paper XXV arXiv:1303.5085

	$G\mu/c^2$
Abelian-Higgs	3.2×10^{-7}
USM	1.5×10^{-7}

Summary & Conclusions

- UETCs calculated directly from Nambu-Goto simulations.
- Best fit to analytic USM model (3 parameters: v, α, ξ).
- CMB power spectra computed with CMBACT, for the 'best fit' parameters.
- UETCs used directly into Boltzmann solver, for accurate power spectrum.
- Good concordance between the two type of calculation - both cosmic string power spectra between the standard Abelian-Higgs and USM ones.
- Power spectrum more similar to standard USM one, but with smaller amplitude.
- Constraint on the string tension $G\mu$ with COSMOMC with the 6 standard parameters. Magnitude of the constraint also between the Abelian-Higgs and USM models.
- Possibility of calculating the cosmic string power spectrum at high precision, with an accurate prediction for the cosmic string contribution to the CMB power spectrum relevant for the Planck satellite.

Acknowledgements

I would like to thank the following organizations for financial support for this workshop:

Department of Applied Mathematics and Theoretical Physics (DAMTP)

UNIVERSITY OF CAMBRIDGE

Gonville & Caius College, Cambridge