

# Large parallel cosmic string simulations



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References:

“Large parallel cosmic string simulations:  
New results on loop production” (2011)

“A new parallel simulation technique” (2012)

Plus some new results.

# Introduction

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Nambu-Goto string simulation.

Flat space motion:

$$\mathbf{x} = \frac{1}{2} [\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)]$$

with  $\mathbf{a}'^2 = \mathbf{b}'^2 = 1$ .

$\Rightarrow$  If we know  $\mathbf{a}$  and  $\mathbf{b}$ , we can determine  $\mathbf{x}$  at any time without simulation.

Exact (in flat space). No “minimum simulation resolution”. No “points per correlation length”. Loops can (and do) form any size.

## *Form of $\mathbf{a}$ and $\mathbf{b}$*

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What functional form for  $\mathbf{a}$  and  $\mathbf{b}$ ?

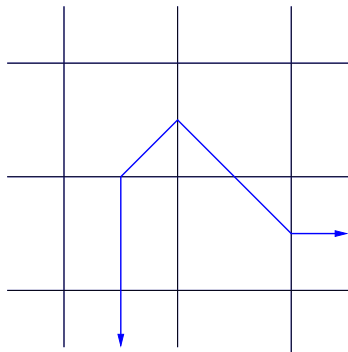
Piecewise linear ( $\mathbf{a}'$  and  $\mathbf{b}'$  piecewise constant).

Initial cond Vachaspati-Vilenkin.

String straight across each cell. Two kinks (one in  $\mathbf{a}$ , one in  $\mathbf{b}$ ) at each face.

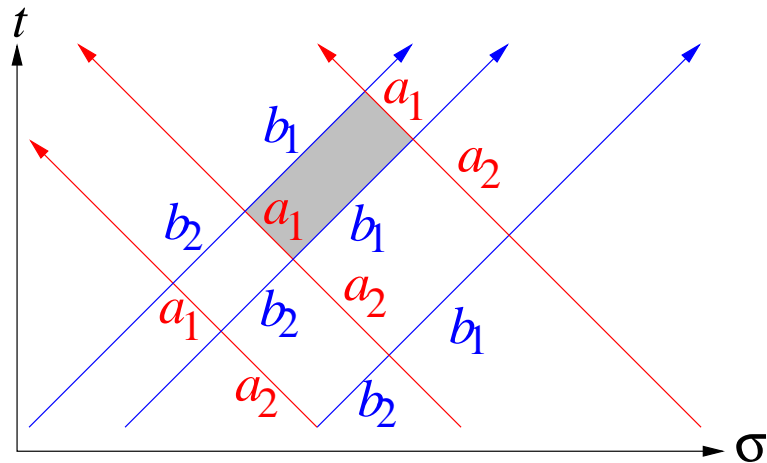
Can use more. No significant effect on results.

Initial conditions have no features below  $\mathbb{V}\mathbb{V}$  cell size, but such features can (and do) develop.



# String world sheet

Linear pieces of  $a$  combine with linear pieces of  $b$  to form flat “diamonds”.



## *Evolution*

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2-D world sheet embedded in 4-D spacetime. Sheets may intersect at points.

We store all diamonds existing at current time.

Evolve world sheet of all strings; look for intersections.

At intersection, split segments that intersected, reconnect.  
Still piecewise linear.

## *Expanding universe*

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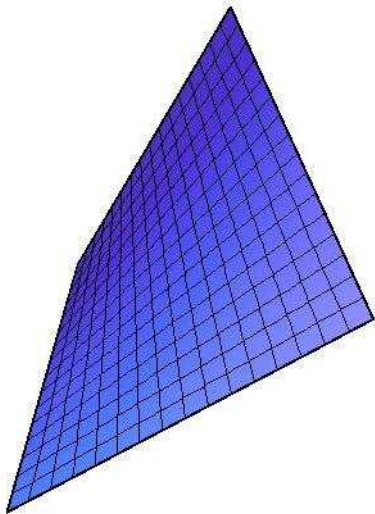
Comoving coordinates: expansion  $\rightarrow$  redshifting.

In exact evolution, string segments become curved. Each segment remembers its entire past: too much data.

Instead, handle redshifting only at first order, so that new edges are straight (but interior curved).

String slows down, diamond curves upward.

First order in  
(segment size)/(Hubble distance),  
so improves rapidly with time.



## *Loop removal*

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Loop of length  $L$  with  $N$  segments each of **a** and **b**.

In one oscillation, each **a** combines with each **b**.

Computational effort  $N^2$ .

Amount of time simulated  $L/2$ . Thus if  $L$  tiny, simulation goes very slowly.

⇒ Small loops must be removed.

