THE NUMBER, SIZE, AND SPEED OF LOOPS

based on Phys. Rev. D 89, 023512 (2014)
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February 3rd
Cosmic Strings 2014

OUTLINE

- What we know and don't know about loop production. $xf(x,p)\mathrm{d}x\mathrm{d}p$ vs. $\alpha f(\alpha,p)\mathrm{d}\alpha\mathrm{d}p$
- The Boltzmann equation: getting n(t,m,p) from f.
- Precision cosmology and agreement with other simulations, e.g., Ringeval, Sakellariadou, Bouchet.
- Conclusion and a bound on $G\mu$.

DEFINITIONS

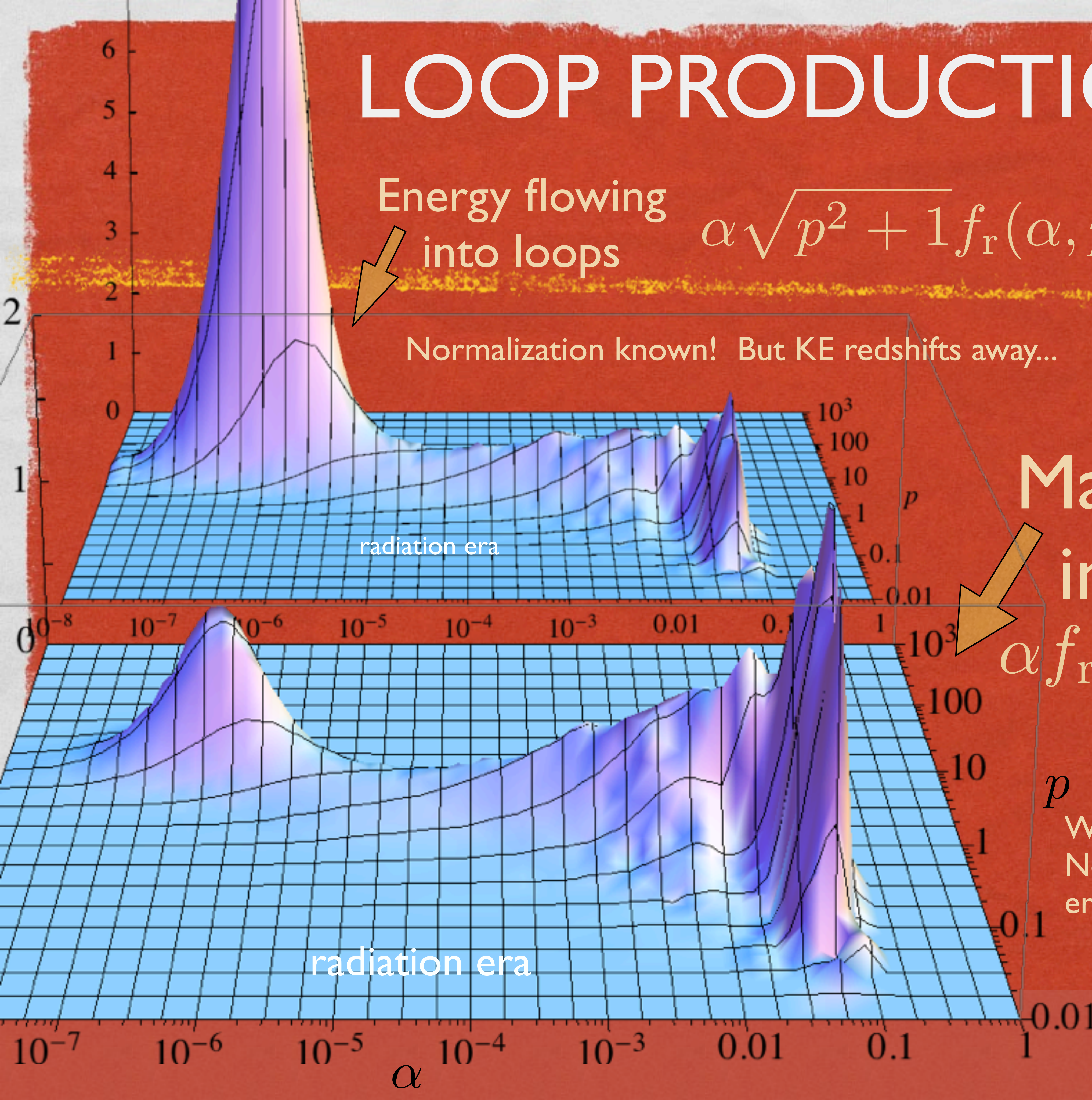
- Scaling: network properties become time-independent when expressed in units of the horizon distance, $d_{\mathrm{h}} = 2t$.
- f is the rate of loop production, $n \sim \int \mathrm{d}t \; f$ is the number of existing loops
- $x = l/d_{\mathrm{h}}$ is the scaling length (energy) of a loop.
- $\alpha = m/\mu d_{\mathrm{h}} = x\sqrt{1-v^2}$ is the scaling mass of a loop.
- $p = v/\sqrt{1-v^2}$ is the momentum (per mass) of a loop.

