Analytic Modelling of Cosmic String & Superstring Networks

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(University of Nottingham)

Based on:

Arxiv: 1312.2123, 1209.2461, 1107.2008, 1012.5014,
0912.4004, 0705.3395,
astro-ph/0512582, hep-ph/0410349

with: A. Achucarro, E. Copeland, M. Leite, A. Lopez, C. Martins, A. Moss, A. Nunes, L. Pogosian, A. Pourtsidou, P. Shellard, D. Skliros, D. Steer, J. Urestilla

In progress w/ Achucarro, Leite, Lopez, Martins, Nunes & Urestilla
Charnock, Copeland, Moss & Skliros
Pourtsidou & Sakellariadou

Motivation

- Generic in a wide range of models (Majumdar & Davis 2002
  Burgess et al 2001,
  Sarangi & Tye 2002,
  Jeannerot et al 2003)

- A number of potential observational effects

- Powerful tool for probing/constraining High-Energy Physics:

Field Theory Strings
Energy scale of
symmetry breaking

Cosmic Superstrings
String coupling
Compactification scale(s)
Warping scale

Important to understand
their cosmological evolution

The VOS model

Kibble 1985
Martins & Shellard 1996/2000

Nambu-Goto action $S = -\mu \int \sqrt{-\gamma}\, d^2\zeta \,, \;\; \gamma_{\alpha\beta} = g_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$

Equation of motion $\nabla^2 x^\mu + \Gamma^\mu_{\nu\lambda}\gamma^{\alpha\beta}\partial_\alpha x^\nu \partial_\beta x^\lambda = 0$

E-M tensor $T^{\mu\nu} = \frac{1}{\sqrt{-g}}\,\mu \int d^2\zeta \sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu\, \delta^{(4)}(x^\lambda - x^\lambda(\zeta^\alpha))$

From E-M tensor define energy: $E(\tau) = \int_{t=\mathrm{const}} \sqrt{h}\, n_\mu n_\nu T^{\mu\nu} d^3\mathbf{x} = a(\tau)\mu \int \epsilon d\sigma$

For a network define: $v^2 = \left\langle \frac{d\mathbf{x}}{d\tau}^2 \right\rangle \equiv \frac{\int \frac{d\mathbf{x}}{d\tau}^2 \epsilon d\zeta}{\int \epsilon d\zeta}$ $\qquad \rho = \frac{\mu}{L^2}$

Differentiate and use eom, find:

$$\dot\rho = -2\frac{\dot a}{a}(1 - 2v^2)\rho$$

$$\dot v = (1 - v^2)\left(\frac{k}{R} - 2\frac{\dot a}{a} v\right)$$

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Differentiate and use eom, find:

$$\dot\rho = -2\frac{\dot a}{a}(1-2v^2)\rho - \frac{\tilde c v \rho}{L}$$

$$\dot v = (1-v^2)\left(\frac{k}{R} - 2\frac{\dot a}{a}v\right)$$

Add interaction term

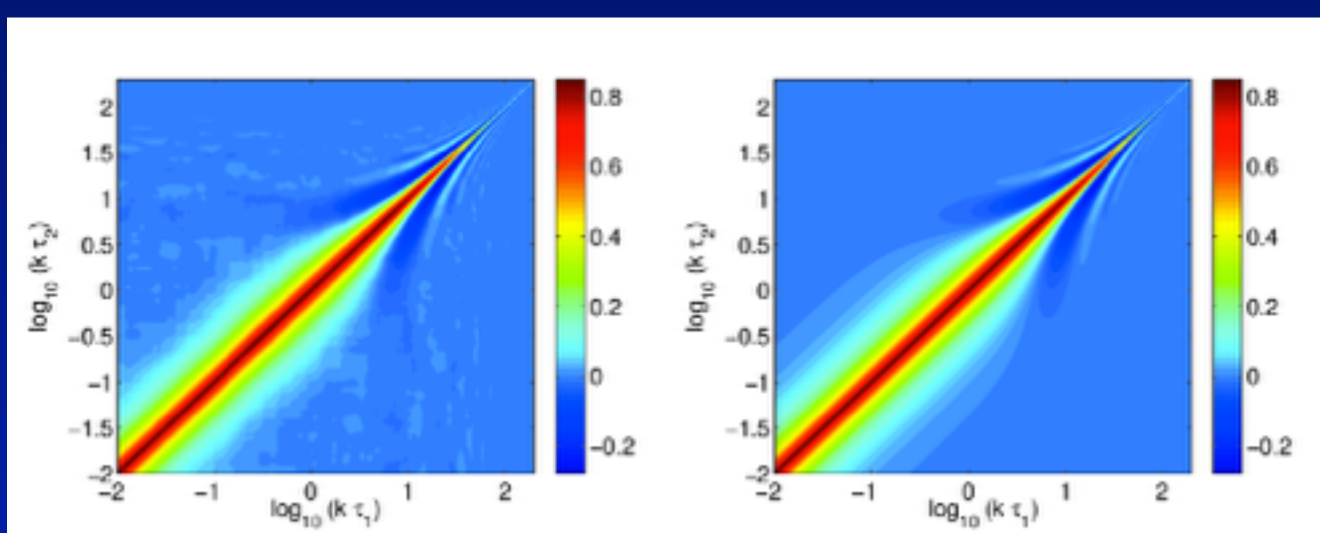
Cf simulations

Computing CMB signals from strings

- Strings are Active, Incoherent sources ⟶ need UETC:

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)$$

- *Standard approach*: model network as K unconnected segments with lengths and velocities given by VOS model
- We have computed integrals analytically: (AA, Copeland, Moss & Skliros, 2012)