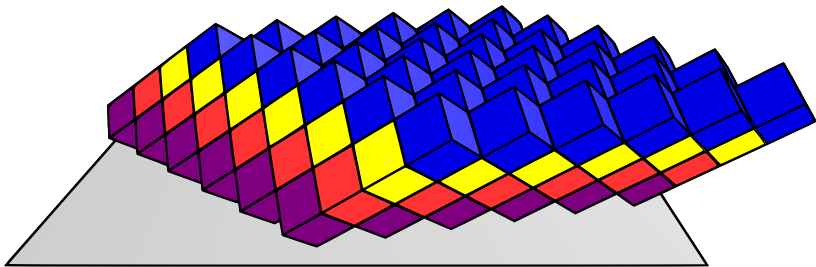
Loop identification procedure: must not intersect itself or rejoin long network for 2 oscillations.

Then record, remove loop.

## *Parallelization technique*

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Rather than dividing up the spatial volume into regions for different processors, we divide spacetime volume into a large number of cubical regions.



Each region can be simulated independently of others. So no coordination between processors is needed. Each region can be run when 4 predecessors have finished.

No particular number of processors required.



## *Scope of simulation*

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In units of initial Vachaspati-Vilenkin cell size (= correlation length of initial conditions) we have done a box of

2000 units on a side in flat space

1500 in the radiation era

1000 in the matter era (but only for 500 units of conformal time)

CPU time not too long because no extra points introduced except at actual intersections. One radiation-era run of size 1500 takes 4000 core-hours, < 1 day real time.

## *Scope of simulation*

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Dynamic range (= conformal end/conformal start).

Depends on initial clock setting. With fixed initial conditions, match network parameters to scaling solution at best-fit time.

Figure of merit: interstring distance approaches scaling values quickly as possible.

Radiation era: start  $t = 6$ , dynamic range 251.

Matter era: start  $t = 9$ , dynamic range 56.

# Results

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Scaling: all linear quantities should be proportional.

Divide everything by horizon distance:

Interstring distance  $d = \sqrt{\mu/\rho_\infty}$ . Scaling:  $d/d_h$ .

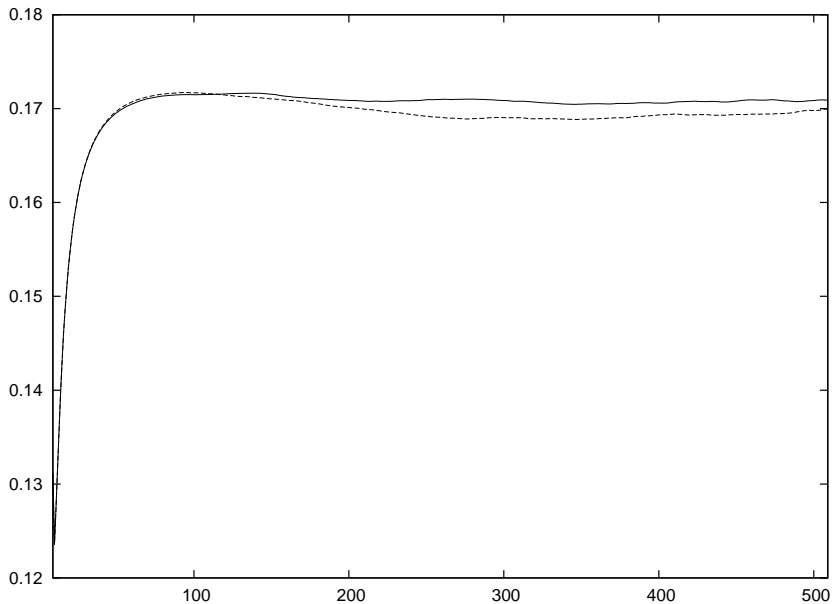
Scaling loop length:  $x = l/d_h$ .

Scaling loop production rate:  $f(x)$

$f(x)dx$  = number of loops formed in volume  $d_h^3$  in time  $d_h$   
with sizes in range  $dx$ .

