Highly Excited Strings in String Perturbation Theory

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Motivation

Highly excited superstrings (HES) form a benchmark of string theory.

They are related to the UV finiteness, play key role in regions of strong gravity (e.g. singularities[1], early universe[2], black holes[3]), provide a source of non-locality (desirable[4] e.g. in resolving information paradox), and their properties may even lead to string theory signatures, e.g. in context of COSMIC SUPERSTRINGS[2]).

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[1] Horowitz, Steif (1997); ...

[2] Sen (1998); Dvali & Vilenkin (2004); Copeland, Myers, Polchinski (2004); Hindmarsh (2011); DS, Copeland, Saffin (2013); ...

[3] Amati, Ciafaloni, Veneziano (1988); D'Appollonio, Di Vecchia, Russo Veneziano (2013);...

[4] Susskind (1995); Low, Polchinski, Susskind, ..., (1997); Giddings (2007); Hartman, Maldacena (2013);...

Cosmic String Zoo

Compactifications of string theory lead to many potential cosmic string candidates (see e.g. Tye's talk):[5]

- F-strings
- D-strings
- (p, q)-strings
- wrapped D-branes
- solitonic strings
- electric and magnetic flux tubes

$\vdots$

[5] Dvali, Vilenkin (2004); Copeland, Myers, Polchinski (2004); Polchinski (2006); Banks, Seiberg (2011)

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[6] Sarangi, Tye (2002); Dvali, Vilenkin (2004); Copeland, Myers, Polchinski (2004); Polchinski (2006); Banks, Seiberg (2011)

Effective Theory Approach

Traditionally one discusses CS in terms of effective theory (EFT).

Although these EFTs often follow from defining string theory equations,[7] the assumptions adopted to derive[8] them also highlight their weaknesses and limited applicability.

E.g., they do not capture truly stringy features, such as:

- couplings to infinite set of oscillator states (e.g. relevant for cusp emission)
- inherently QM processes (e.g. string inter-commutations)
- break down at small scales (e.g. relevant for loop production)

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[7] $\sum_h \int \mathcal{D}(Xg) e^{iS_{\text{Polyakov}}} \simeq \int \mathcal{D}(GB\Phi \dots) e^{iS_{\text{eff}}}$

[8] Tseytlin (1991)

HES in String Theory

Going beyond EFTs . . .

In perturbative string theory one speaks in terms of vertex operators, D-branes, and scattering amplitudes.

New efficient tools appropriate for HES now available, making computations with HES tractable and efficient![9]

$\Rightarrow$ Trick is to consider strings in a coherent state basis where calculations become efficient and tractable[10] ..

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[9] Skliros, Hindmarsh (2011); Hindmarsh, Skliros (2011)

[10] Skliros, Copeland, Saffin (2013)

Coherent Vertex Operators

Definition of closed string coherent state:

(a) is specified by a (possibly infinite) *set of continuous* labels $(\lambda, \bar{\lambda})$, which may be associated to the left- and right-moving modes;

(b) produces a resolution of unity,

$$\mathbb{1} = \sum\!\!\!\!\!\!\int_{\dots} d\lambda d\bar{\lambda} |\lambda, \bar{\lambda}; \dots\rangle\langle\lambda, \bar{\lambda}; \dots|,$$

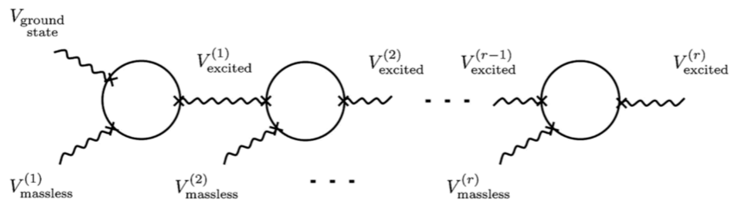
so that the $|\lambda, \bar{\lambda}; \dots\rangle$ span the string Hilbert space. The dots "$\dots$" denote possible additional quantum numbers;

(c) transforms correctly under all symmetries of the string theory

Coherent Vertex Operators

To construct covariant coherent vertex operators (CVO):

1. excite string ground state, $V_{\mathrm{ground\,state}}$, with massless states, V_{massless}, of momenta $k = n_i q$, with $q^2 = 0$, $n_i \in \mathbb{Z}^+$
2. sum over resulting excited states, $V^{(r)}_{\mathrm{excited}}$, ie sum over r



With appropriate combinatorial coefficients, c_r, in sum, resulting state $V = \sum_r c_r V^{(r)}_{\mathrm{excited}}$ satisfies above definition of CVOs

Classical-Quantum Map

Given any classical solution, there is a one-to-one map to the corresponding quantum coherent vertex operators[11]

