The direct assessment support documents consist of a set of documents for each of the two courses (Physics 145 Classical Electrodynamics and Physics 163 Quantum Mechanics) selected by the Department of Physics and Astronomy for the evaluation purpose. Each set consists of (1) the evaluation summary provided by the instructor in the course, (2) the selected problems, and (3) a sample of an excellent performance by a student on each of the selected problems.

Physics 145 Summary of Direct Assessment Fall 2011

The Physics 145 final examination included 5 problems, each based on material from J.D. Jackson's book "Classical Electrodynamics" (3rd edition). The students were given 2.5 hours or about one-half hour per problem. The class included 6 physics graduate students and two engineers. An assessment of the performance of the physics graduate students on two problems from the final exam was requested. The instructor chose problem one as the problem with the highest combined score for all students ("easy") and problem two as the problem with the lowest combined score for all students ("hard"). Each student's work is rated on three criteria: 1) constructing a mathematical representation of the physical system, 2) carrying out the mathematical analysis, and 3) relating the results to the physical problem. In each category the ratings are: Proficient (P), Satisfactory (S), and Needs Improvement (N). The results were:

Problem 1 (easy)	Criterion 1	Criterion 2	Criterion 3
Student 1	P	P	S
Student 2	P	P	P
Student 3	P	P	P
Student 4	P	P	S
Student 5	P	P	P
Student 6	P	P	P
Problem 2 (hard)			
Student 1	N	S	P
Student 2	S	N	P
Student 3	N	S	P
Student 4	N	N	N
Student 5	P	P	P
Student 6	P	P	P

Note that some students chose not to do some parts of problem two, and the N rating is applied in these cases. Students were not expected to complete all parts of all problems.

Austin Napier, instructor for Physics 145

Notes on problems 2 and 3 from the Physics 145 final exam from Fall 2011

In problem 2(a), students are expected to use Bessel function expansions to show that only the phi-component of the vector potential is non-vanishing, and derive a simplified integral form. In problem 2(b) they should use the result of 2(a) to derive an exact formula for B(z) (or at least indicate how to do it). In problem 2(c) the exact result from part (b) should be compared to the result obtained from the Biot-Savart Law. If the student did not do parts (a) and/or (b), full credit was given for part (c). Note that use of the Biot-Savart Law in part (c) is a typical problem in PHY 12.

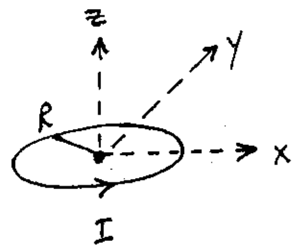
In problem 3(a), students are expected to write the Boundary Conditions for a dielectric cylinder placed in a uniform electric field. The case of the dielectric sphere was done in class. In problem 3(b), they are expected to calculate the electric potential both inside the cylinder and out. In problem 3(c) they should calculate the electric field both inside and out. In part (d) they should determine the polarization vector inside the cylinder and also find the polarization surface charge density (or at least indicate how to find these).

The students were allowed to refer to the textbook during the exam.

Austin Napier, Instructor for Physics 145

"Hard"

2. A circular current loop of radius R carries a steady current I and lies in the x-y plane with its center at the origin.



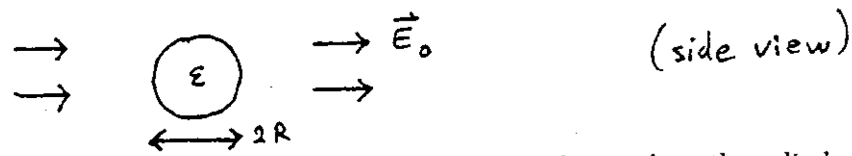
2a. Show that the only non-vanishing component of the vector potential is A_ϕ and show that $A_\phi(\rho, z)$ is given by the formula in Jackson problem 5.10a, or by the formula in problem 5.10b. (Choose one or the other, not both.)

2b. Use your result from part (a) to calculate the exact formula for the magnetic induction $\vec{B}(z)$ along the z-axis.