USM (~8 hours)

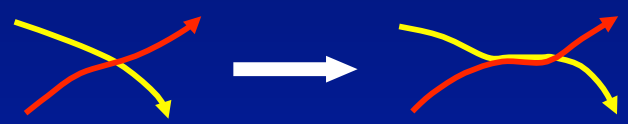
Our analytic result (~20 seconds)

Get Cl's in a few mins:
MCMC analysis including network parameters now possible

(cf Adam's talk, Andrei's & Paul's talks)

Cosmic Superstrings

To describe cosmic superstring networks one must include the following additional effects:

- Strings Evolve in Extra Dimensions (ED)
- Reduced Intercommuting Probabilities
  (Quantum Interactions & ED)
- Multi-tension string components
- New Interactions: Junction formation 
- Non-trivial Kinematic Constraints

VOS in Extra Dims: Microscopics

(AA & Shellard 2004)

- Metric $$ds^2 = N(t)^2 dt^2 - a(t)^2 d\mathbf{x}^2 - b(t)^2 d\mathbf{l}^2$$

Nambu-Goto Action $$S = -\mu \int \sqrt{-\gamma}\, d^2\zeta$$

Equations of motion
$$\dot{\epsilon} = -N^{-2}\epsilon \left\{ N\dot{N} + a\dot{a}\left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon}\right)^2\right] + b\dot{b}\left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon}\right)^2\right]\right\}$$

$$\ddot{\mathbf{x}} + \left\{\frac{2\dot{a}}{a} - N^{-2}\left\{ N\dot{N} + a\dot{a}\left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon}\right)^2\right] + b\dot{b}\left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon}\right)^2\right]\right\}\right\}\dot{\mathbf{x}} = \left(\frac{\mathbf{x}'}{\epsilon}\right)'\epsilon^{-1}$$

$$\ddot{\mathbf{l}} + \left\{\frac{2\dot{b}}{b} - N^{-2}\left\{ N\dot{N} + a\dot{a}\left[\dot{\mathbf{x}}^2 - \left(\frac{\mathbf{x}'}{\epsilon}\right)^2\right] + b\dot{b}\left[\dot{\mathbf{l}}^2 - \left(\frac{\mathbf{l}'}{\epsilon}\right)^2\right]\right\}\right\}\dot{\mathbf{l}} = \left(\frac{\mathbf{l}'}{\epsilon}\right)'\epsilon^{-1}$$

Energy-Momentum Tensor & Energy:

$$T^{\mu\nu} = \frac{1}{Na^3 b^{D-3}}\,\mu \int d\zeta\,(\epsilon \dot{x}^\mu \dot{x}^\nu - \epsilon^{-1} x'^\mu x'^\nu)\,\delta^{(D)}(\mathbf{x} - \mathbf{x}(\zeta,t)\,,\ \mathbf{l} - \mathbf{l}(\zeta,t))$$

$$E = \int_{t=\text{const}} \sqrt{h}\, n_\mu n_\nu T^{\mu\nu}\, d^3\mathbf{x}\, d^{D-3}\mathbf{l}$$

VOS in Extra Dims: Macroscopics

- Energy Density (equiv. correlation length L)

Intercommuting Prob

$$2\frac{\mathrm{d}L}{\mathrm{d}t} = \left[(2+w_\ell^2) + (2-w_\ell^2)\, v_x^2 + (1-w_\ell^2)\, v_\ell^2\right] HL + cP_{\mathrm{eff}}\, v_x$$

Modify 3D terms

New term due to extra dimensional velocities

String Velocities

$$v_x \frac{dv_x}{dt} = \frac{k_x v_x}{R}(1-v^2) - \left(2-w_\ell^2\right) H {v_x}^2 (1-v^2) - H {v_x}^2 {v_\ell}^2$$

$$v_\ell \frac{dv_\ell}{dt} = \frac{k_\ell v_\ell}{R}(1-v^2) - \left(1-w_\ell^2\right) H {v_\ell}^2 (1-v^2) + H {v_\ell}^2 {v_x}^2$$

Effective 3D string motion slows down due to extra dimensional velocities

$$v^2 \equiv v_x^2 + v_\ell^2 \leq 1/2$$

(AA & Shellard 2004)

Intercommutation Probability (Shellard 1987)

- Jackson, Jones & Polchinski 2004 $10^{-3} < P < 10^{-1}$

- Introduce P<1 in 1-scale model:

$$\dot{\rho} \simeq -2H\rho - \rho P/L \quad \rightarrow \quad \rho \propto P^{-2}$$

(Jones, Stoica & Tye 2003)