LOOP PRODUCTION

Energy flowing into loops $\alpha\sqrt{p^2+1}\,f_{\mathrm{r}}(\alpha,p)\mathrm{d}\alpha\mathrm{d}p$

Normalization known! But KE redshifts away...

Mass flowing into loops $\alpha f_{\mathrm{r}}(\alpha,p)\mathrm{d}\alpha\mathrm{d}p$



Which one determines n? Neither. (for radiation era.)

CALCULATING $n(t, m, p)\mathrm{d}m\mathrm{d}p$ FROM $f(t, m, p)\mathrm{d}t\mathrm{d}m\mathrm{d}p$

Without cosmology, the Boltzmann equation is simply

$$n(t, m, p)\mathrm{d}m\mathrm{d}p = \left[\int \mathrm{d}t\, f(t, m, p)\right] \mathrm{d}m\mathrm{d}p$$

We need a cosmology to determine

- dilution → (or use comoving volume)
- redshifting
- evaporation.

how does a loop's p and m flow with time?

CALCULATING $n(t,m,p)\mathrm{d}m\mathrm{d}p$
FROM $f(t,m,p)\mathrm{d}t\mathrm{d}m\mathrm{d}p$

the flow:

$$\boxed{\frac{\mathrm{d}M}{\mathrm{d}t'} = -\Gamma G\mu^2/\sqrt{P^2+1} \qquad \frac{\mathrm{d}P}{\mathrm{d}t'} = -H(t')P}$$

boundary condition

$$M(t';t,m) = m + \Gamma G\mu^2(t-t')$$

(Becomes hypergeometric if we include time-dilation, and matter + radiation cosmology.)

$$P(t';t,p) = p\frac{a(t)}{a(t')}$$

neglecting rocket effect, until we know shapes of loops (see talk by Jose)

The Boltzmann equation:

$$n(t,m,p) = \int_0^t \mathrm{d}t'\, f\left(t', M(t'), P(t')\right) \frac{\partial M}{\partial m}\frac{\partial P}{\partial p}$$