2c. Use the Biot-Savart Law to calculate the magnetic induction $\vec{B}(z)$ along the z-axis. Is your result consistent with (b)?

"EASY"

3. A very long cylinder of radius R is filled with uniform and isotropic material of dielectric constant ϵ. There are no free charges anywhere. The cylinder is placed in an initially uniform electric field $\vec{E}_0$ with the axis of the cylinder perpendicular to the field direction, as shown:



3a. Write down the boundary conditions at $\rho = R$. Take z to be the coordinate along the cylinder axis and choose x along the direction of $\vec{E}_0$.

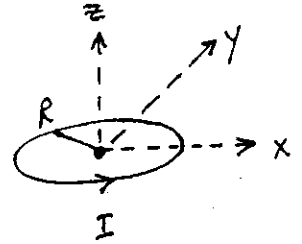
3b. Find the electric potential everywhere, inside and outside of the cylinder.

3c. Use the potential to calculate the electric field inside the cylinder.

3d. Determine the polarization vector $\vec{P}$ inside the cylinder and calculate the polarization surface charge density on the cylinder.

Student 6

2. A circular current loop of radius R carries a steady current I and lies in the x-y plane with its center at the origin.



2a. Show that the only non-vanishing component of the vector potential is A_ϕ and show that $A_\phi(\rho, z)$ is given by the formula in Jackson problem 5.10a, or by the formula in problem 5.10b. (Choose one or the other, not both.)

The vector potential could be obtained by

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x'$$

where for a circular loop

$$\vec{J}(\vec{x}') = I\,\delta(z')\,\delta(\rho'-a)\,\vec{\phi}'$$

and we have

$$\vec{\phi}' = \hat{\rho}\sin(\phi-\phi') + \hat{\phi}\cos(\phi-\phi')$$

The integral expression for the vector potential is then

$$\vec{A}(\vec{x}) = \frac{\mu_0 I}{4\pi} \int \frac{\delta(z')\delta(\rho'-a)\left(\hat{\rho}\sin(\phi-\phi') + \hat{\phi}\cos(\phi-\phi')\right)}{|\vec{x}-\vec{x}'|} \rho' d\rho' d\phi' dz'$$

$$= \frac{\mu_0 I a}{4\pi} \int_0^{2\pi} \frac{\hat{\rho}\sin(\phi-\phi') + \hat{\phi}\cos(\phi-\phi')}{|\vec{x}-\vec{x}'|} d\phi'$$

where the integrand is to be evaluated at $z'=0$ and $\rho'=a$

Note that

$$\frac{1}{|\vec{x}-\vec{x}'|} = \frac{4}{\pi}\int_0^\infty dk \cos[k(z-z')]\Big[\tfrac{1}{2}I_0(k\rho_<)K_0(k\rho_>) + \sum_{m=1}^{\infty}\cos[m(\phi-\phi')]\,I_m(k\rho_<)K_m(k\rho_>)\Big]$$

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$\Rightarrow$ the integral over ϕ' pick out the $m=1$ term in the sum. and the $\hat{\rho}$ component drops out because $\sin(\phi-\phi')$ is orthogonal to $\cos(\phi-\phi')$. Consider the symmetry, we have

$$\vec{A}(\vec{x}) = \frac{\mu_0 I a}{4\pi} \cdot \frac{4}{\pi}\pi\hat{\phi}\int_0^\infty dk \cos(kz)\, I_1(k\rho_<)K_1(k\rho_>)$$

$$= \frac{\mu_0 I a}{\pi}\,\hat{\phi}\int_0^\infty dk \cos(kz)\, I_1(k\rho_<)K_1(k\rho_>)$$ ✓

2b. Use your result from part (a) to calculate the exact formula for the magnetic induction $\vec{B}(z)$ along the z-axis.

