Loops

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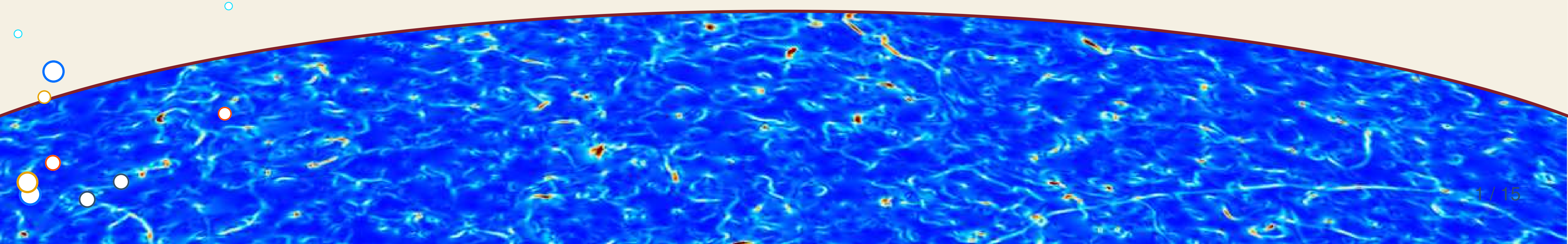

Outline

P. Peter and CR	arXiv:1302.0953
CR and F. R. Bouchet	arXiv:1204.5041
L. Lorenz, CR and M. Sakellariadou	arXiv:1006.0931
CR	arXiv:1005.4842
M. Hindmarsh, CR and T. Suyama	arXiv:0911.1241
A. Fraisse, CR, D. Spergel and F. R. Bouchet	arXiv:0708.1162
CR, M. Sakellariadou and F. R. Bouchet	astro-ph/0511646

Nambu–Goto simulations

- Based on $\longrightarrow$

arXiv.org > astro-ph > arXiv:astro-ph/0511646

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Astrophysics

Cosmological evolution of cosmic string loops

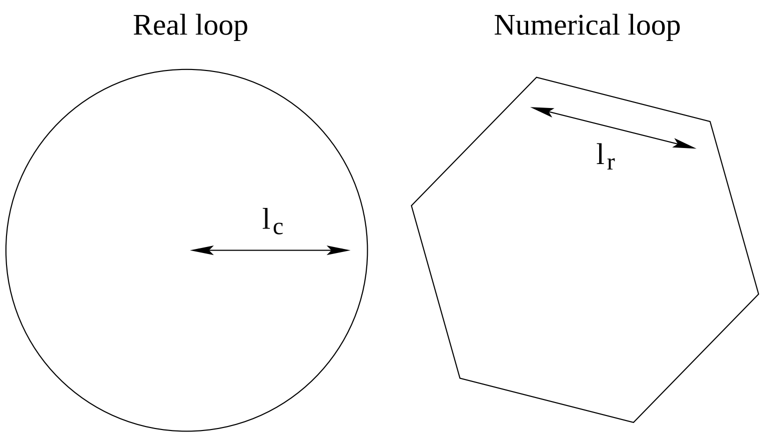
Christophe Ringeval, Mairi Sakellariadou, Francois Bouchet

(Submitted on 22 Nov 2005 (v1), last revised 19 Feb 2007 (this version, v2))

The existence of a scaling evolution for cosmic string loops in an expanding universe is demonstrated for the first time by means of numerical simulations. In contrast with what is usually assumed, this result does not rely on any gravitational back reaction effect and has been observed for loops as small as a few thousandths the size of the horizon. We give the energy and number densities of expected cosmic string loops in both the radiation and matter eras. Moreover, we quantify previous claims on the influence of the network initial conditions and the formation of numerically unresolved loops by showing that they only concern a transient relaxation regime. Some cosmological consequences are discussed.

Comments: 12 pages, 4 figures, uses iopart. Improved statistics, numerical robustness discussed in details, references added, note added. Matches published version

- Numerical parameters



Comoving box size $= 1$

Initial correlation length $\ell_c = 1/100$

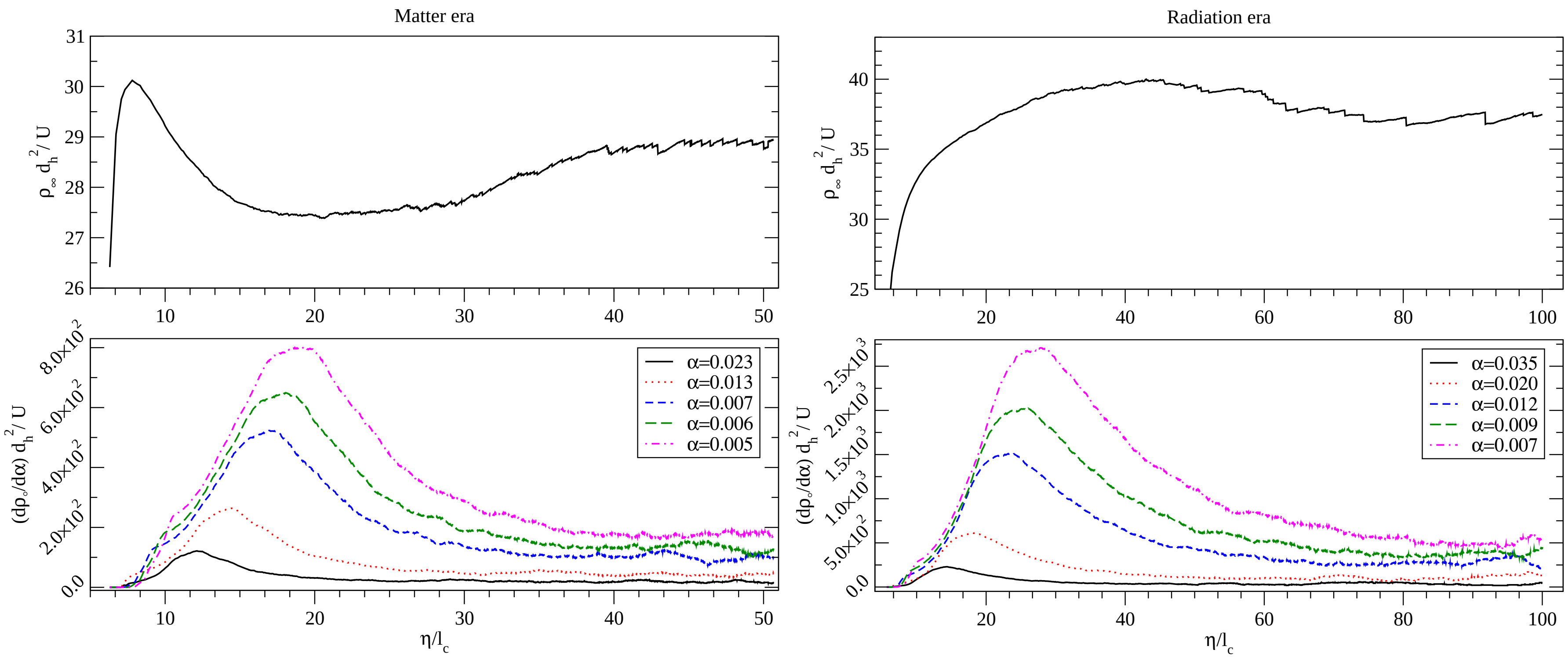
Initial resolution length $\ell_r = 1/2000$