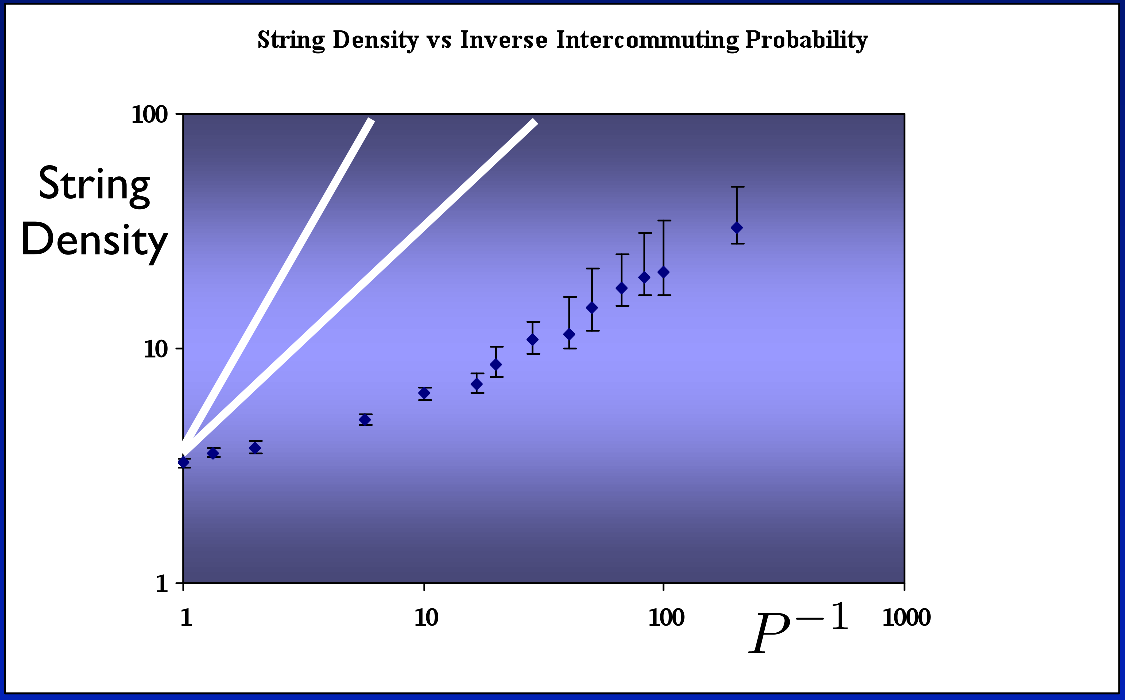
Effect of Small-Scale Structure? Need simulations.

Flat space simulations suggest: $\rho \propto P^{-1}$
(Sakellariadou 2004)

Expanding space: $\rho \propto P^{-0.6\pm0.1}$
(AA & Shellard 2005)

$$P_{\rm eff} \propto P^{1/3}$$

Cf Olum & Vanchurin 2005
Vanchurin 2007



Cosmic Superstrings

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- Multi-tension string components
- New Interactions: Junction formation
- Non-trivial Kinematic Constraints

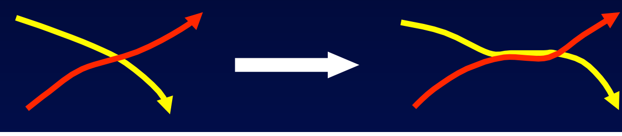
Modelling Superstring Networks

(AA & Shellard 2007)
(cf Tye et al 2006)

For superstring networks, must include:

- N types of string (F, D, FD,...)
- junction formation
- intercommuting probs

- String **Densities**:



$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1-2v_i^2)\rho_i - \frac{\tilde{c}_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{\tilde{d}^k_{ia}\bar{v}_{ia}\mu_i \ell^k_{ia}(t)}{L_a^2 L_i^2} + \sum_{b,\, a\leq b} \frac{\tilde{d}^i_{ab}\bar{v}_{ab}\mu_i \ell^i_{ab}(t)}{L_a^2 L_b^2}$$

Model dep params: depend on P_{ii}

Model dep params: depend on P_{ij}

String **Velocities**:

$$\dot{v}_i = (1-v_i^2)\left[\frac{k_i}{R_i} - 2\frac{\dot{a}}{a}v_i + \sum_{b,\, a\leq b} \tilde{b}^i_{ab}\frac{\bar{v}_{ab}}{v_i}\frac{(\mu_a+\mu_b-\mu_i)}{\mu_i}\frac{\ell^i_{ab}(t)L_i^2}{L_a^2 L_b^2}\right]$$

Energy conservation at junctions

Length of zippers $\ell^k_{ij}(t)$ *are model dependent* $f(L_i, v_i, \mu_i)$

Modelling Superstring Networks

Interaction probs can be computed:

(Jackson, Jones & Polchinski 2004
Hanany & Hashimoto 2005)

Interaction (ij)	$\mathcal{P}_{\rm ij}$
F-F	$g_s^2 \frac{\left(1-\cos\theta\sqrt{1-v^2}\right)^2}{8\sin\theta\, v\sqrt{1-v^2}}$
F-D	$g_s \frac{v^2+\left(\cos\theta\sqrt{1-v^2}\right)^2}{8\sin\theta\, v\sqrt{1-v^2}}$
F-(p,q) , $q \geq 1$	$g_s \frac{\left(q^2v^2+\left(g_s p-\cos\theta\sqrt{(1-v^2)(g_s^2p^2+q^2)}\right)\right)^2}{8\sin\theta\, v\sqrt{(1-v^2)(g_s^2p^2+q^2)}}$
D-D	$\min\left\{\frac{\sqrt{g_s}}{2\pi^{3/4}\theta^{3/4}} e^{2\sqrt{2/3}(\theta/v)} \exp\left[-\frac{4\sqrt{\pi}\theta^{3/2}}{g_s} e^{-4\sqrt{2/3}(\theta/v)}\right], 1\right\}$
(p,q)-(p',q') , $q,q' \geq 1$	$1-(1-\mathcal{P}_{\rm DD})^{qq'}$

Non-perturbative (approximations)

- Evolution Eqns:

$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1-2v_i^2)\rho_i - \frac{\tilde{c}_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{\tilde{d}_{ia}^k \bar{v}_{ia}\mu_i \ell_{ia}^k(t)}{L_a^2 L_i^2} + \sum_{b,a\leq b} \frac{\tilde{d}_{ab}^i \bar{v}_{ab}\mu_i \ell_{ab}^i(t)}{L_a^2 L_b^2}$$