Since $\vec{B} = \vec{\nabla} \times \vec{A}$ and the only non-vanishing component of $\vec{A}$ is A_ϕ

and then

$$B_\rho = -\partial_z A_\phi, \quad B_z = \frac{1}{\rho}\partial_\rho(\rho A_\phi)$$

For the ρ derivative, on the other hand, we could use the Bessel equation identity

$$\frac{d}{dz}X_1(z) + \frac{1}{z}X_1(z) = X_0(z)$$

10 Hence $\frac{1}{\rho}\partial_\rho[\rho X_1(k\rho)] = kX_0(k\rho)$

Then we have $B_\rho = \frac{\mu_0 I a}{\pi}\int_0^\infty dk \sin(kz) I_1(k\rho_<)K_1(k\rho_>)$

$$B_z = \frac{\mu_0 I a}{\pi}\int_0^\infty dk \sin(kz)\begin{Bmatrix} I_0(k\rho)K_1(ka) \\ I_1(ka)K_0(k\rho) \end{Bmatrix}$$

2c. Use the Biot-Savart Law to calculate the magnetic induction $\vec{B}(z)$ along the z-axis. Is your result consistent with (b)?

$$B_z = \frac{\mu_0 I a}{2}\int_0^\infty dk\, k e^{-k|z|} J_0(k\rho) J_1(ka)$$

The z axis corresponds to $\rho = 0$. In this case, we have $J_0(0) = 1$, Then

$$B_z(\rho=0) = \frac{\mu_0 I a}{2}\int_0^\infty dk\, k e^{-k|z|} J_1(ka)$$

$$= \frac{\mu_0 I a}{2} \cdot \frac{a}{(z^2+a^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 I a^2}{2(z^2+a^2)^{\frac{3}{2}}} \checkmark$$

2c). Use the Biot-Savart Law

$$\vec{B_z} = \frac{\mu_0 I}{4\pi}\int \frac{|d\vec{l} \times R|_z}{R^3}$$

$$= \frac{\mu_0}{4\pi}\int \frac{dl\, R \sin\alpha}{R^3}$$

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$$= \frac{\mu_0 I a^2}{2R^3} \Leftarrow \text{where } R^2 = z^2 + a^2$$

$$= \frac{\mu_0 I a^2}{2(z^2+a^2)^{\frac{3}{2}}} \checkmark \text{ is the same as 2b)} \checkmark$$

Very good!

Problem 3 solution Yazhe Zhu

3a) Since the cylinder treat as 2D In this case the potential admits a general expansion

$$\phi = \sum_m [\alpha_m \rho^m + \beta_m \rho^{-m}] \cos(m\phi - \delta_m)$$ ✓

Hence we have

Inside $\phi_{in} = B_0 + \sum_m B_m \rho^m \cos(m\phi)$

Outside $\phi_{out} = A_0 + \sum_m A_m \rho^{-m} \cos(m\phi) - E_0 \rho \cos\phi$

at $\rho = R$, we have

$$-\varepsilon \frac{\partial \phi_{in}}{\partial \rho}\Big|_{\rho=R} = -\varepsilon_0 \frac{\partial \phi_{out}}{\partial \rho}\Big|_{\rho=R}$$ ✓

$$-\frac{1}{R} \frac{\partial \phi_{in}}{\partial \phi}\Big|_{\rho=R} = -\frac{1}{R} \frac{\partial \phi_{out}}{\partial \phi}\Big|_{\rho=R}$$ ✓

3b) Note that when $m \neq 1$, we have $A_m = B_m = 0$ (Including $m=0$)

Thus we may focus on $m=1$ and write

$$\phi = \begin{cases} \phi_{in} = B\rho\cos\phi \\ \phi_{out} = (A\rho^{-1} - E_0\rho)\cos\phi \end{cases}$$

we may obtain the electric field

$$E_\rho = -\frac{\partial \phi}{\partial \rho} = \begin{cases} E_{\rho(out)} = (A\rho^{-2} + E_0)\cos\phi \\ E_{\rho(in)} = -B\cos\phi \end{cases}$$

$$E_\phi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi} = \begin{cases} E_{\phi(out)} = (A\rho^{-2} - E_0)\sin\phi \\ E_{\phi(in)} = B\sin\phi \end{cases}$$