Scaling of the energy density

- Same scaling law for long strings ($\ell > d_\mathrm{h}$) and loops ($\ell \equiv \alpha d_\mathrm{h}$)

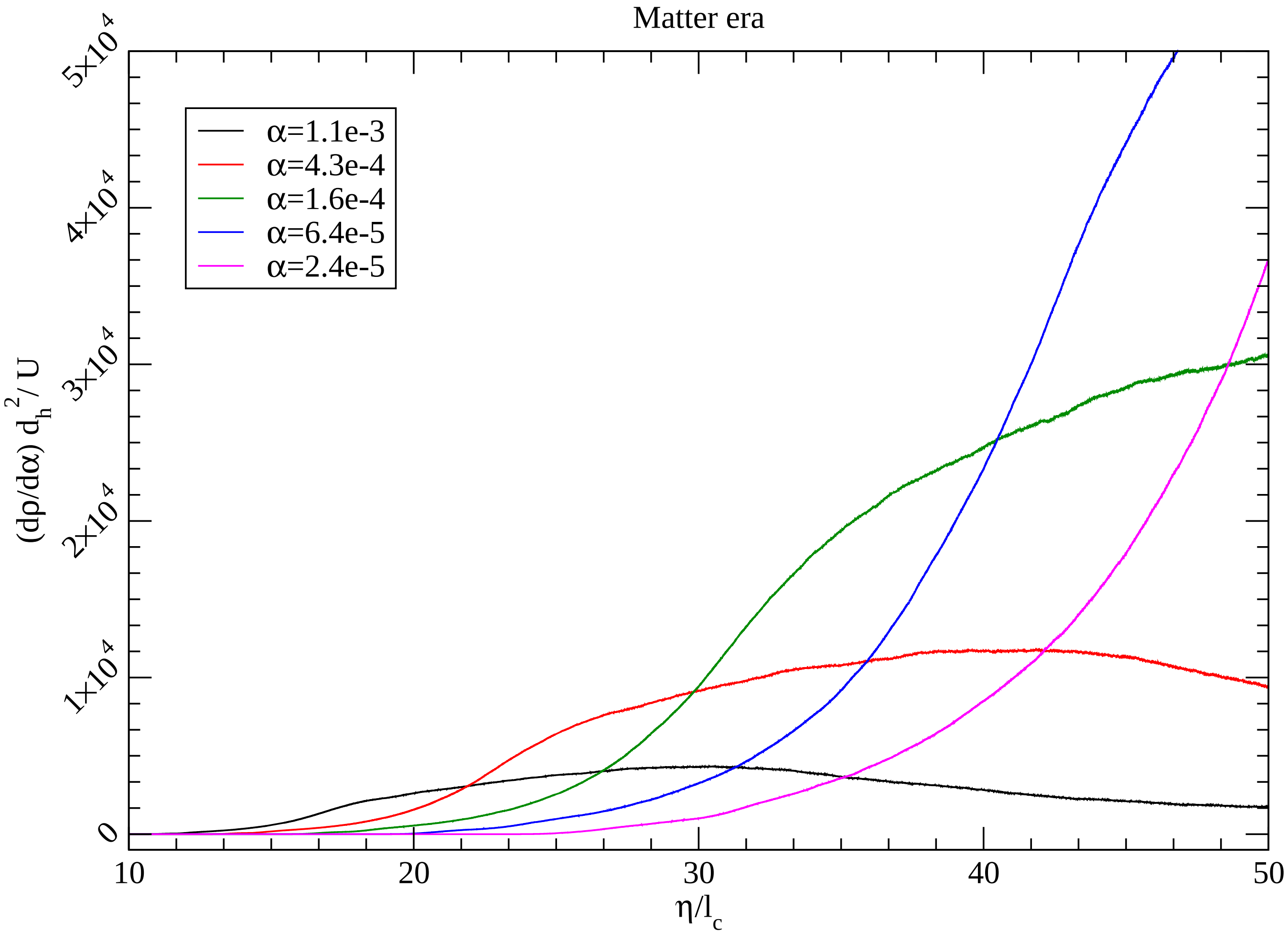
$$\rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{mat} = 28.4 \pm 0.9 \qquad \rho_\infty \frac{d_\mathrm{h}^2}{U}\bigg|_\mathrm{rad} = 37.8 \pm 1.7$$

$$\frac{\mathrm{d}\rho_\circ}{\mathrm{d}\alpha} = \mathcal{S}(\alpha)\frac{U}{d_\mathrm{h}^2} \quad \Rightarrow \quad \frac{\mathrm{d}n}{\mathrm{d}\alpha} = \frac{\mathcal{S}(\alpha)}{\alpha\, d_\mathrm{h}^3}$$