$\tilde{c}_i \propto \mathcal{P}_{ii}^{1/3}$ Numerical Evidence

$\tilde{d}_{ij}^k \propto \mathcal{P}_{ij}^{1/3}$ Need Sims

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Kinematic Constraints

Copeland, Kibble & Steer 2006-7
Copeland, Firouzjahi, Kibble & Steer 2008

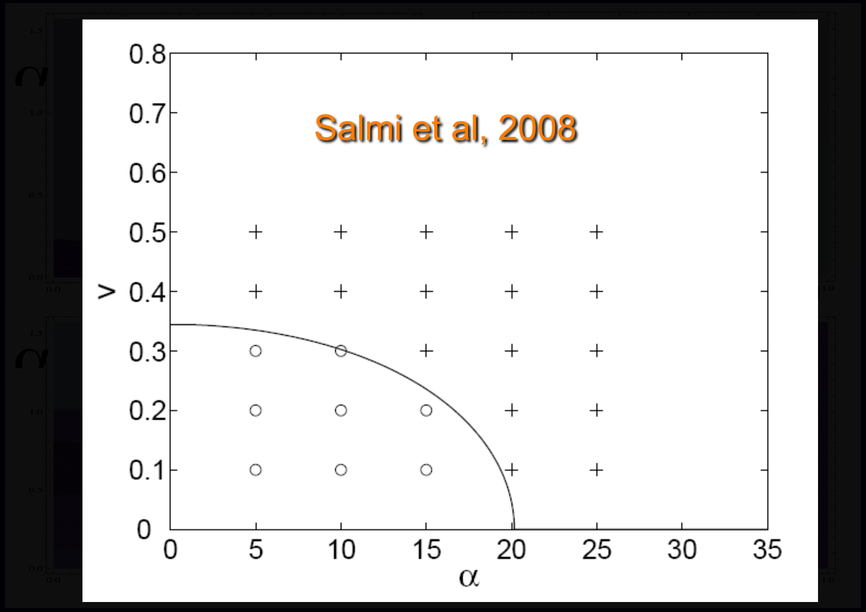
Nambu-Goto eqns for 3 segments with a junction:

$$S = -\sum_j \mu_j \int dt \int d\sigma\, \theta(s_j(t)-\sigma)\sqrt{\mathbf{x}_j'^2(1-\dot{\mathbf{x}}_j^2)} + \sum_j \int dt\, \mathbf{f}_j(t)\cdot[\mathbf{x}_j(s_j(t),t)-\mathbf{X}(t)]$$

imply a kinematic constraint for junction formation: $f_{\vec{\mu}}(v,\alpha) < 0$

Express as window function and integrate in (v,α)-space:

$$S_{ij}^k = \frac{2}{\pi}\int_0^1 \int_0^{\pi/2} \Theta(-f_{\vec{\mu}}(v,\alpha)) \exp[(v-\bar{v}_{ij})^2/\sigma_v^2] \mathrm{d}\alpha \mathrm{d}v < 1$$



Leads to a suppression in the coefficients:

$$\tilde{d}_{ij}^k \to S_{ij}^k \tilde{d}_{ij}^k$$

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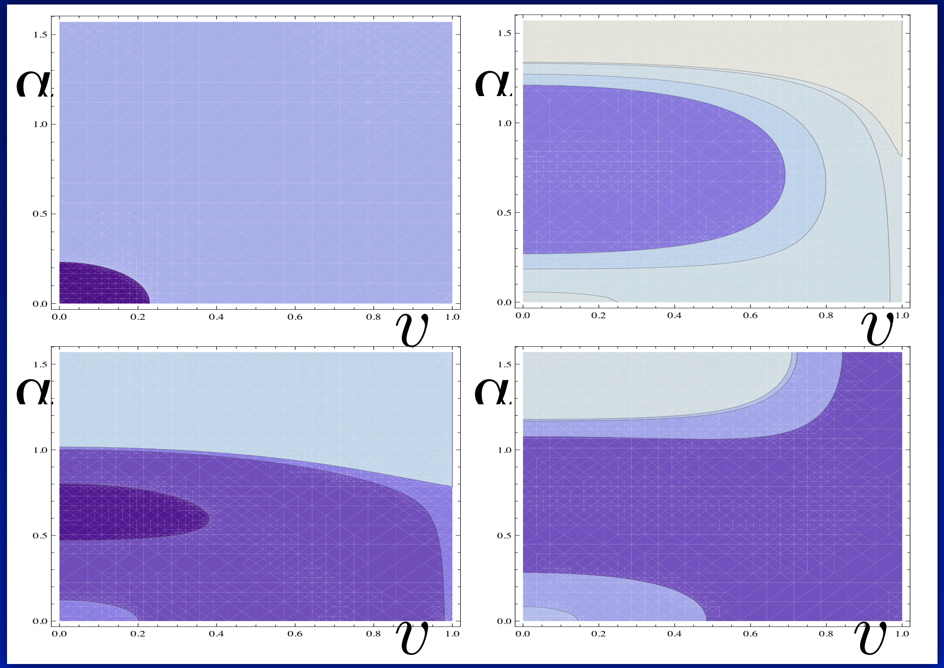


(AA & Copeland, 2010)

Leads to a suppression in the coefficients: $\tilde{d}_{ij}^k \rightarrow S_{ij}^k \tilde{d}_{ij}^k$