loops per *comoving* volume

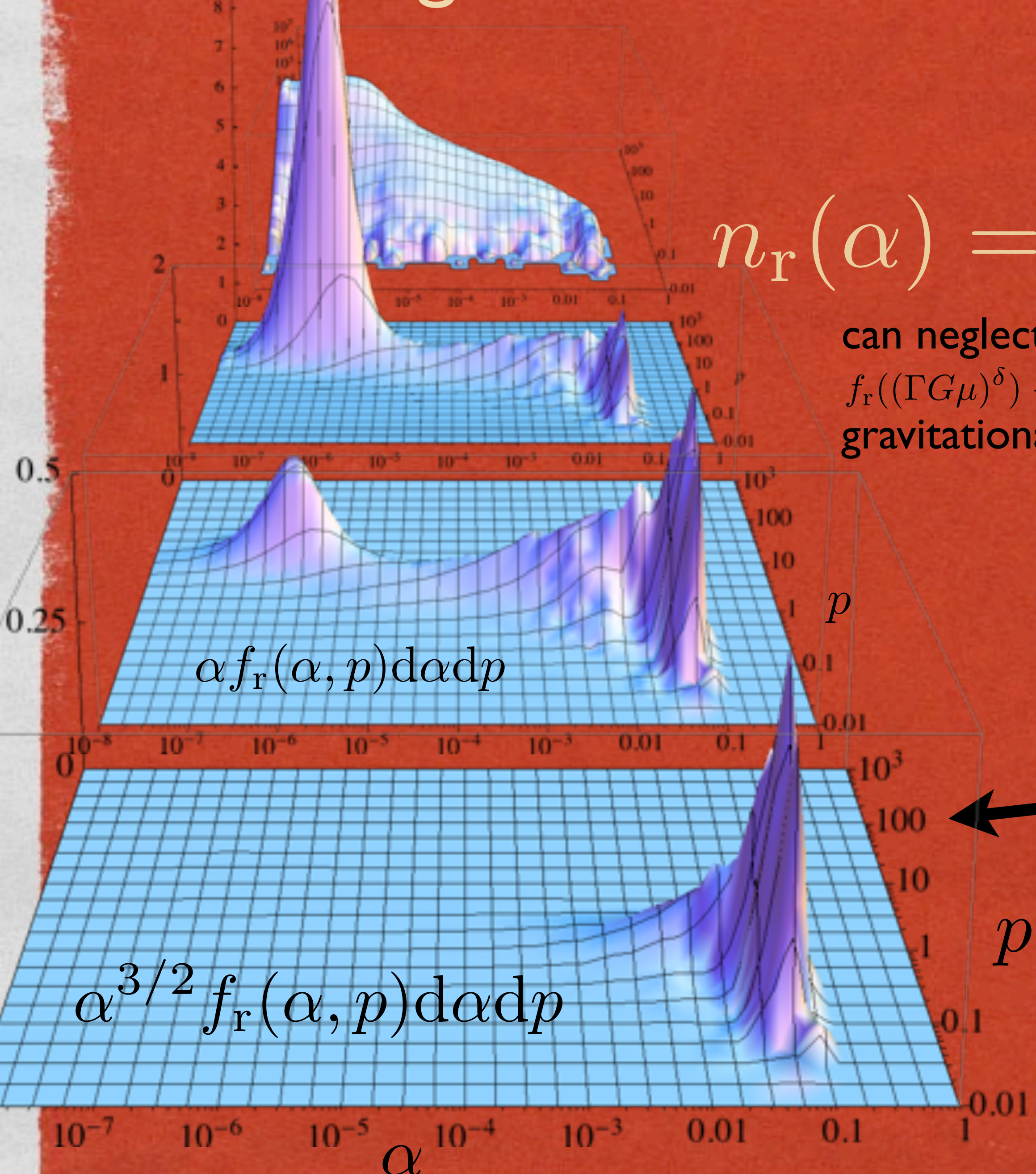
Jacobian determinant

CALCULATING $n(t, m, p)\mathrm{d}m\mathrm{d}p$
FROM $f(t, m, p)\mathrm{d}t\mathrm{d}m\mathrm{d}p$

In scaling coordinates, during the radiation era, this becomes:

$$n_{\mathrm{r}}(\alpha) = \frac{\int_{\alpha}^{\infty} (\alpha' + \Gamma G\mu/2)^{3/2} f_{\mathrm{r}}(\alpha')\mathrm{d}\alpha'}{2\,(\alpha + \Gamma G\mu/2)^{5/2}}$$

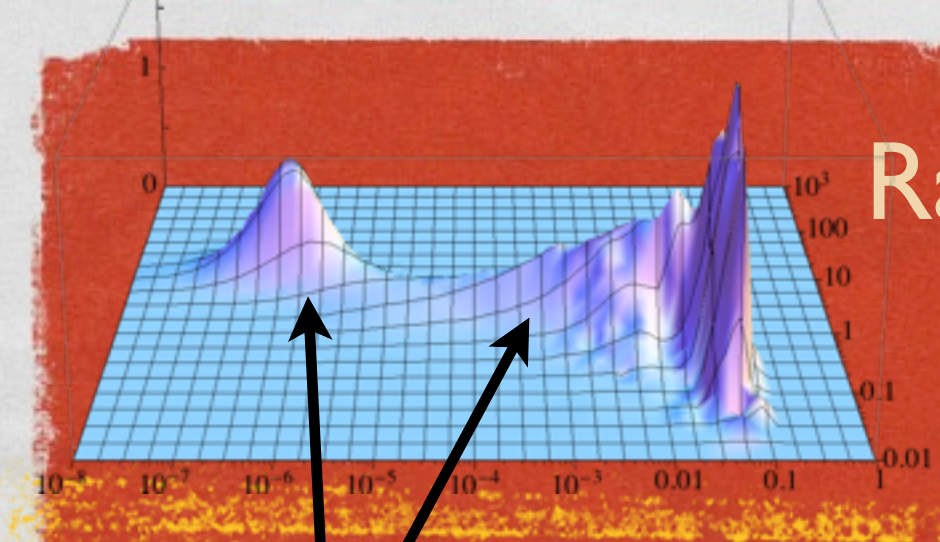
can neglect this *if* $f_{\mathrm{r}}((\Gamma G\mu)^{\delta}) \to 0$ due to gravitational backreaction.



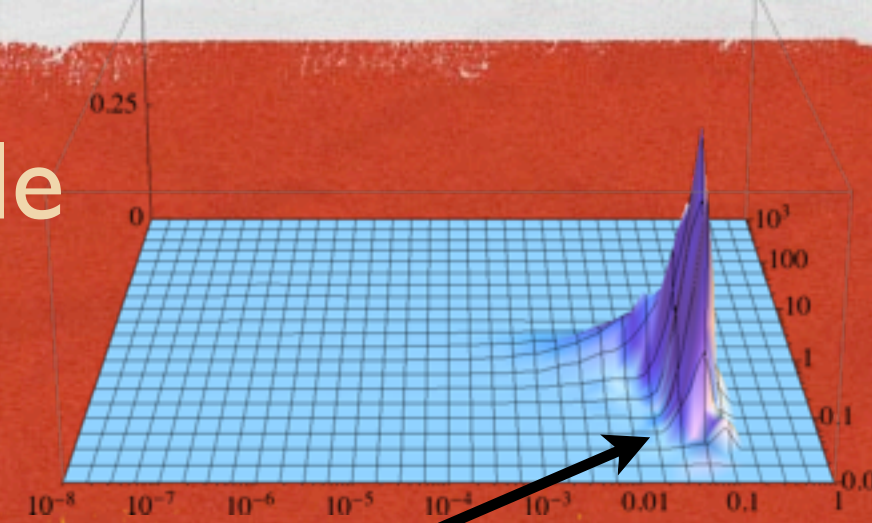
Note extra half-power of α'.

This distribution determines $n_{\mathrm{r}}(\alpha, p)\mathrm{d}\alpha\mathrm{d}p$

(not such a large hierarchy of scales needed during radiation era simulation.)



Radiation era puzzle



If equal amounts of long string are dumped into loops of two different sizes, why do the larger loops contribute much more to $n(\alpha_0)$?