Relaxation towards scaling

- Transient effects last longer for smaller loops

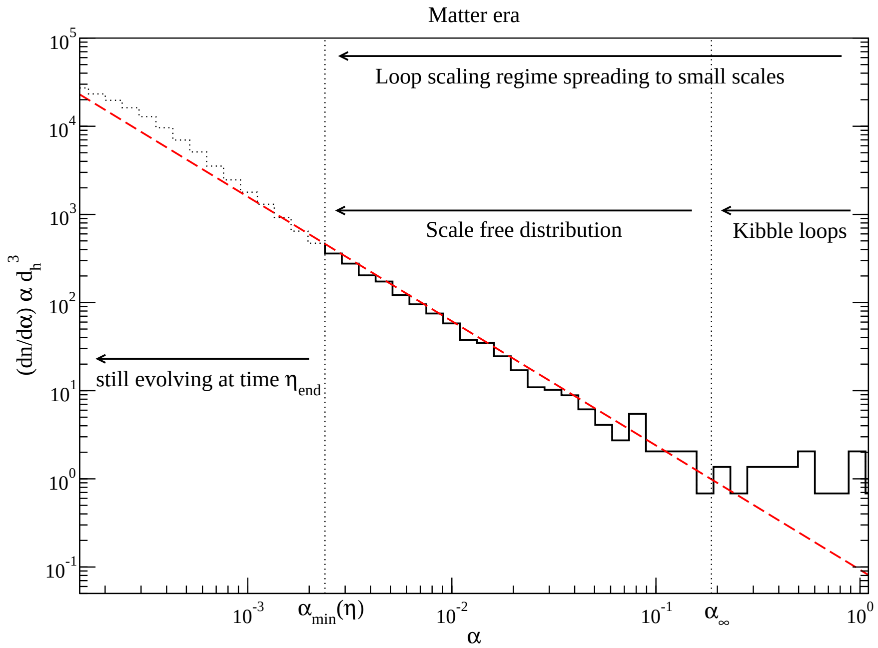


- NG simulations do not incorporate GW ⇒ do not describe $\alpha < \alpha_c$

Loop distribution in scaling

- By the end of the run

Scaling parts



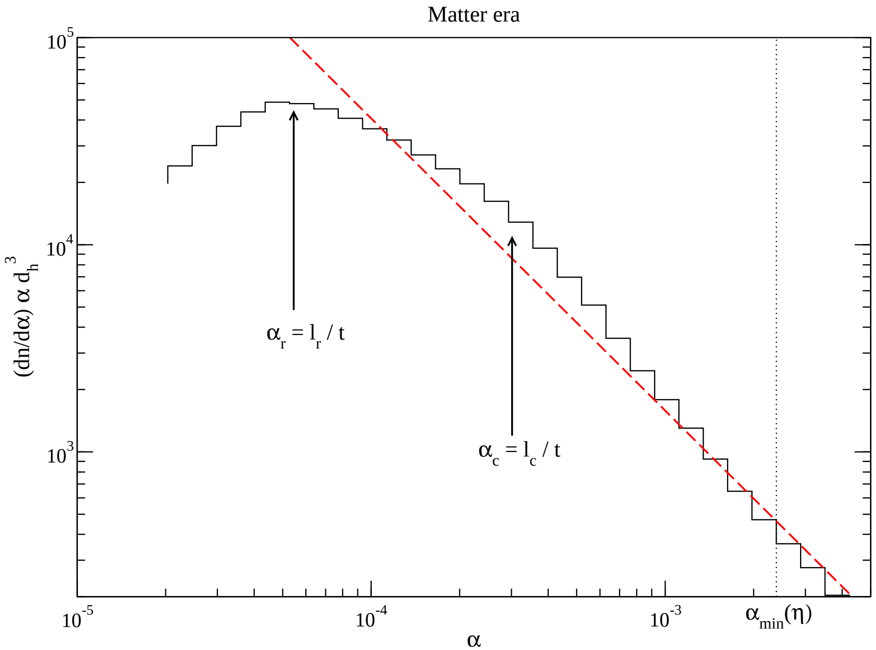
- Scaling form $\mathcal{S}(\alpha) = \dfrac{C_\circ}{\alpha^p}$ with

$$\begin{cases} p \underset{\text{mat}}{=} 1.41\,^{+0.08}_{-0.07} \\ C_\circ \underset{\text{mat}}{=} 0.09\,^{-0.03}_{+0.03} \end{cases} \quad \text{and} \quad \begin{cases} p \underset{\text{rad}}{=} 1.60\,^{+0.21}_{-0.15} \\ C_\circ \underset{\text{rad}}{=} 0.21\,^{-0.12}_{+0.13} \end{cases}$$