For example,[12]

$$X(z,\bar{z}) = \frac{i}{n}\left(\lambda_n\, z^{-n} - \lambda_n^*\, z^n\right) + \frac{i}{m}\left(\bar{\lambda}_m\, \bar{z}^{-m} - \bar{\lambda}_m^*\, \bar{z}^m\right),$$

corresponds to CVO,[13]

$$V(z,\bar{z}) = :C\int_0^{2\pi} ds\, \exp\left(\frac{i}{n}\, e^{ins}\lambda_n \cdot D_z^n X\, e^{-inq\cdot X(z)}\right)$$
$$\times \exp\left(\frac{i}{m}\, e^{-ims}\bar{\lambda}_m \cdot \bar{D}_{\bar{z}}^m X\, e^{-imq\cdot X(\bar{z})}\right) e^{ip\cdot X(z,\bar{z})}:$$

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[11] Hindmarsh & Skliros PRL (2011)

[12] X spacetime embedding; $z, \bar{z}$ worldsheet coordinates; p momenta; $\lambda_n, \bar{\lambda}_n$ polarisation tensors; n harmonics

[13] Skliros & Hindmarsh (2011)

HES Decay and Radiaton

We here compute decay rates and power associated to massless emission for special class of HES states in IR (on $\mathbb{R}^{D-1,1} \times T^{26-D}$)

Some History First

A handful of references on decay rates of HES:

- Wilkinson, Turok, Mitchell (1990): leading Regge (bosonic) states, $\mathbb{R}^{25,1}$, (numerical), $\Gamma_{d=4} \propto L$ and $\Gamma_{d=26} \propto L^{-1}$
- Dabholkar, Mandal, Ramadevi (1998): higher genus bound on leading Regge Heterotic states, $\mathbb{R}^{3,1} \times T^6$, $\Gamma \lesssim M^{-1}$
- Iengo, Russo (2002-6); Chialva, Iengo, Russo (2004-5): leading Regge superstring states, $\mathbb{R}^{D-1,1} \times T^{10-D}$, (numerical),

$$\Gamma \sim G_D \mu^2 L^{5-D}, \qquad \mu = 1/2\pi\alpha'$$

- Gutplerle & Krym (2006); leading Regge Heterotic states, $\mathbb{R}^{8,1} \times S^1$, (numerical)

$\vdots$

Simplest Example

When only first harmonics are present,

$$V = \int_0^{2\pi} \text{đ}s \exp\left(e^{is} i\zeta \cdot \partial_z X\, e^{-iq\cdot X}\right) \exp\left(e^{-is} i\bar{\zeta} \cdot \partial_{\bar{z}} X\, e^{-iq\cdot X}\right) e^{ip\cdot X}$$

These correspond to *leading Regge* trajectories, with,

$$\zeta_\mu \equiv \lambda^i(\delta^i_\mu - p^i q_\mu), \qquad L^2 = \frac{16\pi}{\mu}\big(|\zeta| - 1\big), \qquad |\zeta| \in \mathbb{R}^+$$

The classical analogue reads:

$$X(z,\bar{z}) = i\big(\lambda\, z^{-1} - \lambda^*\, z^1\big) + i\big(\bar{\lambda}\, \bar{z}^{-1} - \bar{\lambda}^*\, \bar{z}^1\big),$$

Massless Radiation: Results

For massless radiation (i.e. $m_1^2 = 0$) from above CVO vertices.
In the IR the result ressums:[14]

$$\frac{dP}{d\Omega_{S^{D-2}}}\Big|_{m_1^2=0} = \sum_N \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}}\, \omega^{D-4-\delta} N^2$$
$$\Big[J'^2_N(A) + \Big((N/A)^2 - 1\Big) J_N^2(A) + \mathcal{O}\big(1/(\alpha' M^2)\big)\Big]$$
$$\Big[J'^2_N(\bar{A}) + \Big((N/\bar{A})^2 - 1\Big) J_N^2(\bar{A}) + \mathcal{O}\big(1/(\alpha' M^2)\big)\Big]$$

where the frequency of emitted radiation,[15]

$$\omega = \frac{4\pi N}{L}, \qquad \text{with} \qquad N = 1, 2, \dots$$

Taking $\delta = 1$ yields a decay rate, $\delta = 0$ yields a power.

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[14] DS, Copeland and Saffin (PRL 2013)
[15] Here $A = N\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_{\mathbf{1}}|$, $\bar{A} = N\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_{\mathbf{1}}|$, the $J_{\boldsymbol{n}}(z)$ are Bessel and $M = \mu L$, $\mu = 1/(2\pi\alpha')$

Effective Description

The above power was shown[16] to agree precisely with that associated to the effective theory,

$$S_{\text{eff}} = \frac{1}{16\pi G_D}\int d^D x\sqrt{-G}\, e^{-2\Phi}\Big(R_{(D)} + 4(\nabla\Phi)^2 - \frac{1}{12}H_{(3)}^2 + \dots\Big)$$
$$- \mu \int_{S^2} \partial X^\mu \wedge \bar{\partial} X^\nu \big(G_{\mu\nu} + B_{\mu\nu}\big) + \dots,$$

where Φ, $G_{\mu\nu}$ and $H_{(3)}$ are the dilaton, spacetime metric and 3-form field strength, $H = dB$, respectively

(We plug classical solutions for X (from classical-CVO map) and compute perturbations in G, B and Φ)

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[16] DS, Copeland and Saffin (PRL 2013)

Higher Harmonics

…the above correspondence acts as a guiding principle to write down the general result for arbitrary harmonics (n, m):[17]

$$\frac{dP}{d\Omega_{S^{D-2}}}\Big|_{m_1^2=0} = \sum_N \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}} \omega^{D-4-\delta} (Nuwg)^2 \left[J'^2_{Nw}(A) + \left((Nw/A)^2 - 1\right) J^2_{Nw}(A)\right] \left[J'^2_{Nu}(\bar{A}) + \left((Nu/\bar{A})^2 - 1\right) J^2_{Nu}(\bar{A})\right]$$

with $n \equiv gu$, $m \equiv gw$, integers and u, w relatively prime. (g can be interpreted as a winding number: $M \sim g\mathcal{R}/\alpha'$, with effective radius, $\mathcal{R}$, determined by *dynamics*.)