Recap

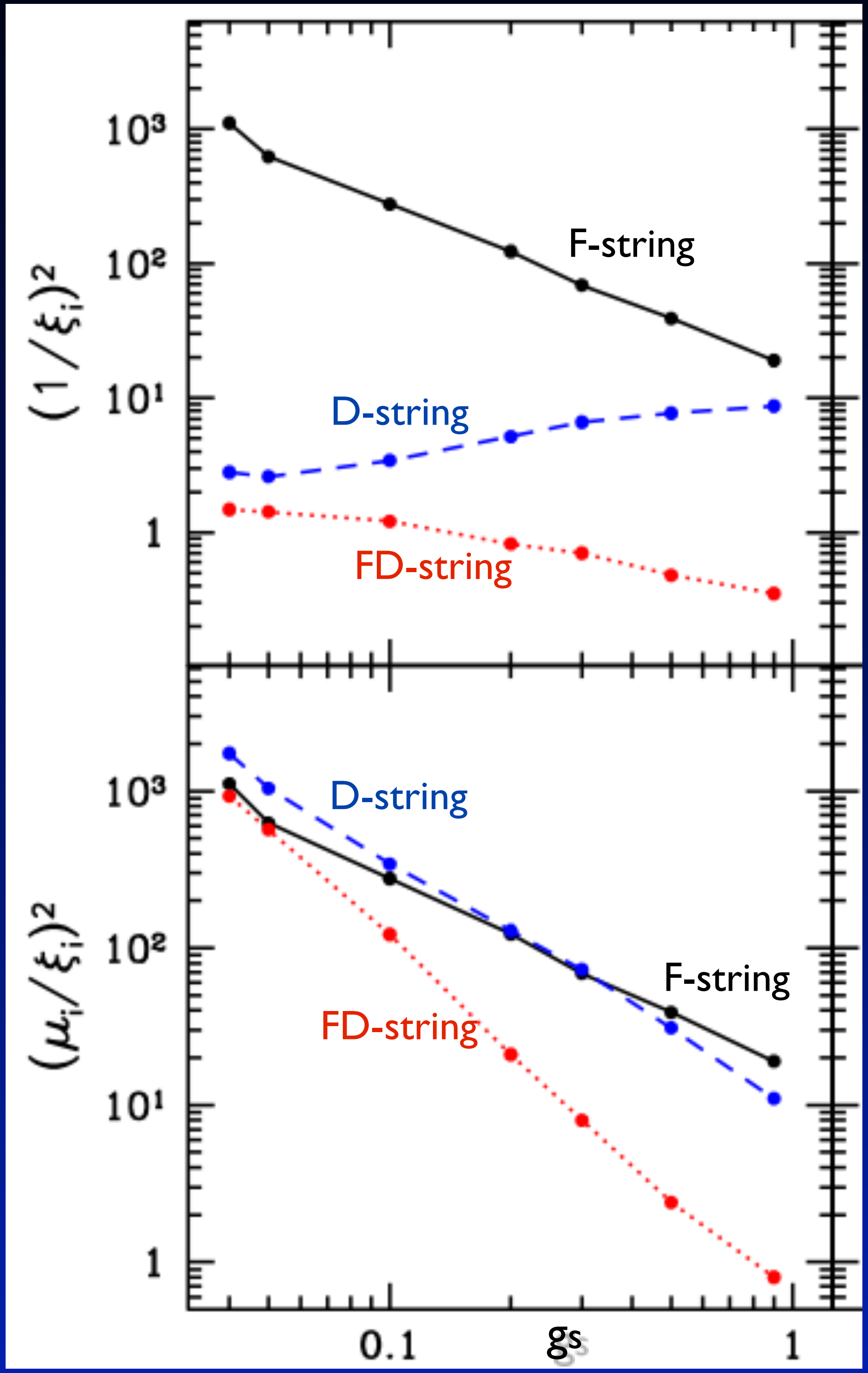
Describe interacting networks of cosmic F, D, (p,q) strings.

Started HD ⟶ Effective 3D model with extra ingredients:

- ✓ Reduced Intercommuting Probabilities
  (ED & Quantum Interactions)
- ✓ Multi-tension string components
- ✓ New Interactions: Junction formation
- ✓ Non-trivial Kinematic Constraints