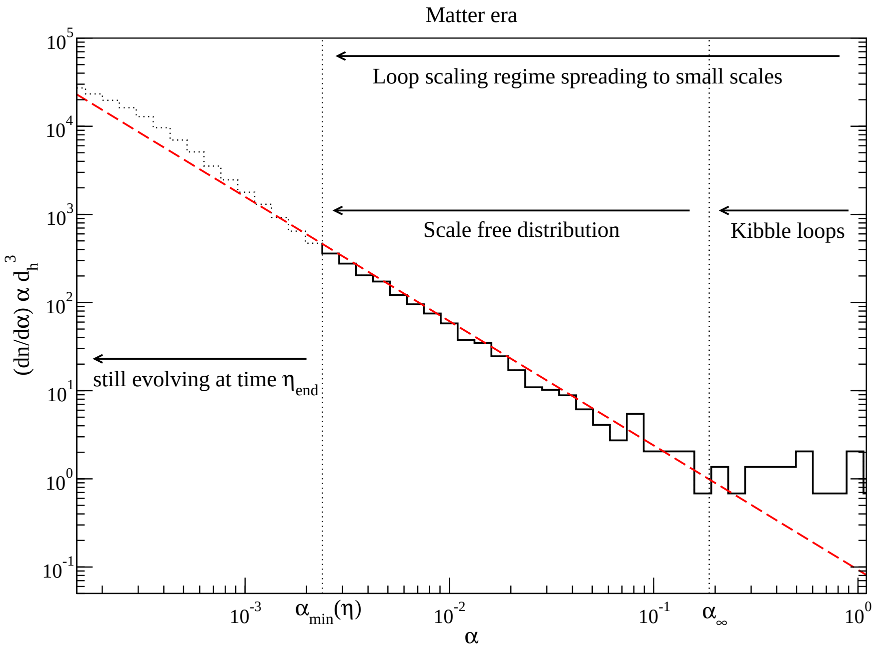
Loop distribution in scaling

- By the end of the run

Non-scaling parts



Scaling parts



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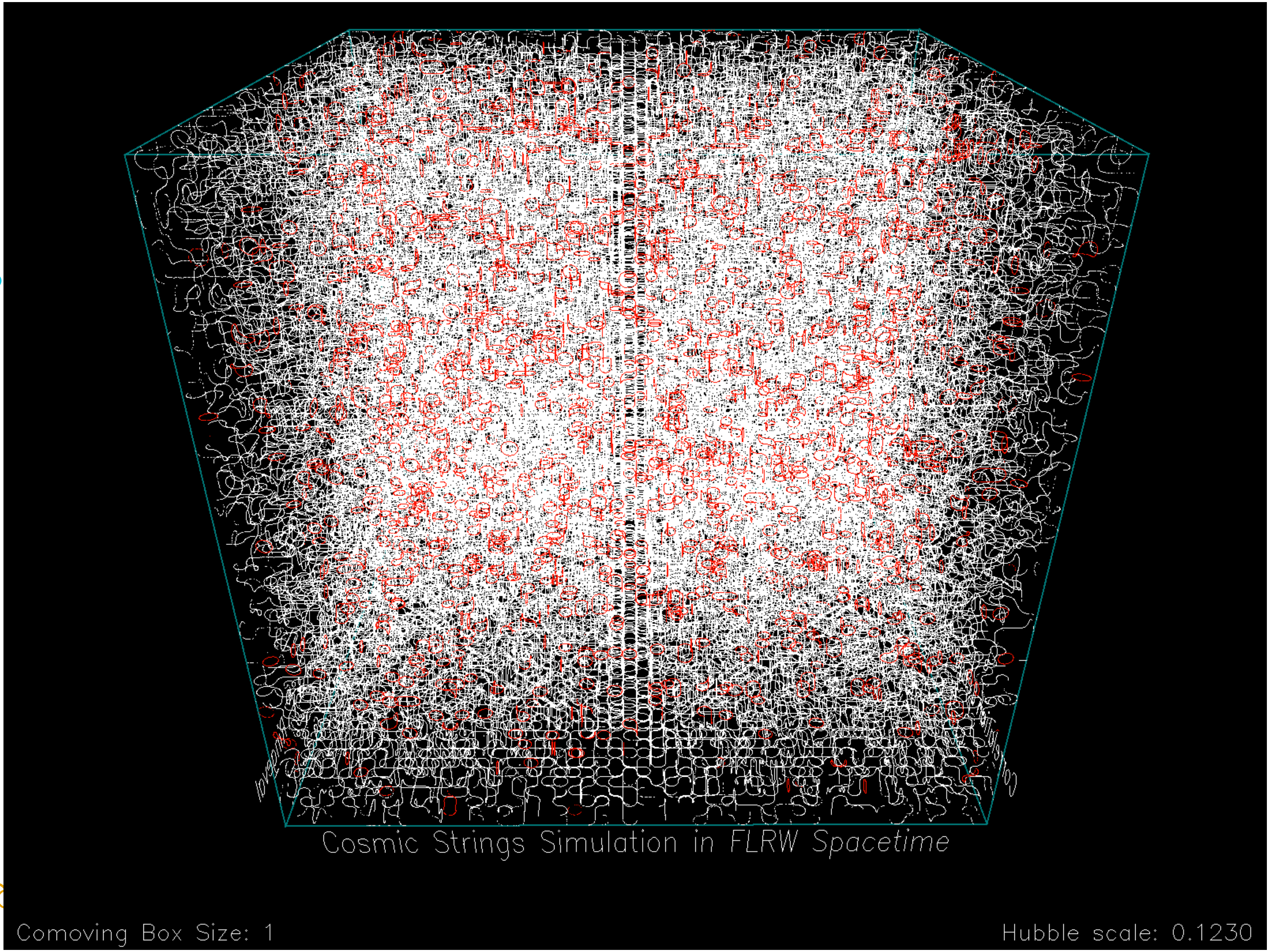
Movie


Cosmic Strings Simulation in FLRW Spacetime
Comoving Box Size: 1
Hubble scale: 0.1230

Polchinsky-Rocha model

- No fragmentation, no reconnection, loops from long string only

$$\mathcal{S}(\alpha) \propto \alpha^{2\chi-2} \Rightarrow p = 2(1-\chi)$$

- Parameter χ inferred from long string scaling
  - From Martins & Shellard simulations, they independently found[1]

$$p_{\mathrm{mat}} \simeq 1.5, \qquad p_{\mathrm{rad}} \simeq 1.8$$

  - Related to two-point functions

$$\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma')\right\rangle = \frac{1}{2}\delta^{AB}T(\sigma-\sigma') \qquad T(\sigma) \simeq \bar{t}^2 - c_1\left(\frac{\sigma}{\hat{\xi}}\right)^{2\chi}$$

- Suggests that all neglected effects mostly renormalise $C_\circ$

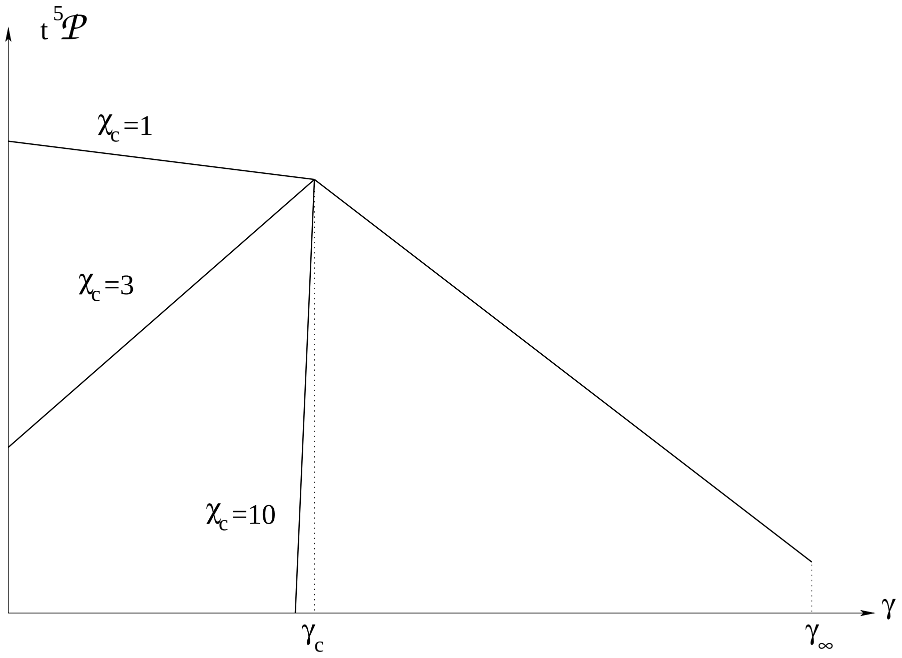
[1] hep-ph/0606205, arXiv:0709.3284

Inclusion of gravitational backreaction

- PR model + GW emission + GW backreaction to extrapolate numerical simulations to small $\ell \equiv \gamma t$
  - Boltzmann equation ($\gamma_\mathrm{d} = \Gamma G U$)

$$\frac{\partial}{\partial t}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) - \gamma_\mathrm{d} \frac{\partial}{\partial \ell}\left(a^3 \frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3 \mathcal{P}(\ell, t).$$