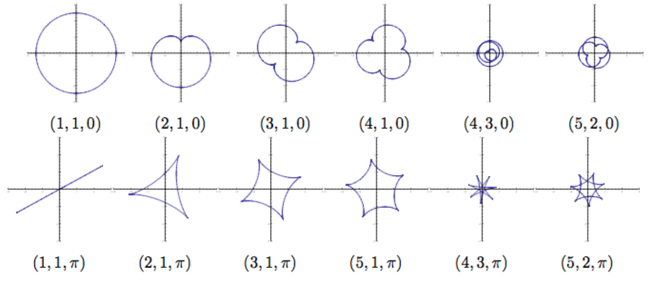
[17] Here $A = Nw\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_{\boldsymbol{n}}|$, $\bar{A} = Nu\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_{\boldsymbol{m}}|$

Duality of 2-Point Amplitudes

In general, all string decay rates (and mass shifts) are invariant under:

$$\lambda_n \to \lambda'_n = (-)^n \lambda_n^*, \qquad \bar{\lambda}_n \to \bar{\lambda}'_n = (-)^n \bar{\lambda}_n^*, \qquad \text{for} \qquad n = 1, 2 \ldots$$

$\to$ distinct string trajectories have the same decay rates and mass shifts!



Summary

- Discussed construction of generic covariant coherent vertex operators and their classical analogues
- Analytically computed decay rates and powers associated to massless emission for special class of HES states in IR (on $\mathbb{R}^{D-1,1} \times T^{26-D}$)
- Found effective field theory that reproduces the leading terms of these decay rates and powers
- Discovered a duality setting decay rates of arbitrary strings equal to those of the dual strings

Future Prospects

- It is now possible to explore truly stringy signatures of cosmic superstrings
- Radiative backreaction is naturally included
- The possibility of massive emission is naturally incorporated
- Can be used to derive effective theories (e.g. for massive particle emission)
- The corresponding superstring version is under construction
- Applies at both small and macroscopic scales
- $\vdots$

High Frequencies

At high frequencies, ie large N, (when backreaction and massive radiation is neglected),

$$\frac{dP_N}{d\Omega_{S^{D-2}}} \propto (Nuw)^{D-4-\delta}\left(\frac{uw}{N}\right)^{2/3} g^{D-2-\delta},$$

generalising the $D = 4$, $u = w = g = 1$ result of Vachaspati and Vilenkin (1986).

String Decay Rates

From unitarity, $S^\dagger S = 1$, one can show that decay rates can be extracted (to leading order in g_s) from:

$$\Gamma = \frac{1}{M}\,\mathrm{Im}\int đ^D\mathbb{P}\,\mathcal{M}_1(\mathbb{P}).$$

The imaginary part is computed by searching for pinch singularities, where the $\mathbb{P}^0$ contour of integration is pinched between two poles as we vary k^0 in the complex plane, leading to:

$$\Gamma = \frac{1}{M}\int đ^D\mathbb{P}\sum_{\{m_j,\,k^\mu\}}|\ldots|^2\,\delta(\mathbb{P}^2+m_1^2)\delta\big((k-\mathbb{P})^2+m_2^2\big)$$

with $m_1^2 = \left(\frac{N}{R}\right)^2 + \left(\frac{M'R}{2}\right)^2 + r + \bar{r} - 2$, $m_2^2 = \ldots$

Two comments:

1. The continuous quantum numbers may (conveniently) be associated with the *classical* polarisation tensors of string embeddings,[18]

$$X(z,\bar{z}) = x - ip\ln|z|^2 + \sum_{n\neq 0}\frac{i}{n}\big(\lambda_n z^{-n} + \bar{\lambda}_n\bar{z}^{-n}\big),$$

so that:

$$|\lambda,\bar{\lambda};p\rangle = |\lambda_1,\lambda_2,\dots,\bar{\lambda}_1,\bar{\lambda}_2,\dots;p\rangle \simeq V(\lambda,\bar{\lambda})$$

2. Coherent states are *not* be eigenstates of annihilation operators in general,

$$\alpha_{n>0}|\lambda,\bar{\lambda};\dots\rangle \neq \lambda_n|\lambda,\bar{\lambda};\dots\rangle$$

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[18] X spacetime embedding; $z,\bar{z}$ worldsheet coordinates; p momenta; $\lambda_n,\bar{\lambda}_n$ polarisation tensors; n harmonics