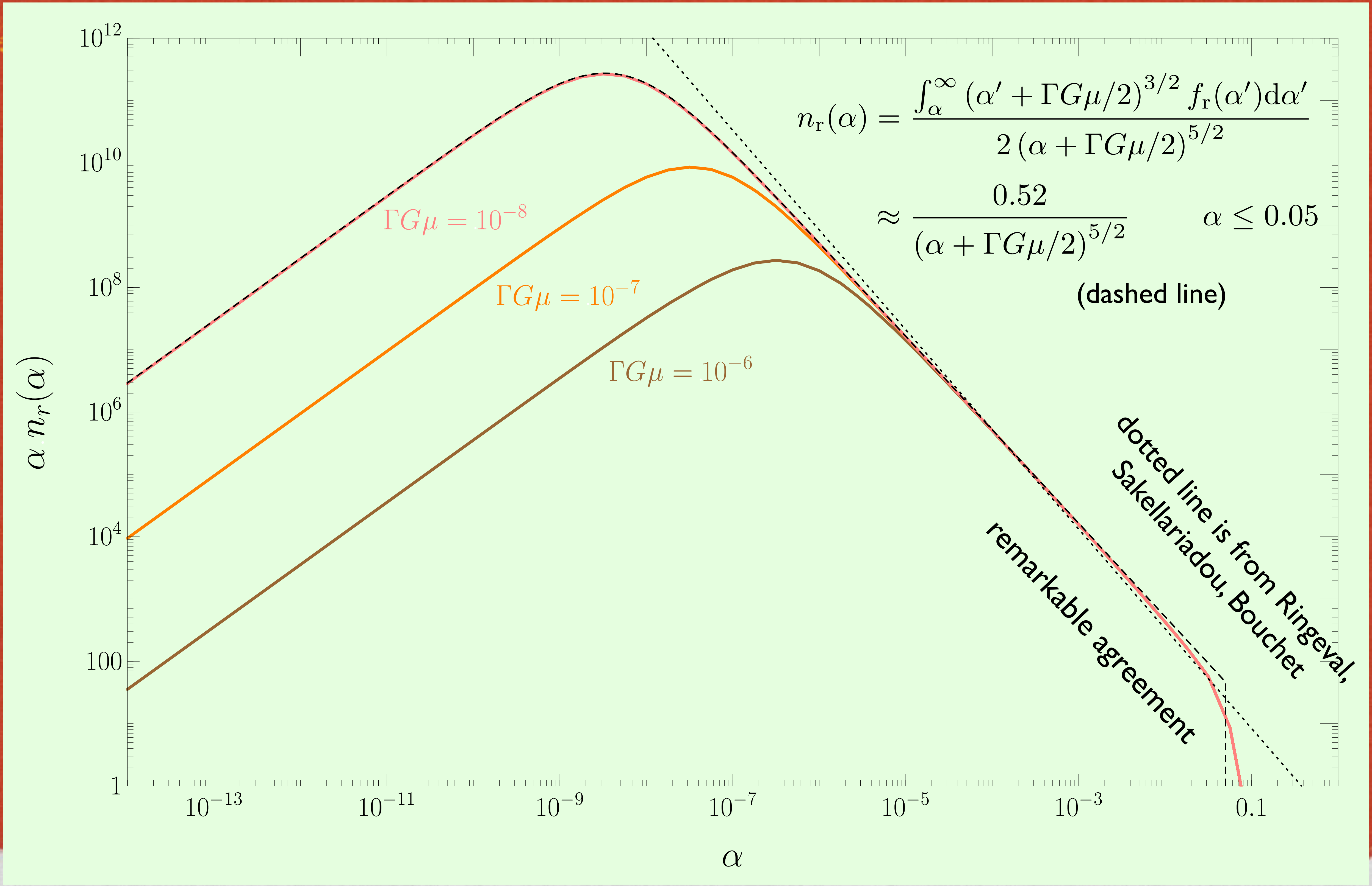
Because these loops are older by the time they contribute to a point in $n(\alpha_0)$, and older loops are from a time when the network was <u>much</u> more dense.

The network loses energy density to loops like 1/a^4, but loops are only diluted like 1/a^3. An existing loop with some fixed mass is more likely to have been produced a long time ago at the (then) horizon scale than recently as a small loop.

This is why loops dominate the string energy density during the radiation era.