  - Postulated piecewise scaling loop production function



$$t^5 \mathcal{P}\left(\gamma = \frac{\ell}{t}, t\right) \propto \gamma^{2\chi - 3}$$

$$\gamma_\mathrm{c} \ll \gamma_\mathrm{d} \ll \gamma_\infty \lesssim 1$$

Cosmological loop distribution

- Can be completely solved analytically (see arXiv.1006.0931)
- From any initial loop distribution $\mathcal{N}_{\rm ini}(\ell)$, one gets $\mathcal{F}(\gamma,t) \equiv \dfrac{\partial n}{\partial \ell}(\gamma,t)$

$$t^4\mathcal{F}(\gamma \geq \gamma_c, t) = \left(\frac{t}{t_{\rm ini}}\right)^4 \left(\frac{a_{\rm ini}}{a}\right)^3 t_{\rm ini}^4\, \mathcal{N}_{\rm ini}\left\{\left[\gamma + \gamma_{\rm d}\left(1 - \frac{t_{\rm ini}}{t}\right)\right] t\right\} + C(\gamma+\gamma_{\rm d})^{2\chi-3} f\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\right)$$
$$- C(\gamma+\gamma_{\rm d})^{2\chi-3}\left(\frac{t}{t_{\rm ini}}\right)^{2\chi+1}\left(\frac{a_{\rm ini}}{a}\right)^3 f\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\frac{t_{\rm ini}}{t}\right),$$

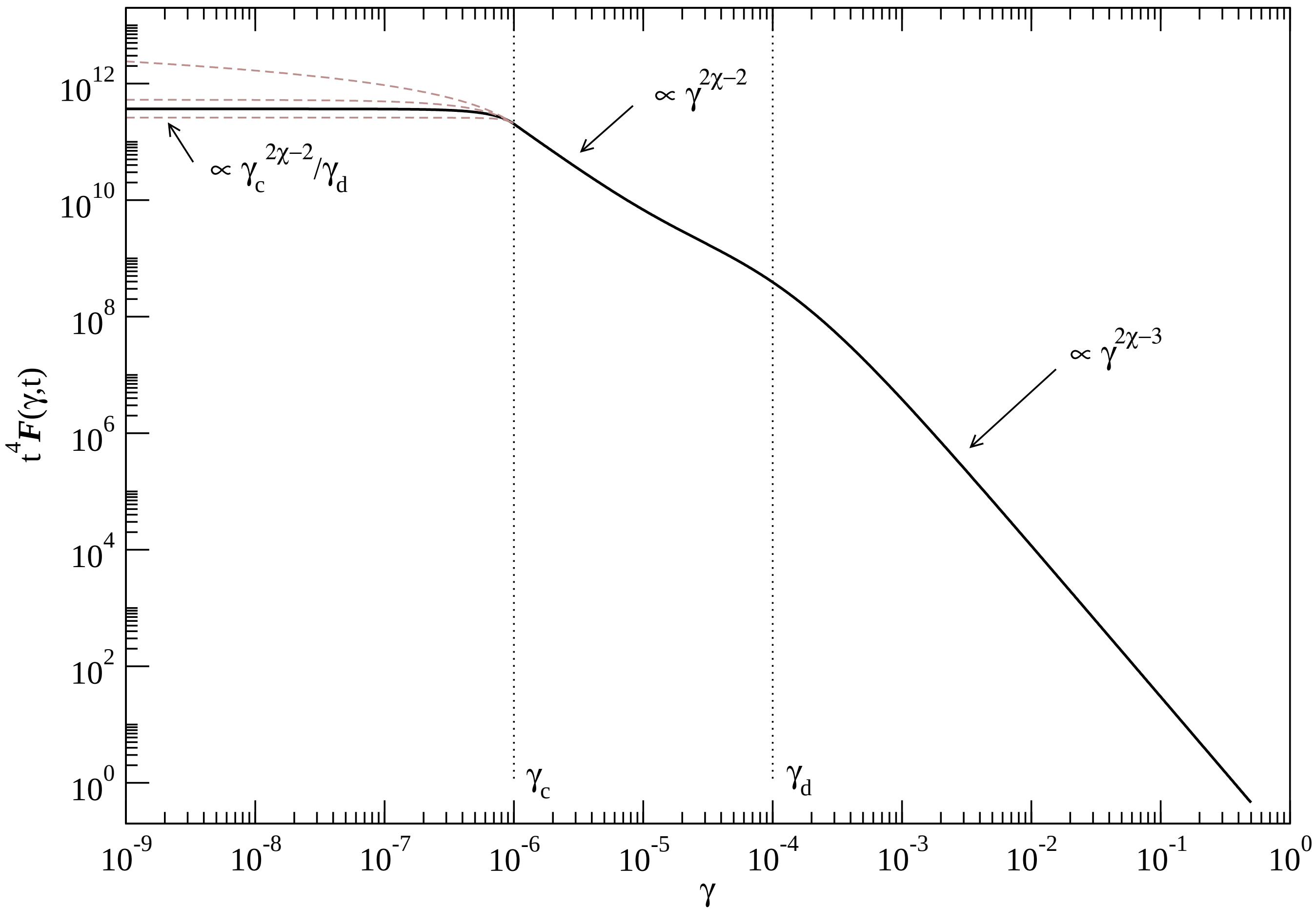
$$t^4\mathcal{F}(\gamma_\mathcal{T} \leq \gamma < \gamma_c, t) = \left(\frac{t}{t_{\rm ini}}\right)^4 \left(\frac{a_{\rm ini}}{a}\right)^3 t_{\rm ini}^4\, \mathcal{N}_{\rm ini}\left\{\left[\gamma + \gamma_{\rm d}\left(1 - \frac{t_{\rm ini}}{t}\right)\right] t\right\} + C_{\rm c}(\gamma+\gamma_{\rm d})^{2\chi_{\rm c}-3} f_{\rm c}\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\right)$$
$$- C(\gamma+\gamma_{\rm d})^{2\chi-3}\left(\frac{t}{t_{\rm ini}}\right)^{2\chi+1}\left(\frac{a_{\rm ini}}{a}\right)^3 f\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\frac{t_{\rm ini}}{t}\right)$$
$$+ K\left(\frac{\gamma_{\rm c}+\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\right)^4 \left[\frac{a\left(\dfrac{\gamma+\gamma_{\rm d}}{\gamma_{\rm c}+\gamma_{\rm d}}\,t\right)}{a(t)}\right]^3,$$

$$t^4\mathcal{F}(0 < \gamma < \gamma_\mathcal{T}, t) = \left(\frac{t}{t_{\rm ini}}\right)^4 \left(\frac{a_{\rm ini}}{a}\right)^3 t_{\rm ini}^4\, \mathcal{N}_{\rm ini}\left\{\left[\gamma + \gamma_{\rm d}\left(1 - \frac{t_{\rm ini}}{t}\right)\right] t\right\} + C_{\rm c}(\gamma+\gamma_{\rm d})^{2\chi_{\rm c}-3} f_{\rm c}\left(\frac{\gamma_{\rm d}}{\gamma+\gamma_{\rm d}}\right)$$

$$\gamma_\mathcal{T}(t) \equiv (\gamma_{\rm c}+\gamma_{\rm d})\frac{t_{\rm ini}}{t} - \gamma_{\rm d}, \qquad \mu \equiv 3\nu - 2\chi - 1$$
$$f(x) \equiv {}_2{\rm F}_1\left(3-2\chi, \mu; \mu+1; x\right) \qquad f_{\rm c}(x) \equiv {}_2{\rm F}_1\left(3-2\chi_{\rm c}, \mu_{\rm c}; \mu_{\rm c}+1; x\right)$$