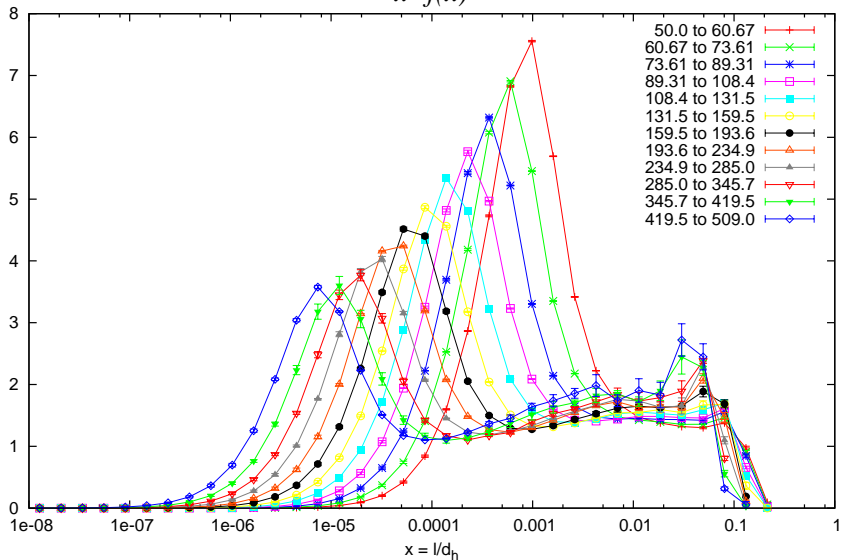
Interstring distance  $d/d_h$ .

Two runs of size 1000 in the matter era

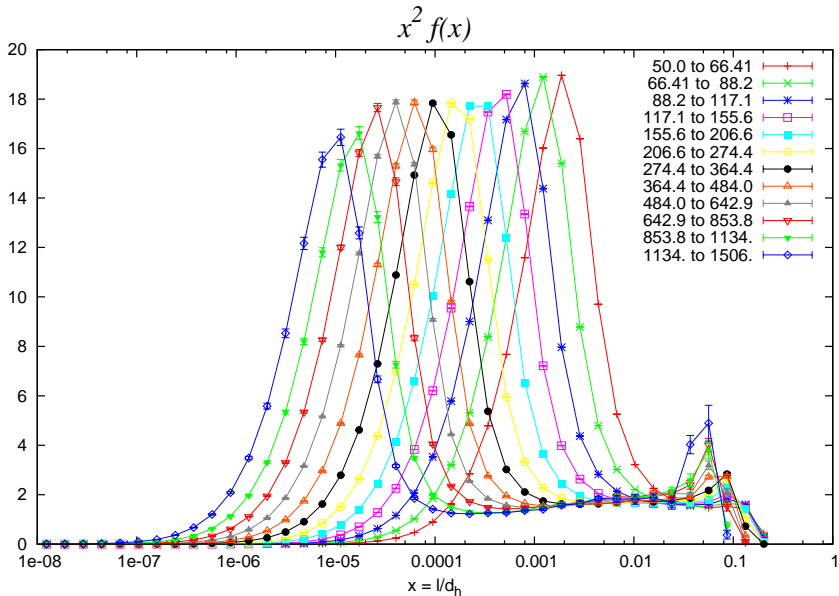


# Loop production function: 3 runs of size 1000 in the matter era

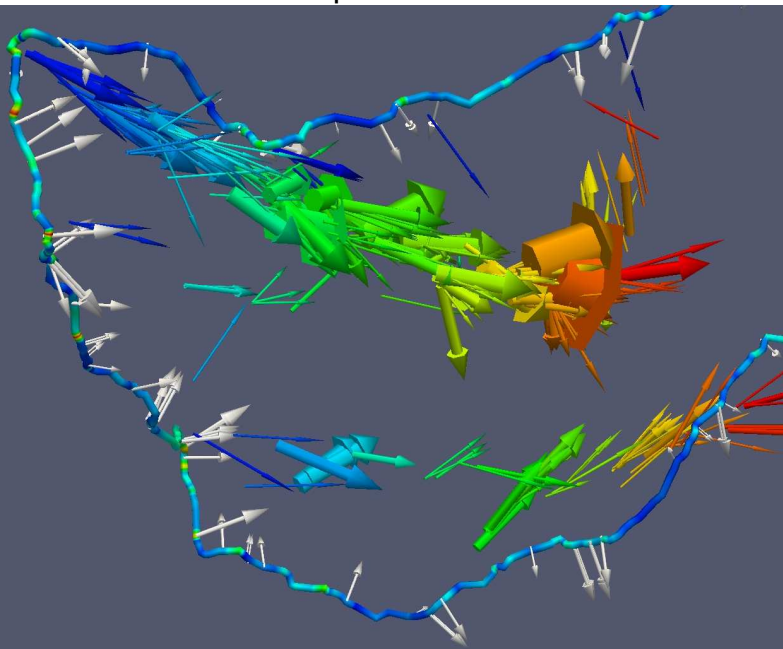
$$x^2 f(x)$$



# Loop production function: 7 runs of size 1500 in the radiation era



Where do the little loops come from?



# That's all, Folks!

Coming up:

Ben on loop number distribution.

Jose on shapes of loops and presence of cusps.