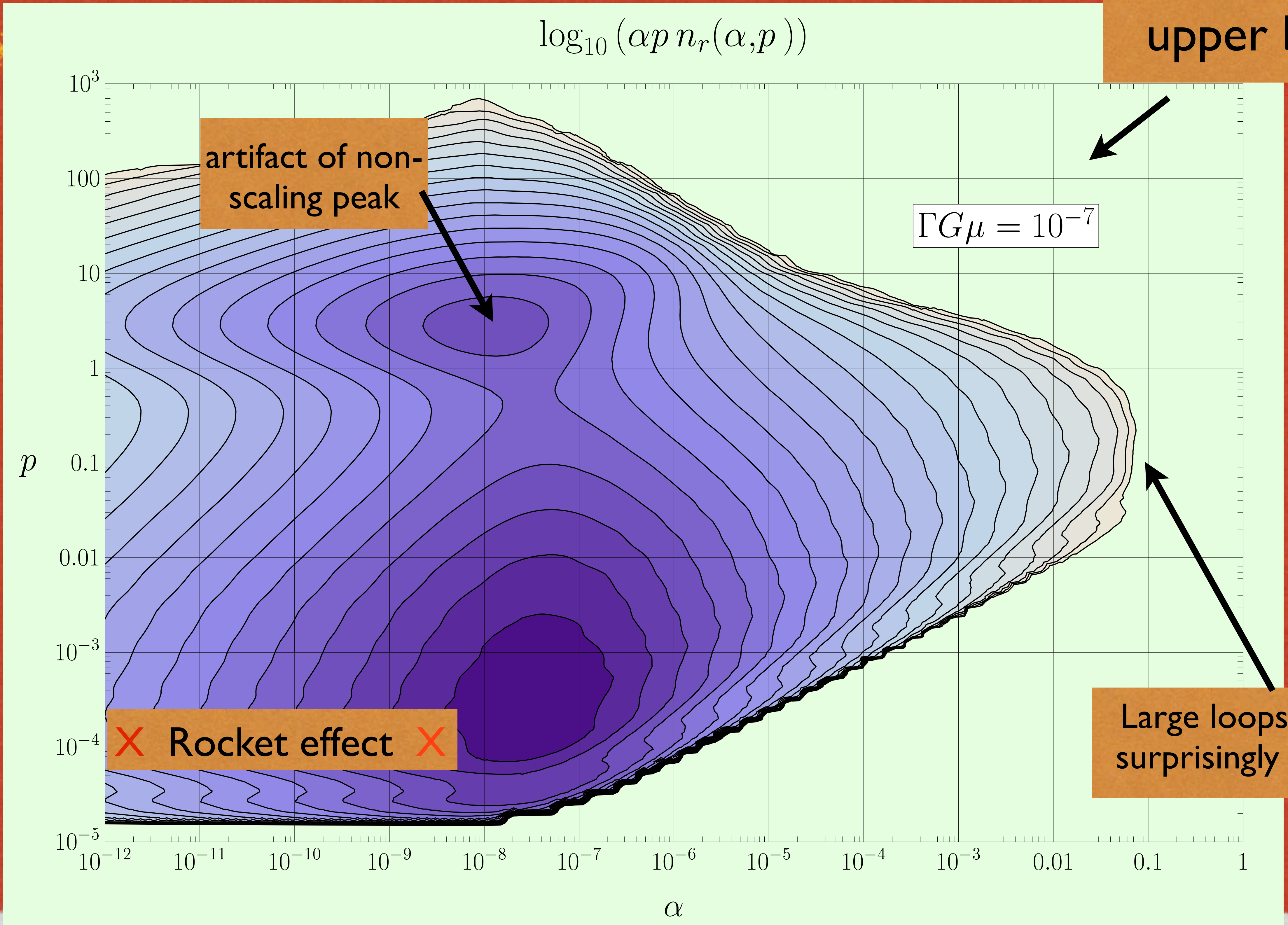
$$\frac{\rho_{\rm r}^{\rm loops}}{\rho_{\rm r}^{\infty}} \approx 100\sqrt{\left(\frac{50}{\Gamma}\right)\left(\frac{10^{-9}}{G\mu}\right)}$$