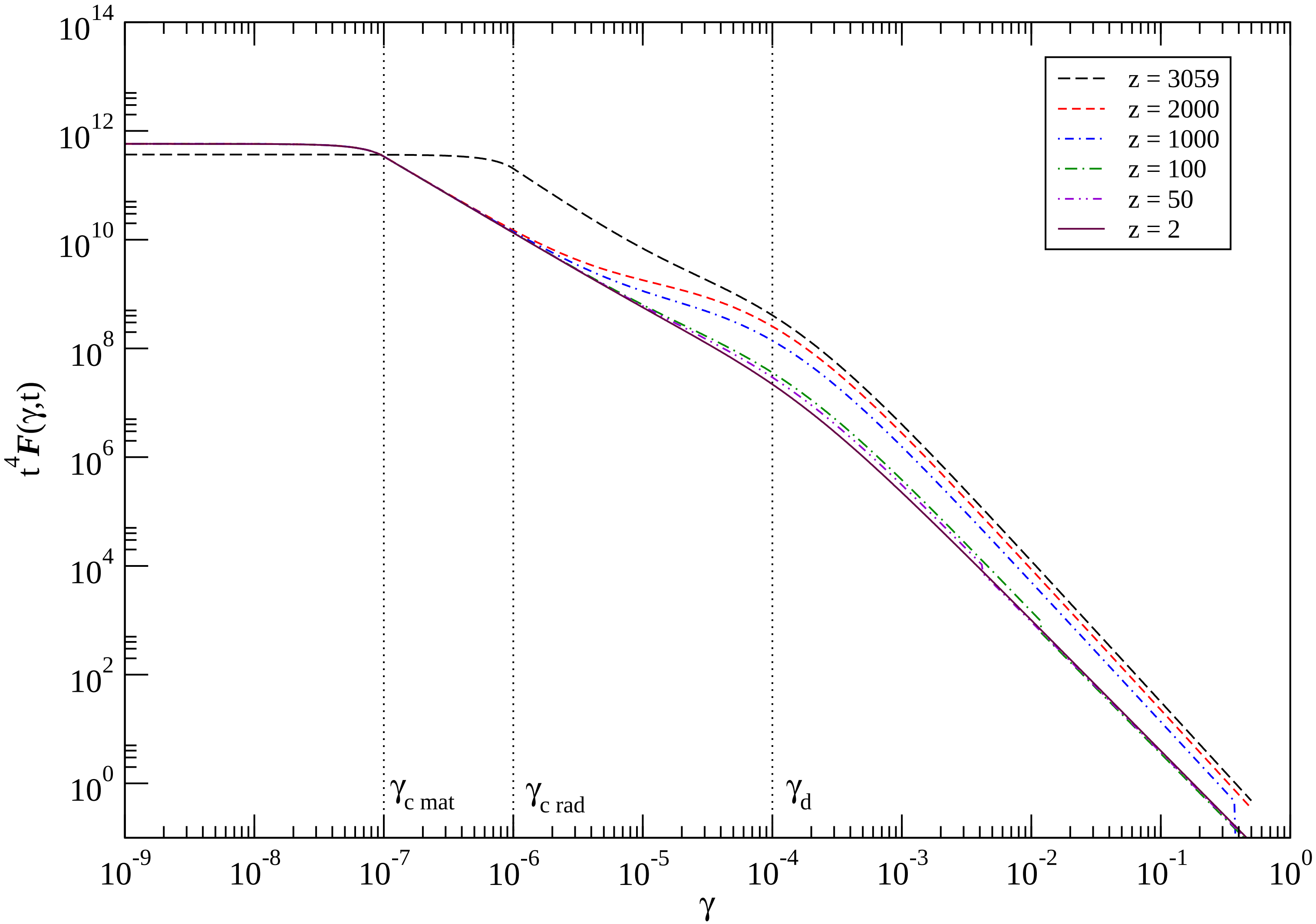
Cosmological loop distribution

- Can be completely solved analytically (see arXiv.1006.0931)
- Scaling attractor does not depend on $\mathcal{N}_{\text{ini}}$ nor on GW backreaction details