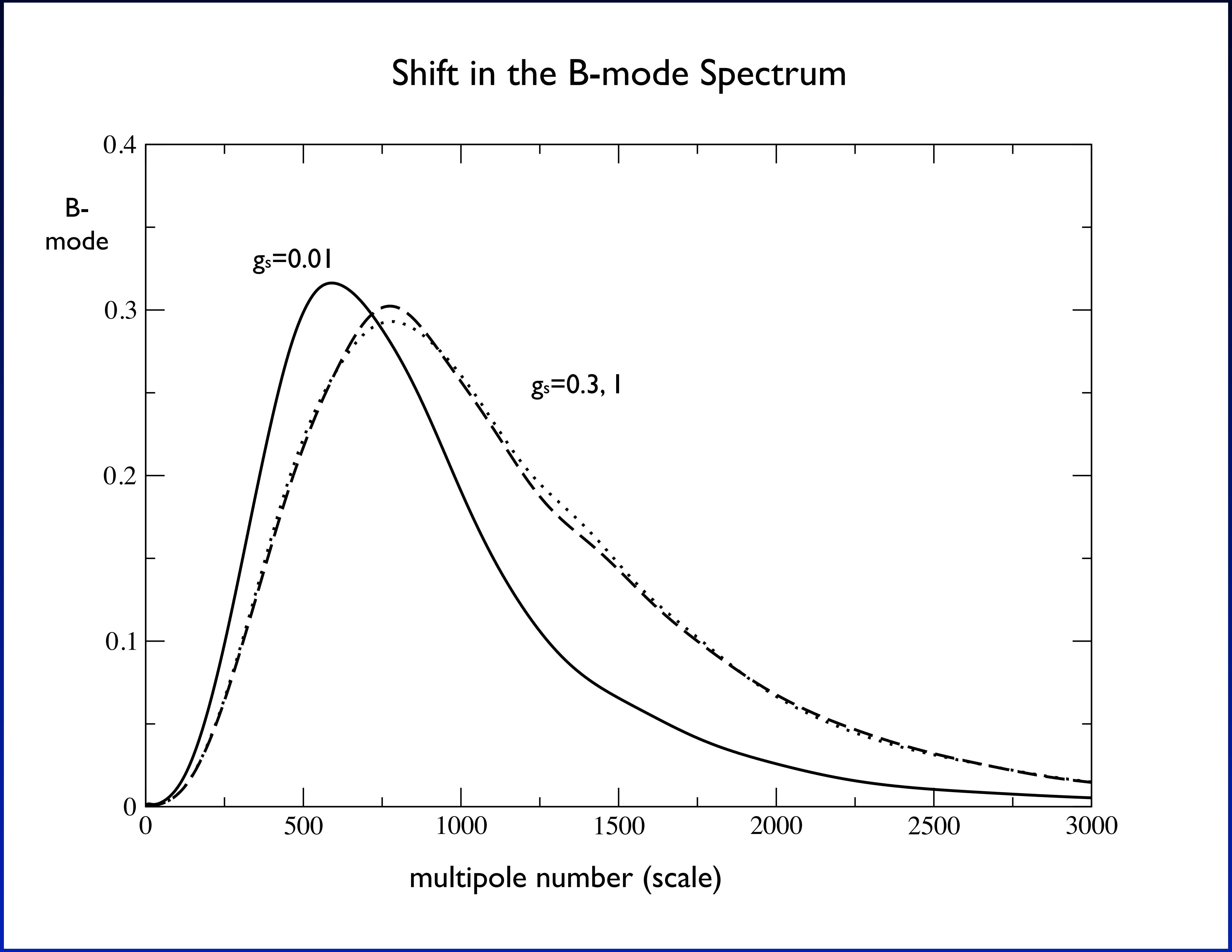
General Network Behaviour

- Scaling for all string types
  (though we keep the first 7 lightest strings)
- Only 3 lightest components
  (F, D, FD strings)
- Hierarchy in number densities
  $N_F > N_D > N_{FD}$
- Hierarchy in tensions
  $\mu_{FD} > \mu_D > \mu_F$
- Number density vs "CMB" density
  Competition depending on g_s



Potentially Observable Shift in B-mode

Pourtsidou et al, 2011 (PRL)



Some Problems and Open Issues

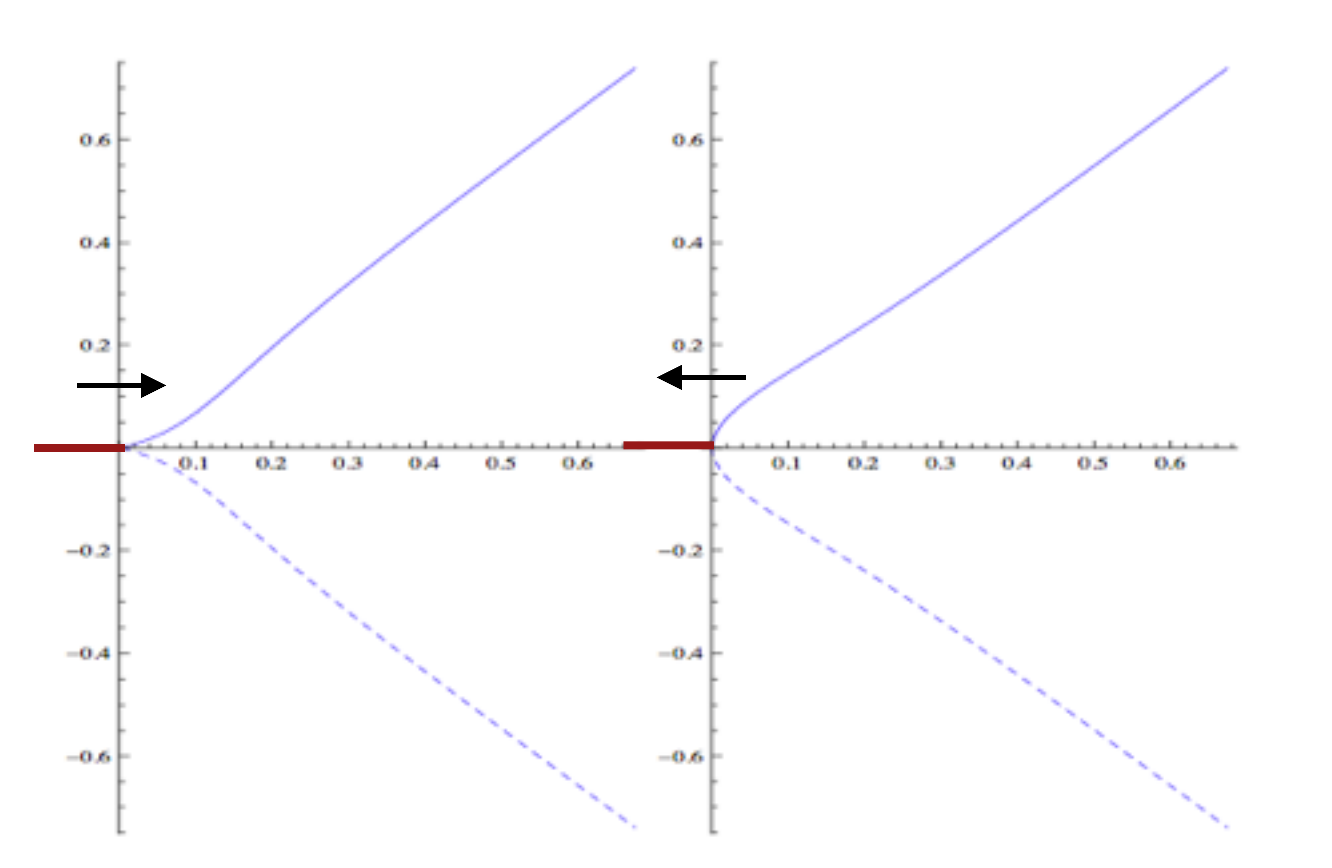
- No simulations to compare/calibrate with!
- Non-perturbative amplitudes poorly understood
- Spontaneous unzipping???