The matching at $\rho = R$ is

$$\varepsilon_0 E_{\rho(out)} = \varepsilon E_{\rho(in)}$$

and $E_{\phi(out)} = E_{\phi(in)}$

$$A = \frac{1 - \frac{\varepsilon}{\varepsilon_0}}{\frac{\varepsilon}{\varepsilon_0} + 1} \cdot E_0 R^2$$

$$B = -\frac{2E_0}{\frac{\varepsilon}{\varepsilon_0} + 1}$$

Hence

$$\phi_{in} = -\frac{2E_0}{\frac{\varepsilon}{\varepsilon_0}+1}\rho\cos\phi$$ ✓

$$\phi_{out} = -\frac{2\frac{\varepsilon}{\varepsilon_0}}{\frac{\varepsilon}{\varepsilon_0}+1} E_0 \rho\cos\phi$$ X

8

3c) 3d) see back!

3c) The potential inside the sphere describes a constant electric field parallel to the applied field with magnitude

$$E_{in} = \frac{2}{\frac{\varepsilon}{\varepsilon_0}+1} E_0 \quad ✓$$

10

~~Outside.~~

~~E_{out}~~ =

3d) $\vec{P} = (\varepsilon - \varepsilon_0)\vec{E} = 2\varepsilon_0 \left(\frac{\frac{\varepsilon}{\varepsilon_0} - 1}{\frac{\varepsilon}{\varepsilon_0}+1} \right) E_0$ ✓ . (inside)

$$\delta_{pol} = \frac{(\vec{P}\cdot\vec{r})}{r} \quad ✓$$

$$= 2\varepsilon_0 \left(\frac{\frac{\varepsilon}{\varepsilon_0} - 1}{\frac{\varepsilon}{\varepsilon_0}+1} \right) E_0 \cos\phi \quad ✓$$

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December 27, 2011
W. Anthony Mann

Evaluation of Student Performance in Physics 163 (Quantum Mechanics)

The evaluations below are based upon the Final Exam for the course, held on Monday December 19, 2011 from 2:00 pm to 6:00 pm. The Exam was a closed book exam, however one ``formula sheet" prepared by each student was allowed. The exam cover sheet listing the topic of each of its six questions is attached. I have selected questions #3 and #6 to be ones which are appropriate for this evaluation; the questions are original - to my knowledge they do not appear in any textbook.

The rating criteria are as follows:

1. Constructing a mathematical representation of the physical system.
2. Carrying out the mathematical analysis.
3. Relating the results to the physical problem.

The rating uses a three-level scale: **P**roficient or **S**atisfactory or **N**eeds **I**mprovement .

The seven students who completed the course are rated as follows:

Student	**Criterion 1**	**Criterion 2**	**Criterion 3**
#1	**P**	**P**	**P**
#2	**S**	**NI**	**NI**
#3	**P**	**S**	**S**
#4	**P**	**P**	**P**
#5	**S**	**S**	**S**
#6	**S**	**S**	**NI**
#7	**P**	**P**	**S**

General comment: In terms of work ethic, lecture attendance, and substantive questions, the group evaluated here ranks among the top 30% of graduate classes to whom I have taught this course.

PROBLEM #3

It is speculated that virtual reactions involving an unknown force may enable a neutron state $|\,n>$ to transition into an anti-neutron state $|\,\underline{u}>$ and visa versa. The Hamiltonian for this ``neutron oscillation" is

$$\mathbf{H} = E_0(\,|\,n><n\,| + |\,\underline{u}><\underline{u}\,|\,) + \Delta(\,|\,n><\underline{u}\,| + |\,\underline{u}><n\,|\,)$$

where E_0 and Δ are constants having units of energy.

3a) Using the neutron and the anti-neutron states as a basis, express the Hamiltonian **H** as a sum of elemental matrix operator forms.

[5 pts]

3b) Show that the time evolution operator for particle states $|\,\psi>$ of the Hilbert space spanned by $\{\,|\,n>, \,|\,\underline{u}>\,\}$, has the form of an oscillating phase factor times a Drehung involving a time-dependent rotation angle.