Relaxation effects are accounted

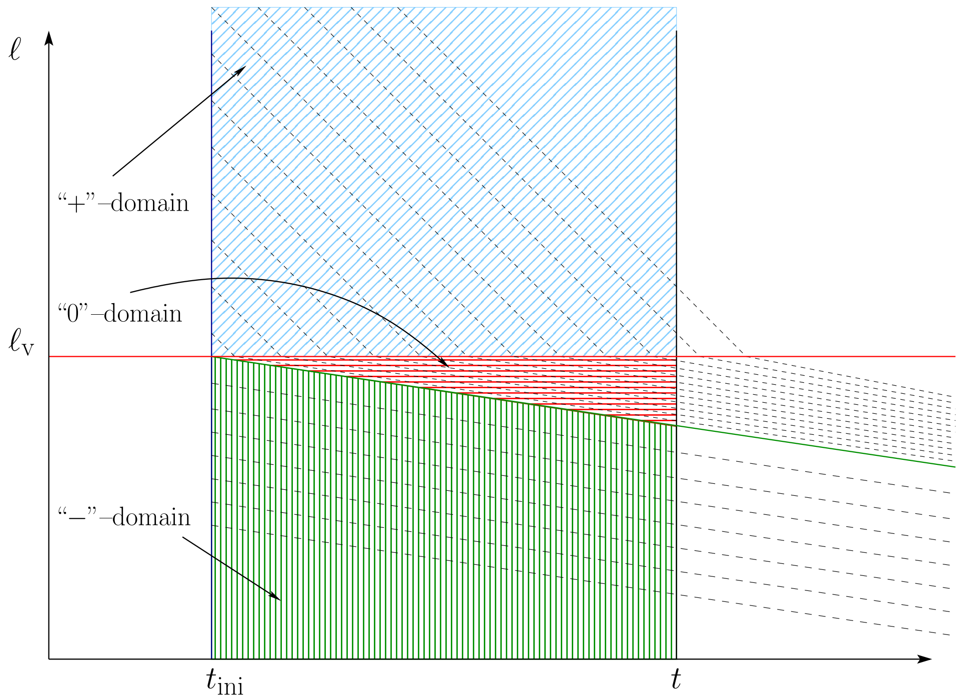
- Example: transition radiation–matter

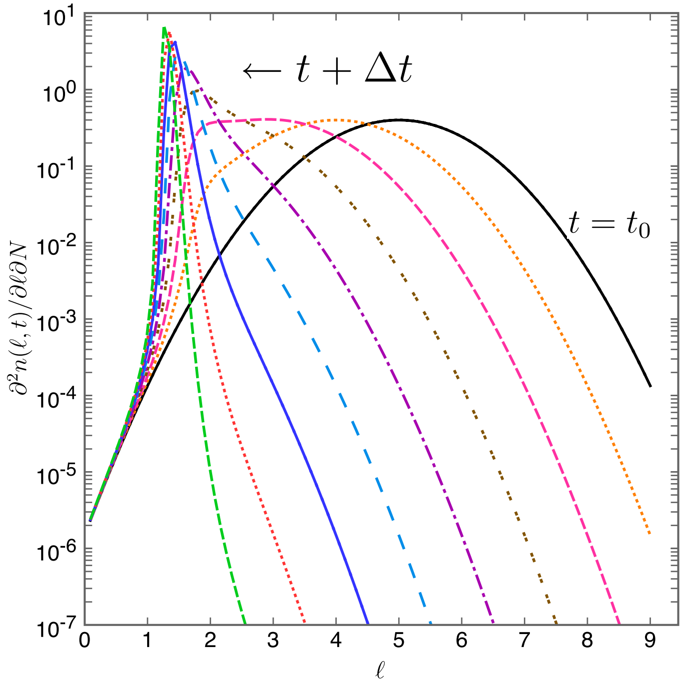


Extension to vortons

- Boltzmann equation for current carrying loops: $n(\ell, t, N)$

$$\frac{\partial}{\partial t}\left[a^3\mathcal{J}(\ell,t)\frac{\partial^2 n}{\partial\ell\partial N}\right] - \left[\gamma_{\rm d}\Theta\left(\ell - \frac{N}{\sqrt{U}}\right) + \gamma_{\rm v}\Theta\left(\frac{N}{\sqrt{U}} - \ell\right)\right]\frac{\partial}{\partial \ell}\left[a^3\mathcal{J}(\ell,t)\frac{\partial^2 n}{\partial\ell\partial N}\right]$$
$$= a^3\mathcal{J}(\ell,t)\mathcal{P}(\ell,t)\delta\left(N - \sqrt{\frac{\ell}{\lambda}}\right)$$





- Again exactly solvable for any $\mathcal{N}_{\rm ini}(\ell)$ (see arXiv:1302.0953)

Loops and CMB trispectrum

- CMB trispectrum from strings is sensitive[2] to $\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma')\right\rangle$

$$T_\infty(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \simeq \epsilon^4 \frac{\bar{v}^4}{\bar{t}^2}\frac{L\hat{\xi}}{\mathcal{A}}\left(c_1\hat{\xi}^2\right)^{-1/(2\chi+2)} f(\chi) g(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$

$$f(\chi) = \frac{\pi}{\chi+1}\Gamma\left(\frac{1}{2\chi+2}\right)\left[4(2\chi+1)(\chi+1)\right]^{1/(2\chi+2)}$$

- Geometrical factor scales as k^ρ: $\rho = 6 + 1/(1+\chi)$

$$g(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) = \frac{\kappa_{12}\kappa_{34}+\kappa_{13}\kappa_{24}+\kappa_{14}\kappa_{23}}{k_1^2k_2^2k_3^2k_4^2}\left[Y^2\right]^{-1/(2\chi+2)}$$

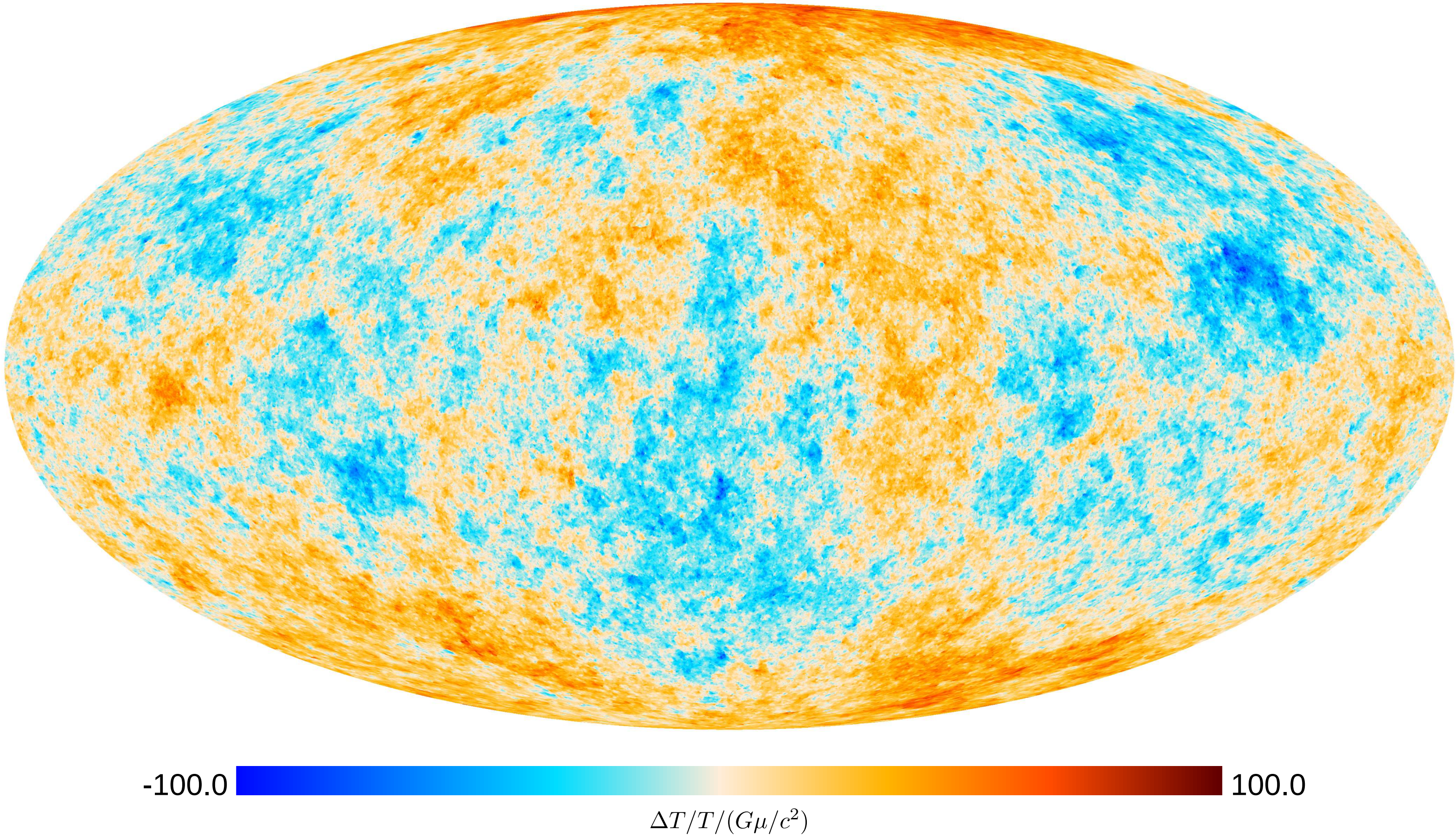
$$Y^2(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \equiv -\kappa_{12}\left(k_3^2k_4^2-\kappa_{34}^2\right)^{\chi+1} + \circlearrowleft,$$