(In progress with Pourtsidou & Sakellariadou, see Mairi's talk)

Unzipping Mechanisms (cf Mairi's talk)

Examined a number of potential unzipping mechanisms in the context of Nambu-Goto strings with junctions:

- Junction Stability / Massive Monopole Dynamics No
- Expanding Background No
- String & Monopole Forces Yes



Semilocal Strings

Non-topological string solutions arising from the action:

$$S = \int d^4x \left[[(\partial_\mu - iA_\mu)\Phi]^2 - \frac{1}{4}F^2 - \frac{\beta}{2}(\Phi^+\Phi - 1)^2 \right]$$

(Achucarro & Vachaspati 1999)

Strings end on monopoles: model as hybrid string-monopole networks:

(Martins & Achucarro 2008)

$$3\frac{dL}{dt} = 3HL + v^2\frac{L}{\ell_d} + c_\star v$$

$$\frac{dl_s}{dt} = Hl_s - v_s^2\frac{l_s}{l_d} + ???$$

But now have high-resolution simulations to calibrate!

(cf Ana's talk)

Semilocal Strings: Analytic Models

Model A

$$\frac{dl_s}{dt} = Hl_s - v_s^2 \frac{l_s}{l_d} + \sigma \left(1 - \frac{L}{l_s}\right) v_m^2$$

σ to be determined by simulations

Model B

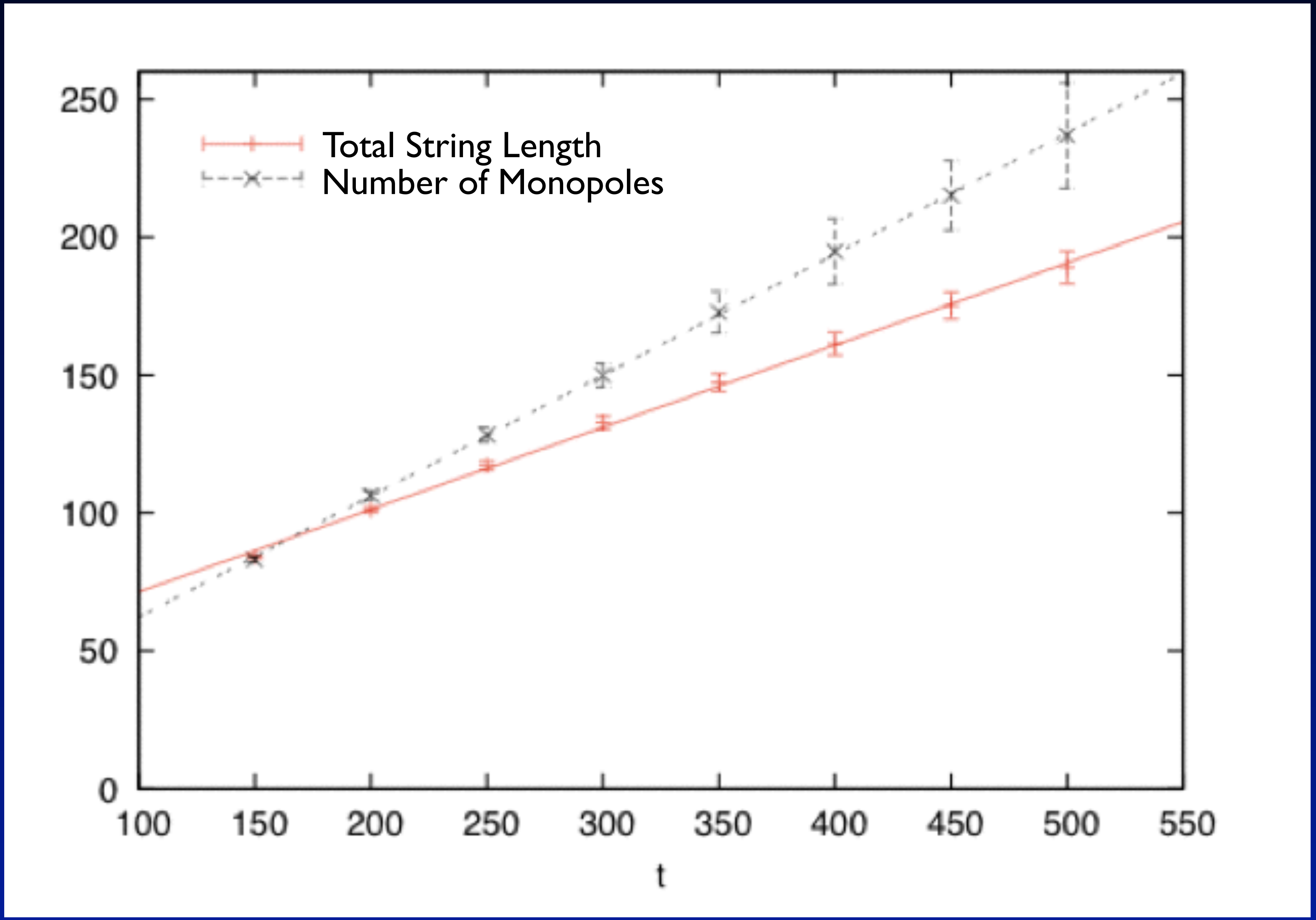
$$\frac{dl_s}{dt} = Hl_s - v_s^2 \frac{l_s}{l_d} + \left(d\frac{v_s l_s}{L} - k_1\right)$$