[20 pts]

3c) At time $t = 0$, a pure neutron state $|\,\psi(0)> \; = \; |\,n>$ is created. Using the time evolution operator or else by other means, determine the shortest time after $t = 0$ when the experimentalist can be **absolutely certain** that the quantum state has become that of a pure anti-neutron $|\,\underline{u}>$.

[25 pts]

PROBLEM #6

The atomic magnetic dipole moment operator is

$$\mathbf{M} = \mu_0 g_L \mathbf{L} + \mu_0 g_S \mathbf{S} \quad .$$

6a) Express **M** as an irreducible tensor operator and relate it to its Cartesian components.

[5 pts]

6b) We wish to evaluate the magnetic moment of a particular atomic state for which ℓ, s, and j have specific values. The relevant expectation value is $< \ell, s; j\ m \mid \mathbf{M} \mid \ell, s: j\ m >$. Based upon this expectation value, what can be said about the relative magnitudes of $< M_x >$, $< M_y >$, and $< M_z >$?

[10 pts]

6c) Briefly discuss the **projection theorem.** What is it based upon ? In what sense does it enable an evaluation to be "projected" ?

[10 pts]

6d) According to the physicist Alfred Lande, atomic magnetic dipole moments are well-described by

$$< M_z >_{j\,m} = (\mu_0/2)\, m\, \hbar \left\{ (g_L + g_S) + (g_L - g_S) [\, \ell(\ell+1) - s(s+1)\,] / [j(j+1)] \right\} .$$

Using angular momentum formalism, derive **Lande's formula.**

[25 pts]

It is speculated that virtual reactions involving an unknown force may enable a neutron state $|n\rangle$ to transition into an anti-neutron state $|\underline{u}\rangle$ and visa versa. The Hamiltonian for this "neutron oscillation" is

$$\mathbf{H} = E_0(|n\rangle\langle n| + |\underline{u}\rangle\langle\underline{u}|) + \Delta(|n\rangle\langle\underline{u}| + |\underline{u}\rangle\langle n|)$$

where E_0 and Δ are constants having units of energy.

3a) Using the neutron and the anti-neutron states as a basis, express the Hamiltonian **H** as a sum of elemental matrix operator forms. [5 pts]

$H_{nn} = \langle n|H|n\rangle = E_0$

$H_{nu} = \langle n|H|u\rangle = \Delta$

$H_{un} = \langle u|H|n\rangle = \Delta$

$H_{uu} = \langle u|H|u\rangle = E_0$

$$\underset{\approx}{H} = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix} = E_0 \underset{\approx}{\mathbb{1}} + \Delta \underset{\approx}{\sigma_x}$$

(5)

3b) Show that the time evolution operator for particle states $|\psi\rangle$ of the Hilbert space spanned by $\{|n\rangle, |\underline{u}\rangle\}$, has the form of an oscillating phase factor times a Drehung involving a time-dependent rotation angle. [20 pts]

$H = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix} = E_0\mathbb{1} + \Delta\sigma_x$

$U = e^{-i\frac{Ht}{\hbar}} = e^{-i\frac{(E_0\mathbb{1}+\Delta\sigma_x)t}{\hbar}}$

since $[\mathbb{1},\sigma_x]=0$ $\quad = e^{-\frac{iE_0\mathbb{1}t}{\hbar}} e^{-\frac{i\Delta\sigma_x t}{\hbar}}$

$e^{-\frac{i\Delta\sigma_x t}{\hbar}} = \sum_{n=0}^{\infty} \frac{1}{n!}\left(-\frac{it}{\hbar}\Delta\right)^n \sigma_x^n$

$= \sum_{n=0}^{\text{all even}} \frac{1}{n!}\left(-\frac{it}{\hbar}\Delta\right)^n + \sum_{n=1}^{\text{all odd}} \frac{1}{n!}\left(-\frac{it}{\hbar}\Delta\right)^n \sigma_x$