Results: $n_{\mathrm{r}}(\alpha)\mathrm{d}\alpha$



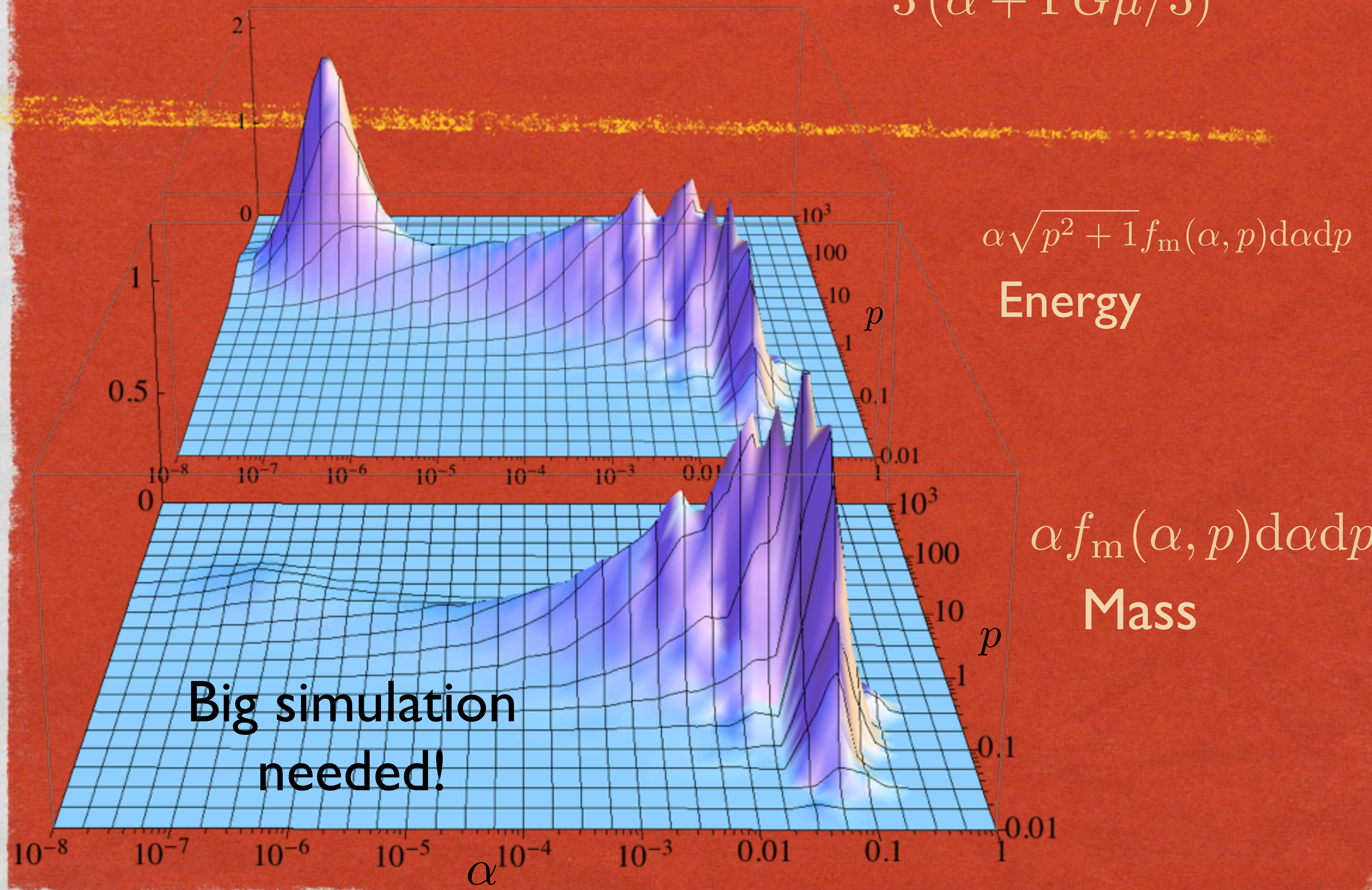
Results:

$$n_{\mathrm{r}}(\alpha, p)\mathrm{d}\alpha\mathrm{d}p$$

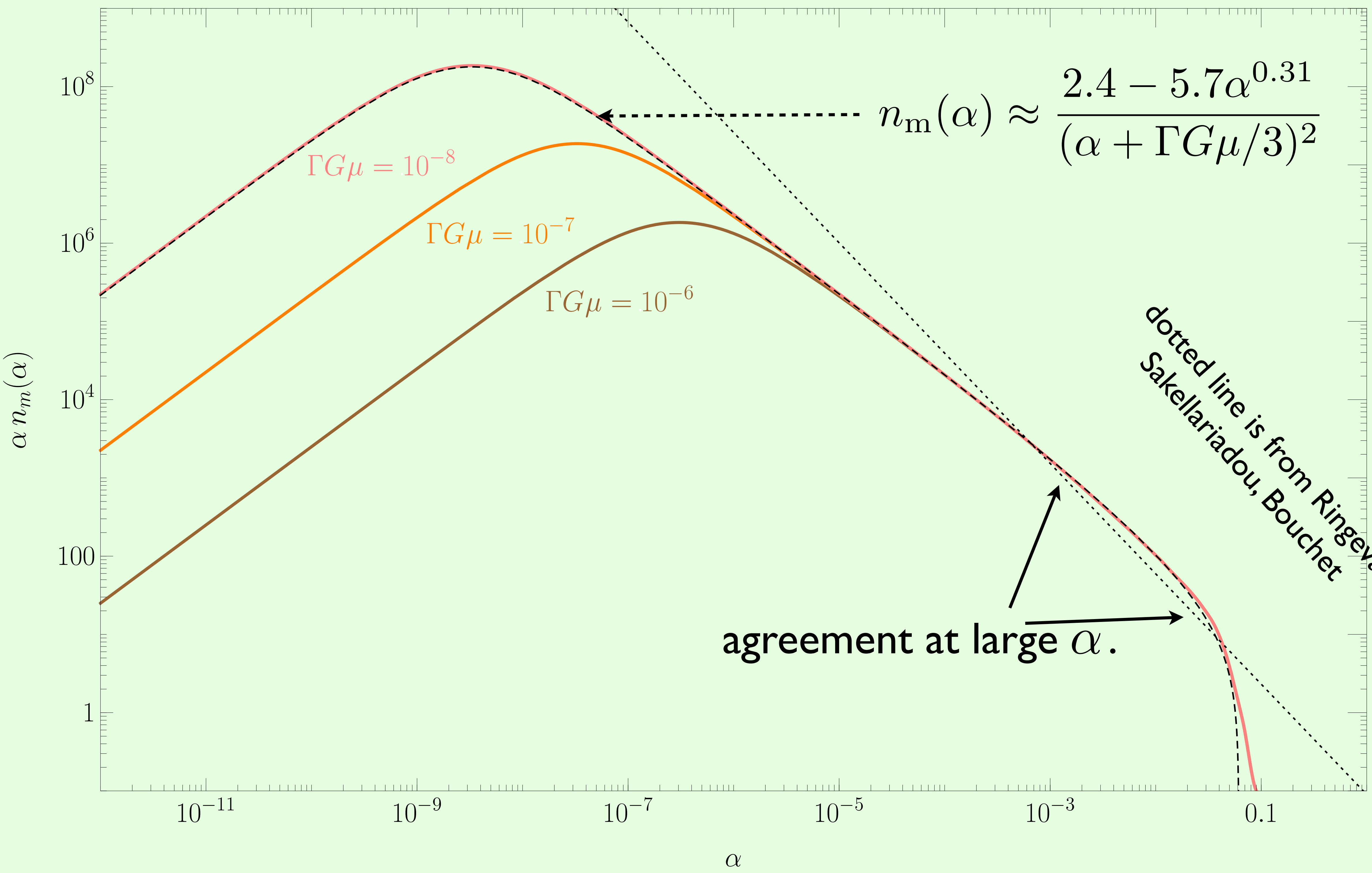


MATTER ERA

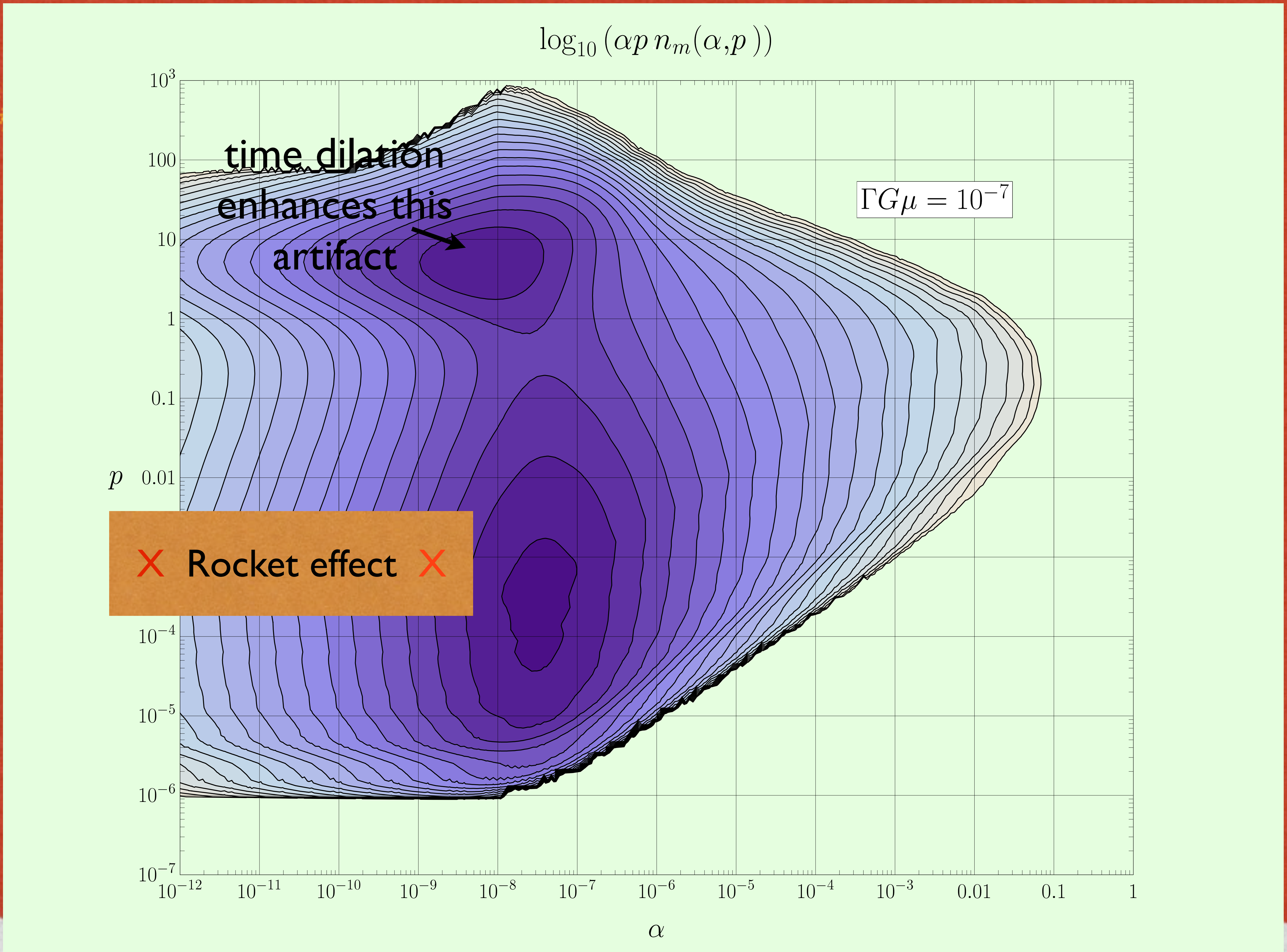
$$n_{\rm m}(\alpha) = \frac{\int_\alpha^{\alpha_{\rm eq}} \left(\alpha' + \Gamma G\mu/3\right) f_{\rm m}(\alpha')\mathrm{d}\alpha'}{3\left(\alpha + \Gamma G\mu/3\right)^2}$$