d, k_1 to be determined by simulations

Semilocal Scaling

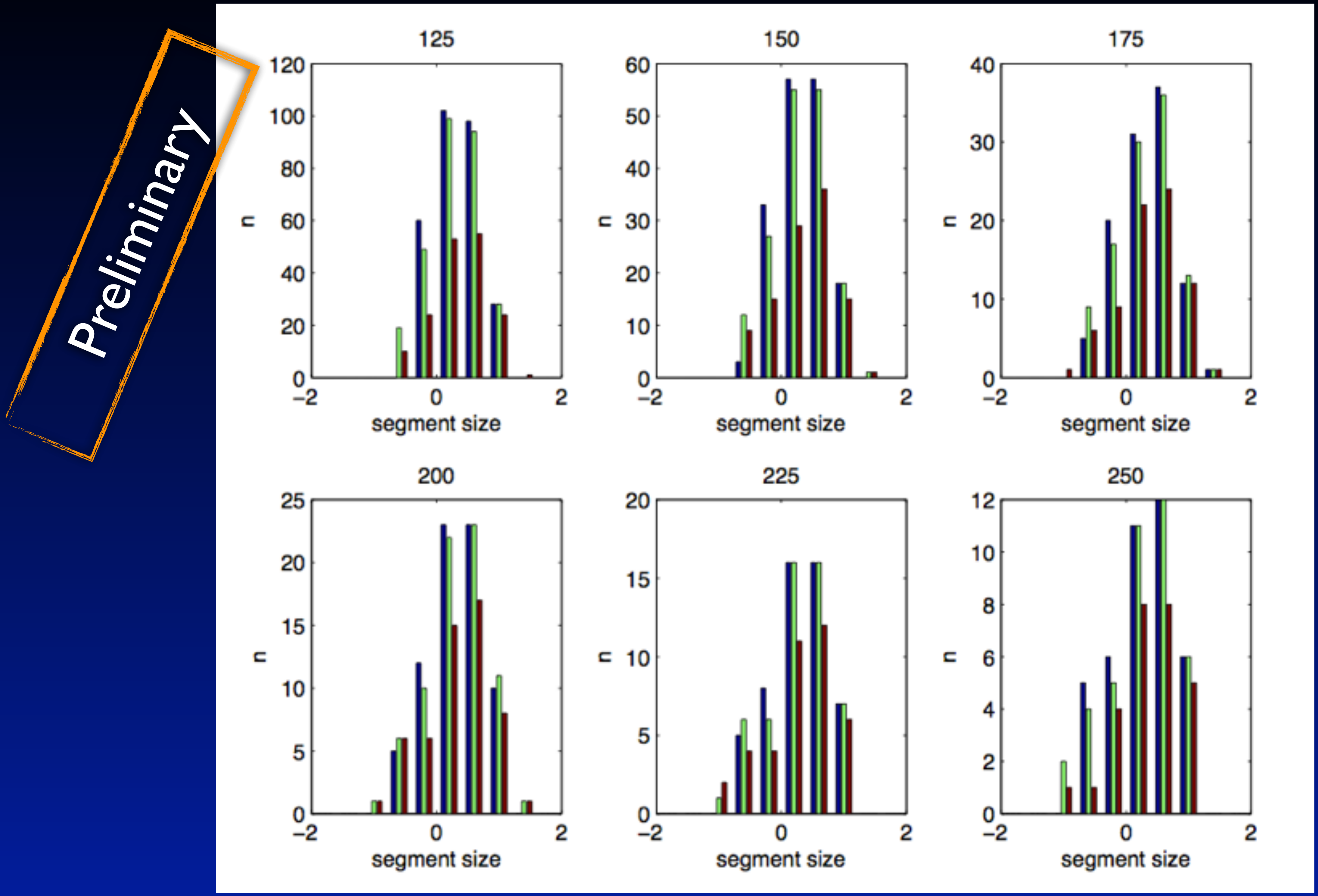
(Achucarro et al Arxiv: 1312.2123)

Simulations: strong evidence for scaling both L and ls



Sort segments in length bins and compare length distributions of Model A, Model B, and Simulations

Semilocal Length Distributions (In progress)



Aim: determine dependence of model parameters on coupling β and cosmology

Conclusions & Outlook

- Field Theory Strings: analytic models demonstrated accurate
  Currently used for efficient CMB predictions

- Cosmic Superstrings: Models developed but need:

  - simulations to test/calibrate

  - better understanding of non-pert interactions

- Semilocals: promising recent progress in both analytic & numerical modelling

  - Confronting models to sims

  - Quantifying velocities