$= \cos\left(\frac{\Delta t}{\hbar}\right)\mathbb{1} - i\sin\left(\frac{\Delta t}{\hbar}\right)\sigma_x$

$\therefore U = e^{-\frac{iE_0\mathbb{1}t}{\hbar}} e^{-\frac{i\Delta\sigma_x t}{\hbar}} = \left(\cos\left(\frac{E_0 t}{\hbar}\right) - i\sin\left(\frac{E_0 t}{\hbar}\right)\right)\left(\cos\left(\frac{\Delta t}{\hbar}\right)\mathbb{1} - i\sin\left(\frac{\Delta t}{\hbar}\right)\sigma_x\right)$

$= e^{-\frac{iE_0 t}{\hbar}}\left(\cos\left(\frac{\Delta t}{\hbar}\right)\mathbb{1} - i\sin\left(\frac{\Delta t}{\hbar}\right)\sigma_x\right)$

where is the Drehung form

(16)

3c) At time $t = 0$, a pure neutron state $|\psi(0)\rangle = |n\rangle$ is created. Using the time evolution operator or else by other means, determine the shortest time after $t = 0$ when the experimentalist can be **absolutely certain** that the quantum state has become that of a pure anti-neutron $|\underline{u}\rangle$. [25 pts]

$P = |\langle\underline{u}|U|n\rangle|^2$

$= \left|\langle\underline{u}|\, e^{-\frac{iE_0 t}{\hbar}}\left(\cos\left(\frac{\Delta t}{\hbar}\right)(|n\rangle\langle n| + |\underline{u}\rangle\langle\underline{u}|) - i\sin\left(\frac{\Delta t}{\hbar}\right)[|n\rangle\langle\underline{u}| + |\underline{u}\rangle\langle n|]\right)|n\rangle\right|^2$

$= \left|e^{-\frac{iE_0 t}{\hbar}}\left(-i\sin\left(\frac{\Delta t}{\hbar}\right)\right)\right|^2$

$= \sin^2\left(\frac{\Delta t}{\hbar}\right)$

If $P = 1$ $\quad \sin\left(\frac{\Delta t}{\hbar}\right) = \pm 1$ $\quad$ +1 will be the first time.

$\frac{\Delta t}{\hbar} = n\pi$ $\quad n = 1, 2, 3\cdots$ $\quad \to \frac{\pi}{2}$

$t = \frac{n\pi\hbar}{\Delta}$

$t_{min} = \frac{\pi\hbar}{\Delta}$ X

(23)

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PROBLEM #6:

The atomic magnetic dipole moment operator is

$$\mathbf{M} = \mu_0 g_L \mathbf{L} + \mu_0 g_S \mathbf{S} \quad .$$

6a) Express **M** as an irreducible tensor operator and relate it to its Cartesian components. [5 pts]

$M_0 = (\mu_0 g_L L_z + \mu_0 g_s S_z) = M_z$

$M_{\pm 1} = \left(\mp \frac{(\mu_0 g_L L_x + \mu_0 g_s S_x) \pm i(\mu_0 g_L L_y + \mu_0 g_s S_y)}{\sqrt{2}} \right) = \left(\mp \frac{M_x \pm i M_y}{\sqrt{2}} \right)$

(5)

6b) We wish to evaluate the magnetic moment of a particular atomic state for which ℓ, s, and j have specific values. The relevant expectation value is $< \ell, s; j\ m \mid \mathbf{M} \mid \ell, s; j\ m >$. Based upon this expectation value, what can be said about the relative magnitudes of $<M_x>$, $<M_y>$, and $<M_z>$? [10 pts]

where $A = \langle j \| M^{(1)} \| j \rangle$

$\langle l s; jm | M_0 | l, s; jm \rangle = \langle j1; m0 | j1; jm \rangle \frac{A}{\sqrt{2j+1}} = \langle M_z \rangle$