Results: $n_{\mathrm{m}}(\alpha)\mathrm{d}\alpha$



Results: $n_{\mathrm{m}}(\alpha, p)\mathrm{d}\alpha\mathrm{d}p$



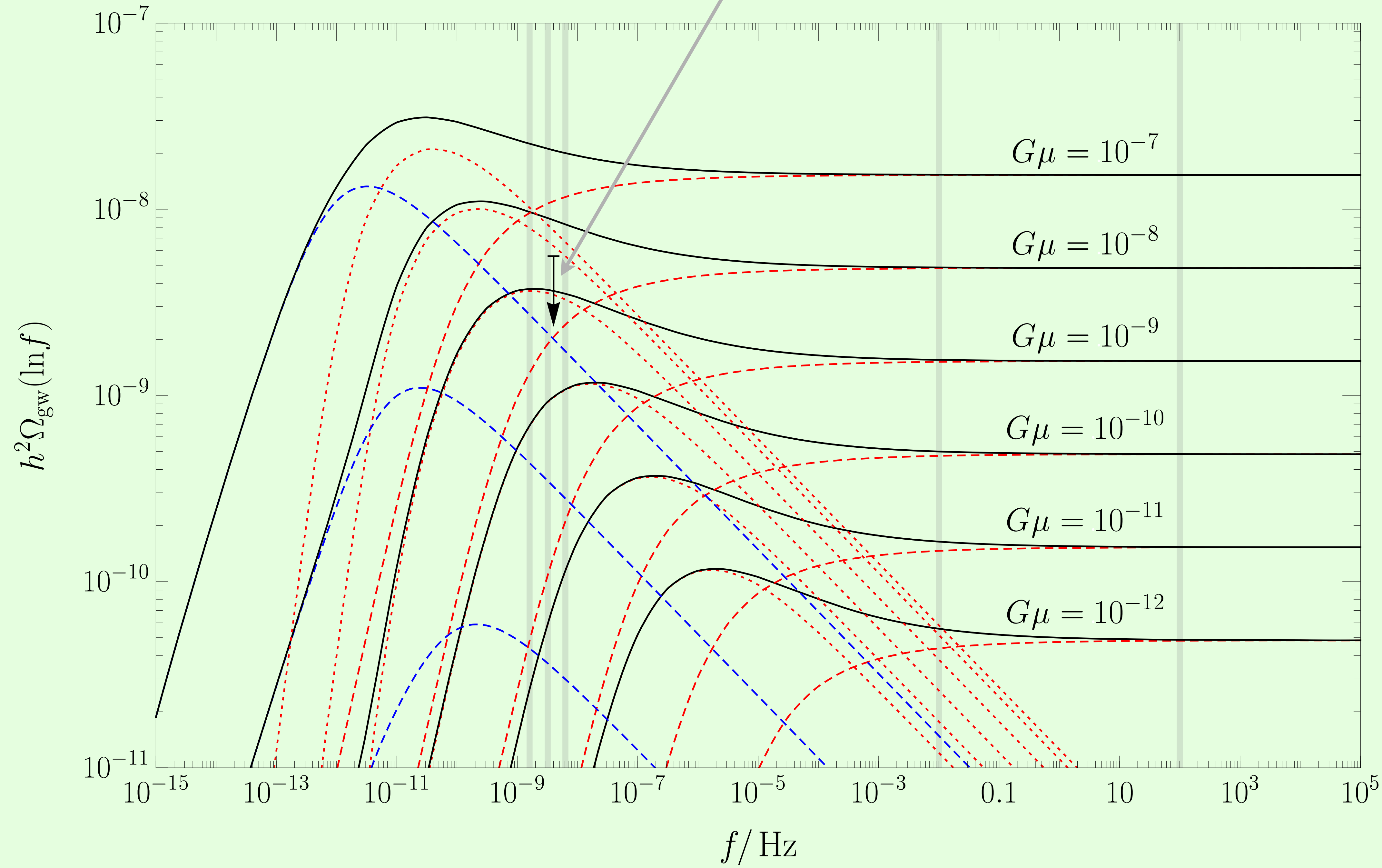
KNOWING LOOP SIZE IS NOT ENOUGH.

Note that energy conservation, and knowledge of the typical loop size $\alpha \sim 0.05$ is insufficient for determining the normalization for $n_{\mathrm{r}}(\alpha)$ or $n_{\mathrm{m}}(\alpha)$. During the radiation era, only about 10% of power flows into the loops that count. This is less of an issue during the matter era.

STOCHASTIC GW BOUND FROM PTA

$\Rightarrow G\mu \leq 2.8 \times 10^{-9}$

Van Haasteren *et al.*
Sanidas, Battye, & Stappers



SUMMARY

- Precision: we know how many loops there are to within a few percent. Loops are very important! $\rho_{\rm r}^{\rm loops}/\rho_{\rm r}^{\infty} \gtrsim 100$
- Consensus: our results are consistent with recent Nambu-Goto simulations, including Ringeval, Sakellariadou, & Bouchet.
- Large loops are somewhat fast at production: $v \sim 0.1$
- PTA limits on stochastic GW background give $G\mu \leq 2.8 \times 10^{-9}$