- This is a consistency relation for loops production mechanism

[2] arXiv:0911.1241, arXiv:0908.0432

Full sky string CMB map

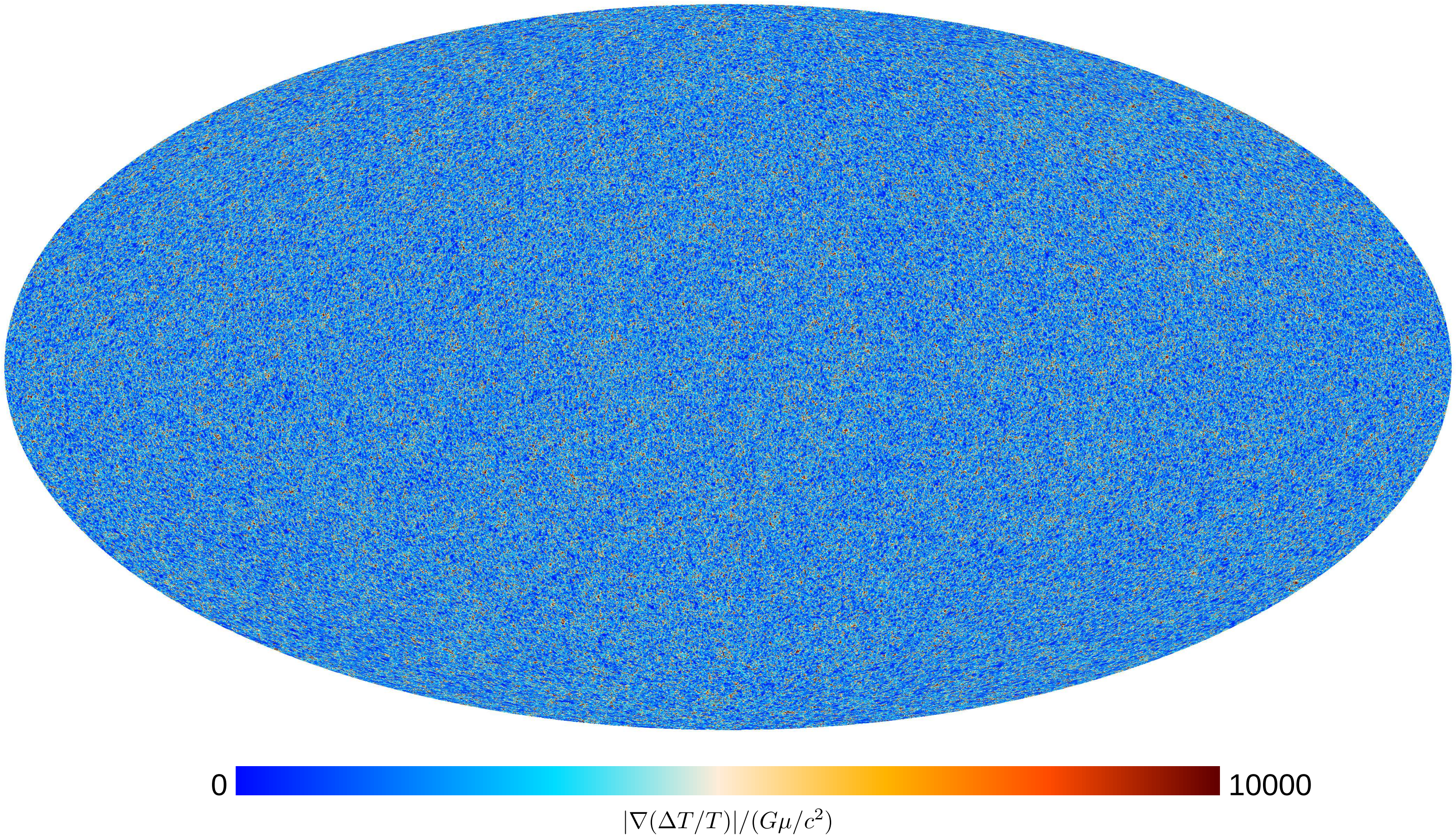
- From arXiv:1204.5041: Temperature anisotropies



- All sky bispectrum computed in Planck XXV (arXiv:1303.5085)
- Trispectrum?

Full sky string CMB map

- From arXiv:1204.5041: Gradient magnitude



- All sky bispectrum computed in Planck XXV (arXiv:1303.5085)
- Trispectrum?

Conclusion

- Nambu–Goto loops in scaling: one length scale $d_\mathrm{h} \propto t$
  - Have a scale-free distribution: $t^4 \dfrac{\mathrm{d}n}{\mathrm{d}\ell}(\gamma, t) \propto \left(\dfrac{\ell}{t}\right)^{2\chi-3}$
  - Without dissipation (GW) $\exists$ non-scaling structures which cascade down to smallest lengths + memory of IC (ℓ_c, ℓ_r)
  - With dissipation (GW), non-scaling structures are radiated away and the loop distribution reaches scaling on all length scales with universal power law exponents uniquely determined by χ

$$\Omega_\circ = \frac{3\pi^2 C}{(1-\chi)\sin(2\pi\chi)} \frac{GU}{\gamma_\mathrm{d}^{1-2\chi}} \simeq 0.10 \times (GU)^{0.59}$$

$$t^3 n_L = \frac{2\chi_\mathrm{c} - 2\chi}{2\chi_\mathrm{c} - 1} \frac{C}{\gamma_\mathrm{d}\gamma_\mathrm{c}^{1-2\chi}} \simeq 6.1 \times 10^{-5} (GU)^{-1.65}$$

- Non-scaling loops
  - Peak at ℓ_r and are loop radiation! (link with String Theory?)
  - Does $\lim_{\ell_\mathrm{r}\to 0} \mathrm{NG} \simeq \mathrm{AH}$? ($n_{H_0} \simeq 5.5 \times 10^{-6}\,\mathrm{Mpc}^{-3}$ is small)