$0 = \langle l, s, jm | M_{\pm1} | l, s, jm \rangle = \langle j1; m\pm1 | j1; jm \rangle \frac{A}{\sqrt{2j+1}} = 0 \left(\mp \frac{\langle M_x \rangle \pm i \langle M_y \rangle}{\sqrt{2}} \right)$

$\therefore \frac{\langle M_z \rangle}{\langle j1; m0 | j1; jm \rangle} = \frac{A}{\sqrt{2j+1}} = \frac{2 \langle M_x \rangle}{\sqrt{2} (\langle j1; m-1 | j1; jm \rangle - \langle j1; m1 | j1; jm \rangle)} = \frac{-2i \langle M_y \rangle}{\sqrt{2} (\langle j1; m-1 | j1; jm \rangle + \langle j1; m1 | j1; jm \rangle)}$

(3)

6c) Briefly discuss the **projection theorem.** What is it based upon? In what sense does it enable an evaluation to be ``projected"? [10 pts]

Projection theorem is $\langle \alpha', jm' | V_q | \alpha, jm \rangle = \frac{\langle \alpha', jm | \vec{J} \cdot \vec{V} | \alpha, jm \rangle}{\hbar^2 j(j+1)} \langle jm' | J_q | jm \rangle$

It is based upon $\langle \alpha', j'm' | T_q^{(k)} | \alpha, jm \rangle = \langle jk; mq | jk; j'm' \rangle \frac{\langle \alpha' j' \| T^{(k)} \| \alpha j \rangle}{\sqrt{2j+1}}$

when V_q is a component of an irreducible vector, (1-rank tensor) and both $|\alpha'\rangle$ $|\alpha\rangle$ have the same j which means the magnitude of total angular momentum.

(10)

6d) According to the physicist Alfred Lande, atomic magnetic dipole moments are well-described by

$$< M_z >_{j\,m} = (\mu_0/2)\, m\, \hbar \left\{ (g_L + g_S) + (g_L - g_S) \left[\ell(\ell+1) - s(s+1) \right] / \left[j(j+1) \right] \right\} .$$

where $\vec{L} \cdot \vec{S} = \frac{1}{2}(J^2 - L^2 - S^2)$ has been used; also $|jm\rangle$ is the eigenket of L^2, S^2 has been considered

Using angular momentum formalism, derive **Lande's formula.** [25 pts]

$\langle M_z \rangle_{jm}$

$= \langle jm | M_0 | jm \rangle$

$\langle jm | M_0 | jm \rangle$

$= \frac{\langle jm | \vec{J} \cdot \vec{M} | jm \rangle}{\hbar^2 j(j+1)} \langle jm | J_z | jm \rangle$

$= m\hbar \frac{\langle jm | \vec{J} \cdot \vec{M} | jm \rangle}{\hbar^2 j(j+1)}$

$= \frac{m \mu_0}{\hbar (j+1) j} \langle jm | (\vec{L} + \vec{S})(g_L \vec{L} + g_s \vec{S}) | jm \rangle$

$= \frac{m\mu_0}{\hbar j(j+1)} \langle jm | (g_L \vec{L}^2 + g_s \vec{L} \cdot \vec{S} + g_L \vec{S} \cdot \vec{L} + g_s \vec{S}^2) | jm \rangle$

$= \frac{m\mu_0}{\hbar j(j+1)} \langle jm | (g_L \vec{L}^2 + \frac{g_s}{2} (J^2 - L^2 - S^2) + \frac{g_L}{2} (J^2 - L^2 - S^2) + g_s \vec{S}^2 | jm \rangle$

$= \frac{m\mu_0}{\hbar j(j+1)} \langle jm | (\frac{g_s + g_L}{2} J^2 + \frac{g_L - g_s}{2} L^2 + \frac{g_s - g_L}{2} S^2 | jm \rangle$

$\langle M_z \rangle_{jm} = \frac{\mu_0}{2} m\hbar \{ (g_L + g_s) + (g_L - g_s) [\frac{l(l+1) - s(s+1)}{j(j+1)}$

(25)

$= \frac{m\mu_0}{\ldots} [\ldots]$

(43)